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Leibniz-Informationszentrum Wirtschaft Leibniz Information Centre for Economics

Where did the GARCH Models Perform Best in Terms of Volatility Forecasting?

Equity vs. Commodities Markets

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Abstract This article aims to compare the performance of GARCH models in terms of volatility forecasting for two asset classes: equity and commodities. The idea behind this research was that GARCH models may perform differently depending on the asset class for which they are used. A comparison based on performance of GARCH, EGARCH and GJR-GARCH for the Romanian equity market, Polish equity market, gold market and Brent Crude Oil market has been done. The results were in line with initial expectations. Both, in-sample and out-of-sample analysis, highlighted the over performance of GARCH models for the equity market compared to the commodity market. Moreover, the performance of GARCH models in terms of volatility forecasting for the gold market decreases as the forecast horizon increases. Thus, it has been proved that there is a bias of classical GARCH models to perform better for equity market. A possible solution would be to implement models that allow regime switching as the Markov Switching GARCH (MRS-GARCH) models do.

Key words Volatility forecasting, commodities, equity markets, statistical loss functions, out-of-sample JEL Codes: C58, C55, C53, C52

1. Introduction

This paper aims to address the issue of volatility in financial markets, whether we are talking about stock markets, commodity markets or the forex markets. Financial markets' volatility is a central issue to the theory and practice of asset pricing, asset allocation and risk management. Therefore, a rising number of scholars has focused in the last decades on the analysis and forecasting of volatility and a plethora of models have been put forward.

Since the introduction of ARCH models by Engle (1982), papers analyzing models of volatility have proliferated. A survey paper by Bollerslev, Chou and Kroner (1992) reports more than 100 articles on this subject. Some of the most popular models of changing volatility have proved to be various forms of GARCH (Generalized Autoregressive Conditional Heteroskedasticity) models. In these models, the volatility process is time varying and is modeled to be dependent upon both the past volatility and past innovations. These models have been used in many applications of stock return data, interest rate data, foreign exchange data etc. One of the latest variants of the GARCH-models is the Regime-Switching GARCH that is supposed to deliver more accurate forecasts due to its flexibility regarding volatility persistence (i.e. not all shocks have to be highly persistent as in the case of non-switching GARCH models).

Given the spare literature on the predictability of stock market volatility in CEE countries and on commodities markets, this paper aims to contribute to filling this gap by focusing upon one aspect of GARCH models (especially of MRS-GARCH), namely, their ability to deliver volatility forecasts. In other words, it investigates the accuracy of the forecasts delivered three types of GARCH models.

The novelty of this paper consists of revealing if there are any differences in performance of GARCH models in terms of volatility forecasting when they are applied for different asset classes. There are many papers that address the issue of GARCH performance on the stock market, as mentioned earlier. There are also papers evaluating the performance of volatility forecasting models on the commodity market such as those drafted by Olga and Serletis (2014), but the idea of looking at the different behaviors of GARCH models according to the asset classes was very little approached in the literature. Therefore, this paper attempts to identify whether there is a clear bias of a higher performance of GARCH models depending on asset classes.

This paper is organized as follows: the first part offers an overview on the empirical research on volatility modeling, forecasting and evaluation of forecasts. The second part presents a brief description of the models employed to forecast volatility and the data used, while the third part contains the results of this study. Last but not least, conclusions and directions for further research are sketched.

2. Literature review

Extending the framework of Engle (1982), Bollerslev (1986) and Taylor (1986) generalized the ARCH (q) model to GARCH (p,q) in which they added the q lags of past conditional variance into the equation. The GARCH (p,q) model allows for both autoregressive and moving average components in the heteroskedastic variance. Empirical findings suggest that GARCH

model is more parsimonious than ARCH model. Although GARCH model has been the most popular volatility model, it has three main problems. Firstly, non-negativity constraint may be violated by the estimated models. Secondly, GARCH model does not take into account the leverage effect and not allow for feedback between the conditional variance and conditional mean (Brooks, 2002).

Since the GARCH model was developed, a huge number of extension models have been proposed as the awareness of GARCH models' weaknesses. Due to the fact that the symmetric GARCH model is unable to account for the leverage effects observed in stock returns, asymmetric GARCH models were proposed that enable conditional variance to respond asymmetrically to rises and falls in innovations. For instance the exponential GARCH (EGARCH) model (introduced by Nelson in 1991) responds asymmetrically to positive and negative value of returns. The EGARCH model always produces a positive conditional variance independently of the signs of the estimated parameters in the model without any restrictions being needed.

Aiming to capture the "leverage effect" Glosten *et al.* (1993) put forward a modified a GARCH model (GJR). This is an asymmetric GARCH model that allows for the conditional variance to respond differently to positive and negative shocks.

It has long been argued that the financial market reacts to large and small shocks differently, and the rate of mean reversion is faster for large shocks (Granger and Poon, 2003).

Despite the extensive variety of GARCH specifications, most of the models seem to be excessively persistent, i.e., react too slowly to movements of the market. It seems like the conditional dependency of the GARCH models helps the model to account for volatility clustering but at the same time it decreases the adaptability to shifts in stock movements (Lamoureaux and Lastrapes, 1990).

Volatility series suffer from shifts that are caused by structural changes but also due to changes in expectations of marketparticipants. For example, the terms "Hausse" and "Baisse" refers to states with large movements of return series that causes these shifts. "Hausse" refers to rapidly increasing stock movements and "Baisse" the opposite. Both these situations are subject to periods of large variance that can be modeled as high-variance regimes. Hence, the situation where neither one of them occurs can be considered as a low-variance regime.

Incorporating regimes or states in a GARCH model makes its mean-reversion state dependent. Thus, how quick the variance will get back to its long-run average will vary between the regimes. Since there is more than one state, a multistate model will always be more flexible since a single-state model's parameters only represent the average meanreversion of the states. Hence, including regimes in a GARCH framework are therefore likely to yield better estimates of the persistence and is therefore of interest (Alexander and Lazar, 2009).

Marcucci's (2005) research is a very important article for the methodology that is going to be used in this paper. To these researches summarized above, there should be added works on the applicability of GARCH models to the commodity market, especially on the oil market such as those published by Wang Y. and Huang D. (2010) or by He L-Y., Zhang Y-J. And Yao T (2015). The same type of approach, but in a broader perspective, analyzing both the oil market and the natural gas market, was applied by Pindyck R.S. (2004), and could be a starting point for further research on volatility for other markets than equity markets.

Also, another little discussed fact is related to the use of GARCH models for less liquid equity markets, such as those in the Central and Eastern Europe. Taking this into account and aiming to add a local flavor to this paper, there was chosen the Romanian stock market, whose evolution is captured by the BET index along with the equity market in Poland, whose evolution is captured by the WIG20 index. For the commodity market, were chosen probably the most traded goods: Brent Crude Oil and Gold.

3. Methodology of research

The methodology followed in this paper is the one presented in the literature and well-known, especially the one proposed by Marcucci (2005). This chapter will be divided into two parts: the description of classical GARCH models (single-state models) and the methodology on which forecasting evaluation will be based (volatility proxies and statistical loss-functions). The first model should be GARCH (p,q), the general form of a GARCH model: **GARCH (p,q)**

$$h_t^2 = \alpha_0 + \sum_{i=1}^q \alpha_i * \varepsilon_{t-i}^2 + \sum_{j=1}^p \beta_j * h_{t-j}^2$$
(1)

Conditional mean

$$r_t = a + \eta_t h_t \tag{2}$$

Where n_t i.i.d with zero mean and unit variance.

GARCH (1,1)

$$h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1}^2 \tag{3}$$

with the following parameter constraints: $\alpha_0 \ge 0$; $\alpha_1 \ge 0$; $\beta_1 \ge 0$, which guarantees a positive conditional variance estimate.

EGARCH (1,1)

$$\log(h_t^2) = \alpha_0 + \alpha_1 \left| \frac{\varepsilon_{t-1}}{h_{t-1}} \right| + \xi \left(\frac{\varepsilon_{t-1}}{h_{t-1}} \right) + \beta_1 \log(h_{t-1}^2)$$
(4)

with no parameter constraints.

The next model is the one implemented by Glosten, Jagannathan and Runkle in 1993, which is aiming to capture the leverage effect, allowing the conditional variance to respond differently to positive and negative shocks. There have been many studies in the literature arguing that the financial market reacts in an asimetric way to shocks of different intensities. Also, it has been discussed that the rate of mean reversion is higher for large shocks than for small shocks (Granger and Poon, 2003).

GJR-GARCH (1,1)

$$h_t^2 = \alpha_0 + \alpha_1 \varepsilon_{t-1}^2 \left[1 - I_{\{\varepsilon_{t-1} > 0\}} \right] + \xi \varepsilon_{t-1}^2 I_{\{\varepsilon_{t-1} > 0\}} + \beta_1 h_{t-1}^2$$
(5)

Where $I_{\{\omega\}}$ is the indicator function which is equal to one when $^{\omega}$ is true and zero otherwise.

The forecasting performances of the models used in this thesis are evaluated by specific statistical loss functions. The most popular way of determining the performance of volatility forecast in current literature is to measure the Mean Squared Error, i.e., MSE. Evidently, the model that performs the best is the one that yields the lowest value of MSE. However, the MSE is rather criticized and there are a lot of loss functions that are argued as better choices. Unfortunately there does not appear to be a superior loss function that alone provides sufficient information of how accurate the models perform compared to each other. The criticism towards evaluating forecast performances are foremost derived from the difficulties of choosing the appropriate loss functions. Following Marcucci (2005), we consider that there is no unique criterion that is best for evaluating forecasting performance. Instead of focusing on a particular loss function that the researchers claim to be superior with regard to others, a wide range of loss functions are chosen. The certain statistical loss functions are:

$$MSE_{1} = n^{-1} \sum_{t=1}^{n} (\sigma_{t+1} - h_{t+1|t})^{2}$$
(6)

$$MSE_{2} = n^{-1} \sum_{t=1}^{n} (\sigma_{t+1}^{2} - h_{t+1|t}^{2})^{2}$$
(7)

$$QLIKE = n^{-1} \sum_{t=1}^{n} (\log(h_{t+1|t}^{2}) + \sigma_{t+1}^{2} h_{t+1|t}^{-2})$$
(8)

$$R2LOG = n^{-1} \sum_{t=1}^{n} \left(\log(h_{t+1|t}^2 \sigma_{t+1}^2))^2 \right)$$
(9)

$$HMSE = T^{-1} \sum_{t=1}^{t} (\sigma_{t+1}^2 h_{t+1|t}^2 - 1)^2$$
(10)

The loss functions provide the researcher the information about how reasonable the performances of the models are. The model which yields the smallest value of one loss function is the model which performed the best. Nevertheless, there is a

high possibility that different models return good results on different loss functions or that some models seem to perform equally well. As a proxy for volatility there will be used the simple daily standard deviation calculated using a rolling-window for last year of observations. The following formula helps us to better present how the proxy for volatility is calculated:

$\sigma_{zilnica\ anualizata} = \sigma_{zilnica} * \sqrt{252}$

Starting from these ideas, this paper will try to compare the results of volatility forecasting models for the two asset classes taken into account (equity market and commodity market). In-sample and out-of-sample evaluations will be done for each of these categories. The results are going be hierarchized in order to obtain a conclusion about the asset class for which volatility forecasting models worked best: the equity market or the commodity market? Is there a clear tendency to overperform for one of the asset classes?

3.1. Data

The first step will be the description of the data series which were used. Daily data will be used for the BET Index, WIG20 Index, Brent Crude Oil and for Gold. The data will be processed according to the following formula in order to obtain return series for each considered asset:

$$r_{t} = 100(\log(p_{t}) - \log(p_{t-1}))$$
(11)

The range for which the data was used was 11/05/2000 - 11/05/2017. Moreover, these data series had to be subdivided into two subintervals to enable the two types of analysis and comparisons, in-sample and out-of-sample as follows:

- BET Index: 130 observations for out-of-sample (last six months) and the rest of the observations for in-sample.
- WIG20 Index: 130 observations for out-of-sample (last six months) and the rest of the observations for in-sample.
- GOLD: 130 observations for out-of-sample (last six months) and the rest of the observations for in-sample.
- Brent Crude Oil: 130 observations for out-of-sample (last six months) and the rest of the observations for in-sample.

Getting back to the way the data is structured, it should be noted that there will be two types of analysis for which the performance of models will be analyzed through different statistical loss functions based on forecasting errors. These will be hierarchized to draw conclusions about their performance so that it can be seen whether a good past performance is a good predictor for a future high performance. This step will be followed by an out-of-sample analysis that will predict volatility for 1 day and 5 day intervals in order to allow us to calculate the prediction error by comparing with a proxy for realized volatility.

4. Results

As previously mentioned in the methodology chapter, there will be estimated 3 types of GARCH models taking into account a normal distribution of errors: GARCH (1,1), EGARCH (1,1) and GJR-GARCH (1,1). Each of these will be estimated for each of the four instruments considered: BET index, WIG20 index, Gold and Brent Crude Oil. It should be remembered that 12 estimates will be obtained in this way, three for each market. The first step in the undertaken approach will be to estimate the coefficients for each model, for each asset considered. The results obtained are presented through the following tables:

| Equity Market | | BET Inde | x | WIG20 Index | | | | | |
|-------------------|------------|-------------|----------------|-------------|-------------|----------------|--|--|--|
| Parameters | GARCH(1,1) | EGARCH(1,1) | GJR-GARCH(1,1) | GARCH(1,1) | EGARCH(1,1) | GJR-GARCH(1,1) | | | |
| δ | 0.072 | 0.081 | 0.061* | 0.052 | 0.062 | 0.054* | | | |
| t-Stat | 3.440 | 4.010 | 1.670 | 2.940 | 2.550 | 1.820 | | | |
| α(0) | 0.651 | -0.122 | 0.080 | 0.655 | -0.152 | -0.164 | | | |
| t-Stat | 11.880 | -5.500 | 10.440 | 12.440 | -9.300 | -10.200 | | | |
| α(1) | 0.172 | 0.241 | 0.182 | 0.191 | 0.220 | 0.033* | | | |
| t-Stat | 20.100 | 11.200 | 17.310 | 21.000 | 9.450 | 1.440 | | | |
| β(1) | 0.825 | 0.881 | 0.890 | 0.880 | 0.830 | 0.910 | | | |
| t-Stat | 88.300 | 3.140 | 51.300 | 92.300 | 77.20 | 95.100 | | | |
| ξ | | 1.033 | 0.189 | | 1.042 | 0.220 | | | |
| t-Stat | | 192.1 | 16.40 | | 177.00 | 21.50 | | | |
| Nr. of parameters | 4 | 5 | 5 | 5 | 6 | 6 | | | |

Table 1. Estimated parameters for equity markets (BET index and WIG20 index)

Note: The parameteres were estimated based on the MLE method. The above table presents for each of them their calculated value for t-Student statistics. Based on these information one could decide if they are statistically significant or not.

"*" denotes that the estimated parameter is not statistically significant for a threshold of 5% (tabelated value is 1.956)

| Commodities Market | | GOLD | | Brent Crude Oil | | | | | | |
|--------------------|------------|-------------|----------------|-----------------|-------------|----------------|--|--|--|--|
| Parameters | GARCH(1,1) | EGARCH(1,1) | GJR-GARCH(1,1) | GARCH(1,1) | EGARCH(1,1) | GJR-GARCH(1,1) | | | | |
| δ | 0.021* | 0.036* | 0.044 | 0.055 | 0.048 | 0.061 | | | | |
| t-Stat | 1.26 | 1.78 | 2.31 | 5.61 | 4.77 | 5.61 | | | | |
| α(0) | 0.45 | -0.18 | -0.22 | 0.3 | -0.167 | -0.144 | | | | |
| t-Stat | 7.88 | -11.200 | -15.300 | 4.55 | -9.510 | -7.520 | | | | |
| α(1) | 0.17 | 0.155 | 0.163 | 0.23 | 0.14 | 0.17 | | | | |
| t-Stat | 15.10 | 11.40 | 15.20 | 25.20 | 7.60 | 16.30 | | | | |
| β (1) | 0.68 | 0.57 | 0.66 | 0.5 | 0.47 | 0.55 | | | | |
| t-Stat | 77.20 | 45.10 | 72 | 39.67 | 30.50 | 42.60 | | | | |
| £ | | 1.01 | 0.331 | | 1.06 | 0.301 | | | | |
| t-Stat | | 142 | 34.10 | | 186 | 29.70 | | | | |
| Nr. of parameters | 4 | 5 | 5 | 4 | 5 | 5 | | | | |

Table 2. Estimated parameters for commodities markets (Gold and Brent Crude Oil)

Note: The parameteres were estimated based on the MLE method. The above table presents for each of them their calculated value for t-

Student statistics. Based on these information one could decide if they are statistically significant or not.

"*" denotes that the estimated parameter is not statistically significant for a threshold of 5% (tabelated value is 1.956)

Following the parameters' estimation for the proposed models for each asset class, we can observe that most of coefficients are statistically significant, but there are very few exceptions. These exception are marked with asterisk (*) in the above tables. Also, a very important aspect is the persistence of volatility for each asset class, which is expressed through the beta parameter: it can be seen that the models show a higher persistence in the equity market (for BET index and WIG20) than for Gold and Brent Crude Oil.

Furthermore, there will be done comparisons on the performance of models for in-sample and out-of-sample, as presented in the methodology chapter. The first step will be to make comparisons and hierarchies based on in-sample estimates and on statistical loss functions. In-sample estimates are shown in the tale below. They are ranked by the score obtained by each model. For each criterion, a ranking from 1 to 12 is made, and the model receives a grade (e.g. 1 being the best). Finally, an arithmetic mean of the marks will be achieved, and the hierarchy will be based on the final score. Through this paper we will try to get a conclusion based on comparisons between equity vs. commodities, not necessarily between models in the same class.

| In-Sample | MSE1 | RANK | MSE2 | RANK | QLIKE | RANK | R2LOG | RANK | HMSE | RANK | TOTAL | RANK |
|-----------------|-------|------|--------|------|-------|------|-------|------|-------|------|-------|------|
| GARCH BET | 1.569 | 10 | 78.12 | 10 | 1.616 | 1 | 8.16 | 2 | 7.351 | 7 | 6.0 | 6 |
| EGARCH BET | 1.535 | 7 | 77.13 | 7 | 1.618 | 3 | 8.172 | 3 | 7.388 | 8 | 5.6 | 5 |
| GJR-GARCH BET | 1.538 | 8 | 77.30 | 8 | 1.617 | 2 | 8.12 | 1 | 6.36 | 1 | 4.0 | 3 |
| GARCH WIG20 | 1.491 | 2 | 74.71 | 3 | 1.632 | 4 | 8.201 | 6 | 6.525 | 3 | 3.7 | 2 |
| EGARCH WIG20 | 1.496 | 3 | 74.60 | 2 | 1.635 | 6 | 8.189 | 5 | 6.474 | 2 | 3.6 | 1 |
| GJR-GARCH WIG20 | 1.48 | 1 | 74.54 | 1 | 1.633 | 5 | 8.225 | 9 | 6.53 | 4 | 4.1 | 4 |
| GARCH GOLD | 1.56 | 9 | 77.44 | 9 | 1.647 | 8 | 8.268 | 10 | 7.239 | 6 | 8.4 | 10 |
| EGARCH GOLD | 1.53 | 5 | 76.48 | 5 | 1.648 | 10 | 8.201 | 6 | 7.724 | 11 | 7.4 | 8 |
| GJR-GARCH GOLD | 1.532 | 6 | 76.65 | 6 | 1.647 | 8 | 8.188 | 4 | 7.708 | 10 | 6.8 | 7 |
| GARCH OIL | 1.497 | 4 | 75.78 | 4 | 1.646 | 7 | 8.275 | 11 | 8.176 | 12 | 7.6 | 9 |
| EGARCH OIL | 1.831 | 12 | 313.95 | 12 | 1.677 | 12 | 8.279 | 12 | 7.049 | 5 | 10.6 | 12 |
| GJR-GARCH OIL | 1.78 | 11 | 250.53 | 11 | 1.651 | 11 | 8.213 | 8 | 7.411 | 9 | 10.0 | 11 |

Table 3. The in-sample analysis (BET, WIG20, Gold and Brent Crude Oil)

Given that loss functions that have been used, the best performing models are those for which the lowest values of these functions have been recorded. Based on the rankings for each criterion, a final ranking was calculated as a simple arithmetic mean for the grades the models received on each criterion

Therefore, we can see that GARCH models performed significantly better in-sample for equity markets. The final ranking highlights that the models which better reflect the leverage effect (EGARCH and GJR-GARCH) perform better, with two of these models located between the top 3. Also, better performance was obtained for the GARCH models used to estimate the volatility for the stock market in Poland. The justification could be brought here by the fact that the Polish market is more liquid. Moreover, there is a clear tendency of models that are used to predict volatility in equity markets to over-perform, at

the expense of those used for commodity markets. Note that these observations were made only based on in-sample analysis.

Further on, we will go to the out-of-sample estimation for the 6-month period chosen as the sample for this type of forecast. The same Garch models will be used. It will again try to highlight a possible model performance difference depending on the asset class to which they applied. Also, there will be used two forecast horizons for GARCH models: 1 day and 5-day intervals.

| Out-of-sample | MSE1 | RANK | MSE2 | RANK | QLIKE | RANK | R2LOG | RANK | HMSE | RANK | TOTAL | RANK |
|-----------------|-------|------|------|------|-------|------|--------|------|------|------|-------|------|
| GARCH BET | 0.361 | 3 | 2.3 | 3 | 0.839 | 4 | 1.4993 | 3 | 0.40 | 6 | 3.8 | 4 |
| EGARCH BET | 0.341 | 2 | 2.0 | 1 | 0.826 | 2 | 1.4632 | 2 | 0.38 | 4 | 2.2 | 2 |
| GJR-GARCH BET | 0.340 | 1 | 2.1 | 2 | 0.823 | 1 | 1.4448 | 1 | 0.39 | 5 | 2.0 | 1 |
| GARCH WIG20 | 0.412 | 6 | 2.7 | 6 | 0.852 | 6 | 1.6626 | 6 | 0.38 | 3 | 3.7 | 3 |
| EGARCH WIG20 | 0.379 | 4 | 2.4 | 4 | 0.840 | 5 | 1.5996 | 5 | 0.38 | 2 | 4.0 | 5 |
| GJR-GARCH WIG20 | 0.380 | 5 | 2.4 | 5 | 0.834 | 3 | 1.5886 | 4 | 0.37 | 1 | 4.1 | 6 |
| GARCH GOLD | 0.496 | 11 | 3.3 | 11 | 0.939 | 11 | 1.9588 | 11 | 0.46 | 11 | 11.0 | 11 |
| EGARCH GOLD | 0.463 | 7 | 2.9 | 7 | 0.920 | 10 | 1.8884 | 10 | 0.44 | 10 | 8.8 | 9 |
| GJR-GARCH GOLD | 0.464 | 8 | 3.0 | 8 | 0.918 | 8 | 1.8833 | 9 | 0.44 | 9 | 8.4 | 8 |
| GARCH OIL | 0.763 | 12 | 7.3 | 12 | 1.045 | 12 | 2.3504 | 12 | 0.51 | 12 | 12.0 | 12 |
| EGARCH OIL | 0.475 | 10 | 3.2 | 9 | 0.918 | 9 | 1.821 | 8 | 0.43 | 8 | 8.8 | 9 |
| GJR-GARCH OIL | 0.473 | 9 | 3.2 | 10 | 0.908 | 7 | 1.7941 | 7 | 0.42 | 7 | 8.0 | 7 |

Table 4. The out-of-sample evaluation (1 day ahead forecast)

After testing out-of-sample the GARCH models used for 1-day ahead forecast, we have obtained results in line with those achieved for testing in-sample regarding the asset class for which the GARCH models perform best. Therefore, it has been noticed that the GARCH models perform better for forecasting volatility on the equity market than on the commodity market using a one-day forecast horizon. The first 6 models in the final standings are those for the equity markets.

Also, the final performance ranking highlights that model that better capture the leverage effect (EGARCH and GJR-GARCH) are more efficient in terms of volatility forecasting. This observation is valid for both, the equity market and the commodity market. Based on the table above, we can see that the last two ranked models are GARCH (1,1) for Gold and Brent Crude Oil. This observation is in line with the previous statement regarding the EGARCH and GJR-GARCH models.

On the other hand, the result obtained in the in-sample analysis that the models have performed better on the Polish stock market has not been respected anymore. This time, two of the top three best-performing models were on the Romanian equity market, represented by the BET index. Thus, for a one-day forecast horizon, it is highlighted that higher stock market liquidity is not necessarily a more conducive environment for GARCH volatility forecasting models.

The next step in this research is represented by the comparisons between models based on predictions for a 5-day interval. The results are presented in the following table:

| Out-of-sample | MSE1 | RANK | MSE2 | RANK | QLIKE | RANK | R2LOG | RANK | HMSE | RANK | TOTAL | RANK |
|-----------------|--------|------|-------|------|-------|------|--------|------|-------|------|-------|------|
| GARCH BET | 6.002 | - 8 | 220.0 | 6 | 3.021 | 8 | 12.05 | 8 | 0.585 | 8 | 7.6 | 8 |
| EGARCH BET | 5.299 | 4 | 181.7 | 4 | 2.964 | 5 | 11.73 | 4 | 0.559 | 5 | 4.4 | 4 |
| GJR-GARCH BET | 5.395 | 5 | 187.7 | 5 | 2.971 | 6 | 11.77 | 5 | 0.561 | 7 | 5.6 | 5 |
| GARCH WIG20 | 0.929 | 1 | 10.9 | 1 | 2.431 | 3 | 8.531 | 1 | 0.232 | 3 | 3.7 | 2 |
| EGARCH WIG20 | 0.972 | 3 | 12.1 | 3 | 2.428 | 1 | 8.599 | 3 | 0.200 | 1 | 2.2 | 1 |
| GJR-GARCH WIG20 | 0.932 | 2 | 11.0 | 2 | 2.430 | 2 | 8.539 | 2 | 0.228 | 2 | 4.1 | 3 |
| GARCH GOLD | 7.359 | 11 | 292.9 | 11 | 3.120 | 11 | 12.816 | 11 | 0.624 | 11 | 11.0 | 11 |
| EGARCH GOLD | 6.540 | 9 | 243.7 | 9 | 3.061 | 9 | 12.480 | 9 | 0.599 | 9 | 9.0 | 9 |
| GJR-GARCH GOLD | 6.662 | 10 | 251.7 | 10 | 3.069 | 10 | 12.529 | 10 | 0.602 | 10 | 10.0 | 10 |
| GARCH OIL | 10.514 | 12 | 543.3 | 12 | 3.284 | 12 | 13.973 | 12 | 0.680 | 12 | 12.0 | 12 |
| EGARCH OIL | 5.929 | 7 | 236.7 | 7 | 2.984 | 7 | 11.808 | 7 | 0.559 | 6 | 6.8 | 7 |
| GJR-GARCH OIL | 5.866 | 6 | 241.7 | 8 | 2.959 | 4 | 11.807 | 6 | 0.540 | 4 | 5.6 | 5 |

Table 5. The out-of-sample evaluation (5-days ahead forecast)

The results did not surprise us with regard to the best performing models in terms of volatility forecasting. Again, the top three models were the ones applied on the equity market. This time, the result obtained for the in-sample analysis for which the best performances were obtained on the Polish market was respected. The explanation is again attributed to the higher liquidity and higher transparency of this stock exchange.

The models that performed well on the Polish market were followed by EGARCH and GJR-GARCH on the Romanian market and then, surprisingly, by EGARCH and GJR-GARCH for Brent Crude Oil. Thus, we can see that the performance of the GARCH models has improved for the commodities market (especially for Brent Crude Oil) when the comparison was made on the basis of a five-day volatility forecasting model. EGARCH(1,1) and GJR-GARCH(1,1) used for the oil market managed to overperform GARCH (1,1) applied to the Romanian equity market. This suggests that for longer periods, models that better capture the leverage effect (EGARCH and GJR-GARCH) can be used with approximately the same accuracy to predict volatility in the oil market.

The results for forecasting volatility in the gold market continued to be the weakest, for all considered criteria. Their performance remained low for both, in-sample and out-of-sample, no matter the forecast horizon length. Note that for the gold market the GARCH models performed best for in-sample analysis (Ranks: 7th, 8th and 9th). For the out-of-sample the performance became worse (Ranks: 8th, 9th and 11th). Also, increasing the forecast horizon from 1 day to 5 days has further increased the prediction error for the GARCH models applied to the gold market (Ranks: 9th, 10th and 11th).

5. Conclusions

This paper intended to answer one question: is there a consistent difference in performance between GARCH models according to the asset classes for which they are applied? This idea started from the experience gained by trading on the oil market and the equity market in Romania, noticing different behaviors and different factors that influence them. There have been done different tests based on in-sample and out-of-sample analysis in order to eliminate the possibility of over-fitting. Several types of loss functions have also been used in order to avoid potential bias from a certain loss function.

The results were in line with the intuition that underpinned this research. Differences in performance were obtained between the applied models on the two asset classes: equity and commodities. These differences have been maintained for the all three models both, in-sample and out-of-sample, reinforcing the idea that GARCH, EGARCH and GJR-GARCH perform better for equity markets than for commodity markets. It has also been observed that models that better capture the leverage effect on financial markets have produced better results when they were compared using loss functions, both, in-sample and out-of-sample.

Although the results showed clearly an outperformance of GARCH models for the equity market, it was noticed that as the horizon forecasting is increased, the performance of forecasting volatility on the stock market falls and gets closer to the one obtained for oil market. This brings into question the possibility of using certain categories of GARCH models to predict oil market volatility under certain conditions and perhaps for longer horizons (10 days, 22 days = one month of trading). On the other hand, the behavior of models for the gold market was a totally unfavorable: performance decreased as the forecast horizon was increased, but also when it passed from the in-sample to the out-of-sample estimation. Thus, for the gold market, the Garch models have recorded the lowest performance of the four markets considered.

This research can be improved by taking into account a larger number of asset classes such as currencies or bonds (sovereign or corporate). Thus, the GARCH models can be applied for a higher number of asset classes in order to obtain conclusions about their relative performance. Also, classical GARCH models with a single state may not be sufficiently suited for markets such as the commodity market where volatility shocks were recorded more often than in the stock market. These jumps can be determined by a series of macroeconomic events or news about production and demand for that commodity. Such problems can be addressed using GARCH models with regime switches, such as those proposed by Marcucci (2005).

These new models can make it easier to switch from one state to another, estimating different parameters for each volatility regime. Marcucci (2005) proposed and implemented models with two volatility regimes, between which the switch was made based on Markov Chains. Further, a third state can be added to increase the performance. Thus, there could be considered thre states: with low volatility, with medium volatility and one with high volatility, and the switch between them could be made again on the basis of Markov Chains. This idea will underline a future research which will aim to identify a more suitable model for each considered asset classes.

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