Günay, Samet

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Fractal Structure of the Stock Markets of Leading Asian Countries

Samet Günay
Assistant Professor, Department of Banking and Finance,
Istanbul Arel University
dr.sgunay@gmail.com

In this study, we examined the fractal structure of the Nikkei225, HangSeng, Shanghai Stock Exchange and Straits Times Index of Singapore. Empirical analysis was performed via non-parametric, semi-parametric long memory tests and also fractal dimension calculations. In order to avoid spurious long memory features, besides the Detrended Fluctuations Analysis (DFA), we also used Smith's (2005) modified GPH method. As for fractal dimension calculations, they were conducted via Box-Counting and Variation $(p=1)$ tests. According to the results, while there is no long memory property in log returns of any index, we found evidence for long memory properties in the volatility of the HangSeng, the Shanghai Stock Exchange and the Straits Times Index. However, we could not find any sign of long memory in the volatility of Nikkei225 index using either the DFA or modified GPH test. Fractal dimension analysis also demonstrated that all raw index prices have fractal structure properties except for the Nikkei225 index. These findings showed that the Nikkei225 index has the most efficient market properties among these markets.

Keywords: Fractal Structure, Long Memory, Fractal Dimension, Self-Similarity, Asian Stock Markets
JEL Classification: C14, C22, G10

I. Introduction

In the last two decades, borrowing and firm value issues receded into the background due to the structured financial instruments and the exotic options coming into prominence. During this period, some models have been improved by financial engineers in order to evaluate these instruments and to model the path of financial time series. However, these models are generally based on the assumptions of conventional financial theory, such as normal distribution and random walk, which are deeply criticized in different studies (Ramasamy and Helmi, 2011).

As the existing conventional theory oversimplifies the complicated structure of financial markets and explains the problems under ideal and normal conditions, we need accurate and credible models in order to build up an effective finance theory (Velásquez, 2009). Conventional theory based on the Bachelier's (1900) study reduced the tail probabilities to an insignificant level using normal distribution. However, this insignificance forms the basis of models such as Markowitz (1952), Sharpe (1964), Fama (1965, 1970) and Black and Scholes (1973), which are the basic studies of conventional theory. The scandals and crises of the last 50 years, including Enron (2001), Parmalat (2003), Black Monday (1987), the collapse of Long Term Capital Management (LTCM) (1998) and the mortgage crisis (2008), demonstrated that the random walk assumption of the Efficient Market Hypothesis is not valid. This is because, if the distribution of price changes were normal, five standard deviation observations would be seen every 7,000 years. However, in this case changes are seen almost every 4 to 5 years. As stated by Mandelbrot and Hudson (2004), the chain is only as strong as its weakest link and the weakest links in conventional theory are the Efficient Market Hypothesis, random walk and normal distribution assumptions.

The first and the most important criticism of conventional theory was made by Mandelbrot regarding the ignoring of self-similarity and long memory properties of the financial time series. It should be noted that while the long memory issue was associated with Mandelbrot, the first person who adressed this topic was Hurst (1951). That is why Mandelbrot (1972) used the H notation as the long memory parameter in honor of Hurst[1]. In that period, Hurst showed that there were significant correlations between the floods. According to his studies, a good flood year had a tendency to be followed by another good flood year, and similarly a bad flood year was followed by another bad flood year. That is, the movements of good and bad years are not in a random process (Grabbe, 2001). Hurst explained this correlation via the power law. One of the most crucial observations of Hurst can be explained as follows. Natural events generally appear to follow a normal distribution but only when the occurrence order of the events is not taken into account (Mandelbrot and Hudson, 2004). The statistic introduced by Hurst measures the persistency between the events and it is known as the Rescaled Range (R/S). The Rescaled Range is the range

[1] Hurst was a hydrologist appointed by the British government to Cairo in order to conduct some research about the Nile River in 1906.

of the partial sums of the deviations from the mean of any time series (Chaitip et al., 2011). Inspired by the Bible, Mandelbrot and Wallis (1968) named the persistency situations the "Joseph Effect" and adapted them into the financial time series theory as the long memory issue (Grabbe, 2001).[2] Mandelbrot (1963, 1983) also contributed to the emergence of self-similarity and the fractal dimension as new topics in finance theory. In fact, all of these models paved the way for the evolution of conventional finance theory.

According to conventional finance theory based on the random walk assumption of Fama (1965), there is no correlation between asset returns, and therefore asset prices develop with the geometric Brownian motion. Almost all models of conventional finance theory are formed under this assumption and probability distributions of the returns are accepted as normal. For instance, in the Black and Scholes (1973) option pricing model, distributions of the log-returns are assumed as normal, and asset prices follow a geometric Brownian motion. However, as stated by Mandelbrot (1963 and 1972) the autocorrelations in financial asset returns decay hyperbolically rather than exponentially, and the tail structure of probability distributions of financial asset returns obeys power law features. Under these circumstances, the acceptance of conventional finance theory in regard to obtaining systematic returns is ill-advised due to short memory.

As stated by Peters (1994), big changes are the sum of many small changes in a Gaussian world. In fact, large jumps can frequently be seen in the asset prices in a panic situation; therefore, discontinuities cause big changes and generate fat tails in the probability distributions of asset returns. These discontinuities are the result of liquidity crunch which arises from the single period investment horizon of the market participants. Consequently, an investor who has this information does not create single period effect over the prices; instead, information may be internalized by different investors with different investment horizons. Longer periods mean longer time to internalize the information and so there will be more consensus on the fair price, that is, longer time horizon creates smoother time series, and this produces the long memory features. In case of the existence of long memory, the weak form of the Efficient Market Hypothesis will be invalid. In that case, investment horizon in the portfolio decisions will also be more important and derivative pricing based on the martingale methods will be unreliable. On the other hand, short memory

[2] In the Bible, the story of Prophet Joseph refers to seven plentiful years followed by another seven years of severe famine.

models such as ARCH, GARCH, ARMA and also Capital Asset Pricing Model (CAPM) and Arbitrage Pricing Model (APT) will lose validity.

Many studies showed that the return and volatility structures of financial assets demonstrated long memory and power law properties. The power law issue is as important as the risk and return topic in financial modeling, as it can provide for the more accurate estimations in the characterization of the financial risk considering extreme price movements. While in short memory processes such as ARMA and GARCH, autocorrelation functions decay exponentially; for long memory processes, the same function decays as a power law (Farmer and Geanakoplos, 2002). The autocorrelation at lag *k* of a time series with a long memory property follows the process below:

$$\rho(k) \sim c\,\rho(k)^{-\gamma}, \qquad k \to \infty \tag{1}$$

where $\rho(k) = \frac{cov[x(t),\ x(t+k)]}{var[x(t)]}$ is an autocorrelation function. As for the gamma value, it is in relation with Hurst exponent H as follows: $\gamma = 2 - 2\mathrm{H}$. In the case of short memory property (H=0.5) gamma will be equal to 1. Otherwise, this statistic will be between 0 and 2.

This hyperbolic decay can be accepted as the sign of a long memory property, as opposed to an exponential rate of the Brownian motion. On the other hand, we can demonstrate the same process with the self-similarity parameter *H*, and the fractional differencing parameter *d*, respectively (LeBaron B., 2001).

$$\rho(k) \sim c\,\rho(k)^{2H-2} \text{ and } \rho(k) \sim c\ \rho(k)^{2d-1} \tag{2}$$

In the case of long memory, the *H* values will be between 0.5 and 1, and if it is *H=0.5*, it gives us the short memory process or random walk. As for the d value, we can define it for different ranges as follows. For $-0.5<d<0$, the process has a short memory and the sum of the absolute values of the autocorrelations of the process tends to be a constant. For $0<d<0.5$, the process has long memory features and all the autocorrelations are positive. Under these circumstances the autocorrelations of the process will decay hyperbolically instead of decaying exponentially as it is seen in the I(0) process. Finally, for $0.5 \leq d<1$ the process is a mean-reverting return series (Baillie, 1996).

By considering these facts, in this study the fractal structure of four Asian countries' stock markets was tested via the long memory and the fractal dimension analyses. All fractality analyses were conducted with log-return, absolute return and raw price series in the empirical section. Distinct from the existing literature regarding long memory, we examined the fractal dimensions of the stock indexes. There may be structural breaks in the time series as the studied period includes different financial crisis such as Asia 1997, Russia 1998, .com 2001, mortgage crisis 2008; therefore, we used the modified GPH model that considers level shifts in the time series in conjunction with the Detrended Fluctuations Analysis. There are some recent studies such as Lee et al. (2014) which restrict the time period of empirical analysis in order to avoid spurious effects. Recently Kang and Yoon (2010) demonstrated that there is a clear relationship between structural breaks and long memory features. Considering this fact, we used quite a long period of time and considered structural breaks without any manual smoothing in the time series unlike Choi et al. (2010).

The rest of the study is structured as follows: Section two will provide literature reviews, section three will give theoretical information about the DFA, Smith's (2005) modified GPH model and the fractal dimension methods, and section four will present the findings of the econometrics tests.

II. Literature Review

The seminal papers of Hurst (1951) and Mandelbrot (1972) aroused researchers' interest in the topics of modeling of long memory and of fractional differencing. Following the introduction of the ARFIMA model by Granger and Joyeux (1980) and Hosking (1981), Geweke and Porter-Hudak (1983) presented a semi-parametric method, known as GPH model, in discrete time. These studies showed that the differencing parameter d did not need to be an integer, that is, it had values between 0 and 1. Similarly, Baillie et al. (1996) proposed a fractionally integrated process for the GARCH model.

Related to these pioneer studies, much research has been conducted on the long memory issue for return and volatility, such as Ding et al. (1993), Greene and Fielitz (1977), Lobato and Savin (1998), Cheung (1993) and Engel and Hamilton (1990). Since the classical *R/S* analysis of Mandelbrot is not robust for heteroscedasticity or short-term dependence, Lo (1991) developed a modified *R/S* test that was robust to short-range dependence. More recently, Willinger et al. (1999) reported that because the modified *R/S* statistic showed a strong preference for accepting the null hypothesis with no long-range dependence, Lo's acceptance of the hypothesis for the Center for Research in Security Prices

(CRSP) data is less conclusive than it is usually regarded in econometrics literature. Moreover, Hiemstra and Jones (1997) applied the modified rescaled range test to the return series of 1,952 common stocks. They concluded that there was some evidence consistent with persistent long memory in the returns of a small proportion of stocks. Alternatively, Giraitis et al. (2001) proposed a method for the (global) Hurst exponent determination based on wavelets. Using this method, they analyzed synthetic data with predefined Hurst exponents and fracture surfaces.

Besides Mandelbrot's *R/S* model (1972) that estimates Hurst exponent, after the introduction of DFA by Peng et al. (1994) to test mosaic structure of DNA nucleotides, it has been used in financial markets in computing the Hurst exponent. Some of the studies showed that the DFA model outperforms the classical *R/S* model. For example, Kim et al. (2014) discuss the advantages and disadvantages of the different long memory analyses. The results showed that DFA is the most appropriate technique for the Hurst exponent estimation. Similarly, Weron (2002) used the *R/S* analysis, DFA and periodogram regression methods, constructing empirical confidence intervals for all three methods. He stated that the DFA test provided the most successful statistics. Furthermore, according to Rege and Martin (2011), while linear trend at DFA analysis is still robust, the quadratic trend used in the DFA method overestimates the value of the Hurst exponent. Despite that, Kristoufek (2010) showed that the *R/S* still remains a useful and robust method even when it is compared to the DFA.

There is also a great deal of interest in structural breaks in long memory literature. For instance, Mikosch and Starica (1998), Granger and Hyung (1999) and Hamilton and Susmel (1994) demonstrated that structural breaks can produce spurious long memory property, which is why some modified methods have been generated, such as Smith's (2005) modified GPH model and Baillie and Morana's (2009) Adaptive FIGARCH model. Ohanissiana et al. (2008) also proposed a statistical test to distinguish between true long memory and spurious long memory based on the invariance of the long memory parameter for temporal aggregates of the process under the null of true long memory. Some papers, such as Choi and Wohar (1992), showed that there is serious bias with regard to the classical GPH estimator. Due to similar reasons, Robinson (1995) and Phillips (1999a, 1999b) presented a modified GPH model, and Guggenberger and Sun (2006) introduced a new, computationally attractive estimator $\hat{d}_{WLP}(r)$ by taking a weighted average of $\hat{d}_{LP}(0)$ estimators over different bandwidths as an alternative to the classic GPH method.

Some studies on the long memory and fractality of Asian financial markets can be exemplified as follows. Kang and Yoon (2006) exhibited long memory features of the volatilities of four different Asian countries; Japan, South Korea, Hong Kong and Singapore. However, they could not find any evidence of long memory in the log returns of these countries. Han (2011) showed that when the structural breaks are taken into account via Adaptive FIGARCH model versus classical FIGARCH, Korean exchange returns displayed lower long memory features. Considering this fact, Choi et al. (2010) manually modified the time series after the determination of break dates. In another interesting study, Tan et al. (2011) examined the long memory features of the developed and emerging Asian markets' stock returns. According to the results, developed markets do not present long memory features in the returns, whereas emerging markets' stock returns (Indonesia, Malaysia and Philippines) appear predictable. More recently, Gil-Alana et al. (2014) analyzed the long memory features of bull and bear periods of some European and Asian stock indexes and showed that bull period volatilities of the FTSE, Dax, HangSeng and STI indexes are more persistent than the bear periods. Bhattacharya and Bhattacharya (2012) investigated the existence of the long memory properties in ten emerging stock markets (Hungary, China, Brazil, Chile, Malaysia, Korea, Russia, Mexico, India and Taiwan) using the classical *R/S* statistic, Lo's statistic and the GPH statistic as well as the modified GPH statistic of Robinson (1995). Sriboonchitta et al. (2011) examined the behavior of both Thailand's and India's international tourism market by using the *R/S*, modified *R/S* and GPH methods. They showed that while there are findings of long memory in Thailand's tourism market, evidence for India's tourism market supports the existence of short term memory. Cajueiro and Tabak (2004) tested the Efficient Market Hypothesis for China, Hong Kong and Singapore by means of the long memory dependence approach. They stated that liquidity and capital restriction factors can help explain the results of market efficiency tests.

III. Econometric Methodology

As there are assumption difficulties in the parametric models, in the long memory analysis of return series we preferred to use non-parametric and semi-parametric methods (DFA and modified GPH model of Smith) in the empirical section of study.

The primary reason DFA stands out among the non-parametric methods is that it has proven to be a robust method for the non-stationary time series. When we review the related literature, it is seen that there is a common accord concerning the performance of DFA; Vandewalle et al. (1997), Weron (2002), Jiang et al. (2005) Witt and Malamud (2013). After the seminal papers of Geweke and Porter-Hudak (1983), GPH model attracted intensive attention as a semi-parametric model in measuring long memory properties and received wide acceptance in the literature. Despite this, as stated by Andersson and Gredenhoff (1998), although semi-parametric methods are demonstrably robust against the non-normal distributions and heteroscedasticity (ARCH effects) issues seen in financial time series; in case of the existence of the trends and structural breaks in these time series, reliability of the estimated statistics can drop. With all of these factors in mind, in this study we preferred to use the modified GPH test of Smith (2005) which considers the structural breaks in the time series. As stated before, the purpose of the study is to examine the fractality features of four Asian markets' stock index returns. Long memory analysis is not sufficiently informative to make a decision concerning fractality, so we conducted fractal dimension tests via price series, in addition to the long memory tests for log returns and absolute returns. At that stage to assess these factors, we used Box-Counting and Variation (p=1) methods following the suggestion of Gneiting et al. (2012).

1. Detrended Fluctuations Analysis

In order to test the long memory property of the log and absolute returns, we performed the DFA proposed by Peng et al. (1994) which tests the mosaic structure of DNA nucleotides. The first reason for preferring this method is that it can be applied to non-stationary time series, in comparison to the *R/S* and modified *R/S* analysis. Secondly, the DFA outperforms the *R/S* analysis, avoiding spurious long memory property owing to its calculation process.

More recently, Weron (2002) outlined the following steps to carry out the DFA test:

1. Firstly, the times series *Y* is divided into *d* intervals of length n and for each interval $i = 1,\ldots,n$, and a cumulative time series $Y_{i,m} = \Sigma_{j=1}^{i} X_{j,m}$ is obtained
2. In order to calculate the local trends, a line is fit to $\{Y_{1,m},\ldots, Y_{n,m}\}$ using the OLS estimation for each *d* interval, $=1,\ldots,d,\ \widetilde{Y}_m(x) = c_m + \beta_m x.$
3. In the next step, the time series is detrended by subtracting the local trends from each interval. The root mean square (RMS) fluctuation of the integrated

and detrended time series is computed as follows

4.

$$F(m) = \sqrt{\frac{1}{n}\sum_{i=1}^{n}[\,Y_{i,m} - (c_m + \beta_m i\,)]^2}\,. \tag{3}$$

According to the order of the filtering trend: first order polynomials are denoted by DFA1 (linear trend) and second order polynomials are denoted by DFA2.

5. Finally, the mean values of the RMS fluctuation are calculated for every *d* interval of the length *n* to obtain the DFA fluctuation:

$$F(n) = \frac{1}{d}\sum_{m=1}^{d}\sqrt{\frac{1}{n}\sum_{i=1}^{n}[\,Y_{i,m} - (c_m + \beta_m i\,)]^2}\,. \tag{4}$$

The Hurst exponent is calculated from the following scaling relationship:

$$F(n)\ \sim c\,n^H \tag{5}$$

where c is a constant and H is the Hurst exponent.

2. Smith's Modified GPH Method

Smith (2005) suggested a modification to the classic GPH model in order to reduce the bias when the data generating process exhibits level shifts. The classic GPH (1983) model's d estimater can be expressed below,

$$\hat{d} = d_* + \frac{\Sigma_{j=1}^{J}(X_j - \overline{X})\log(\hat{f}_j/f_j)}{\Sigma_{j=1}^{J}(X_j - \overline{X})^2} \tag{6}$$

where $d_* + \dfrac{\Sigma_{j=1}^{J}(X_j - \overline{X})\log(f_j)}{\Sigma_{j=1}^{J}(X_j - \overline{X})^2}$. For consistency, Geweke and Porter-Hudak (1983) recommended the rule of $J/T \to 0$ as $T \to \infty$. Smith (2005) offered to add an extra regressor, $-log\,(p^2 + w^2)$, to the GPH model in order to reduce the bias caused by level shifts.

$$\log\hat{f}_j = \alpha + dX_j + \beta Z_{jk} + \hat{u}_j \tag{7}$$

where $k > 0$, $p_T = \frac{kJ}{T}$ (J is the number of the periodograms in d estimation, T is the sample size and k is a scalar between 1 and 5) and $Z_{kj} = -\log\left[\frac{(kJ)^2}{T^2} + w_j^2\right]$. Smith's modified GPH estimator can be stated as:

$$\hat{d}^k = d_*^k + (\widetilde{X}' M_Z \widetilde{X})^{-1} \widetilde{X}' M_Z \log\left(\frac{\hat{f}}{f}\right) \tag{8}$$

where $\widetilde{X} \equiv X - \overline{X}$, $M_Z = I - \widetilde{Z}_k (\widetilde{Z}_k{}' \widetilde{Z}_k)^{-1} \widetilde{Z}_k'$, $\widetilde{Z}_k \equiv Z_k - \overline{Z}_k$, $\overline{X} = J^{-1} \sum_{j=1}^{J} X_j$, $\overline{Z}_k = J^{-1} \sum_{j=1}^{J} Z_{kj}$. Smith used the spectrum instead of the periodogram; d_*^k denotes this stuation.

3. Fractal Dimension

Different methods are used to calculate fractal dimension; the most popular and most practical approach is the Box-Counting method. However, Gneiting et al. (2012) stated that the Variation ($p = 1$) (madogram) can provide more efficient results for the time series. For this reason we used both of these methods to calculate the fractal dimensions of the related indexes.

If the N_0 is a set of points distributed in the Euclidian space, this set can be covered with square boxes with edge length ε. $N(\varepsilon)$ demonstrates the minimum number of the boxes in order to cover the set. Using this notation, the Box-Counting dimension is defined by:

$$\hat{D}_{BC} = \lim_{\varepsilon \to 0} \frac{ln\ N(\varepsilon)}{\ln\left(\frac{1}{\varepsilon}\right)}. \tag{9}$$

In order to compute the dimension value, we count $N(\varepsilon)$ for different edge lengths. Finally we plot $N(\varepsilon)$ versus $\ln \frac{1}{\varepsilon}$; the slope of the linear function will be equal to the Box-Counting dimension value (Murdzek, 2007).

Using the procedure of Gneiting et al. (2012), the madogram estimator of the fractal dimension can be defined in the following way: a stochastic process $\gamma_p(t)$ with stationary increments is:

$$\gamma_p(t) = \frac{1}{2} E\,|X_u - X_{t+u}|^{\,p}. \tag{10}$$

In this process, for order $p = 1$ we recover the madogram. For the time series data, the power variation of order p defined by:

$$\widehat{V}_p\left(\frac{l}{n}\right) = \frac{1}{2(n-1)} \sum_{i=l}^{n} |X_{i/n} - X_{(i-l)/n}|^p. \tag{11}$$

Variation estimator of order p for the fractal dimension is as follows:

$$\widehat{D}_{V;p} = 2 - \frac{1}{p} \frac{log\,\widehat{V}_p\left(\frac{2}{n}\right) - log\,\widehat{V}_p\left(\frac{1}{n}\right)}{log 2}. \tag{12}$$

IV. Empirical Analysis

This paper examines the fractality feature of selected Asian countries' stock markets, which are Japan (Nikkei225), China (HangSeng and Shanghai Stock Exchange) and Singapore (Straits Times Index), during the period between 01/04/1991 and 03/10/2014. Fractality analyses have been performed for log returns, absolute returns and price series of all indexes using DFA, Smith's (2005) modified GPH analysis and fractal dimension methods. All of the time series were provided by finance.yahoo.com.

1. Descriptive Statistics

It is useful to examine the descriptive statistics in order to have a general idea about the characteristics of data before continuing to the long memory analysis of the stock index series. As it can be seen from the descriptive statistics, the highest and the lowest mean returns are seen in the log returns of the SSE and the STI, respectively. Likewise, the same results are also valid for the standard deviation statistics. According to the skewness and kurtosis values, all log returns are leptokurtic, and they are positively skewed, which means they tend toward lower returns. Obtained statistics demonstrate that the highest skewness and kurtosis values belong to the SSE index. We know that for the normal distribution, skewness and kurtosis statistics have to be 0 and 3 respectively, so it is clear that all index returns are far from the normal distribution. More accurate and certain observations can be made with the Jarque-Bera (J-B) test statistics. Accordingly, all the obtained J-B test statistics are statistically significant in 99% confidence level. These results also confirm the deviation from normal distribution. Absolute returns also exhibit nearly identical results for all indexes, except for the negative skewness of Nikkei 225 index.

Table 1. Descriptive Statistics

	Return type	Nikkei 225	HangSeng	SSE	STI
Mean	Log returns	0.004800	0.005094	0.006126	0.003855
	Abs returns	-3.23E-05	0.000240	0.000263	9.59E-05
Std. Dev.	Log returns	0.004591	0.005563	0.008687	0.004353
	Abs returns	0.006643	0.007540	0.010627	0.005814
Skewness	Log returns	2.586526	3.222534	10.78612	3.676517
	Abs returns	-0.210409	0.033588	5.307449	0.315879
Kurtosis	Log returns	16.66521	22.39339	296.0993	27.93205
	Abs returns	8.037998	11.82545	146.7163	14.52193
Jarque-Bera	Log returns	50732.57**	99242.32**	20524277**	160557.1**
	Abs returns	6073.341**	18509.33**	4934770**	31640.70**

2. Detrended Fluctuations Analysis

With the definition of Tapiero (2010), we can say that the Hurst exponent identifies statistical bias in the time series and arises from the self-similar power law. Despite the fact that the Hurst exponent can be measured by different types of methods, Kim et al. (2014), Weron (2002) and Kristoufek (2010) showed that the DFA outperforms other non-parametric methods, such as the Rescaled Range (*R/S*) analysis. As said by Kantelhardt et al. (2001), the advantage of the DFA is that it removes trends in different orders systematically and can be performed on apparently non-stationary time series. The DFA estimates the Hurst exponent (*H*) as such in the *R/S* analysis. Accordingly, $0.5 < H < 1$ indicates the long memory process, $0 < H < 0.5$ shows the mean reverting short memory process, and finally, $H=0.5$ indicates the random walk. Table 2 shows the Hurst exponent values of the log and absolute returns of the related indexes.

Table 2. Hurst Exponent Estimation of DFA

	H (log retruns)	H (Abs. Returns)	RMSE (log retruns)		RMSE(Abs. Returns)	
			0.0050	0.0284	0.0066	0.1871
			0.0075	0.0366	0.0127	0.3723
Nikkei225	0.5276	0.8801*	0.0102	0.0622	0.0253	0.7534
			0.0141	0.0870	0.0482	1.4794
			0.0206	0.0893	0.0976	2.6908
			0.0060	0.0368	0.0075	0.2089
			0.0088	0.0517	0.0144	0.4100
HangSeng	0.5292	0.9106*	0.0122	0.0770	0.0276	0.8353
			0.0182	0.1168	0.0538	1.6083
			0.0268	0.1785	0.1086	2.7525
			0.0088	0.0622	0.0104	0.2618
			0.0128	0.0842	0.0191	0.5228
SSE	0.5919*	0.9393*	0.0178	0.1450	0.0354	0.9535
			0.0294	0.1887	0.0713	2.0773
			0.0406	0.2301	0.1356	4.2535
			0.0048	0.0334	0.0058	0.1566
			0.0073	0.0464	0.0111	0.3136
STI	0.5903*	0.9114*	0.0102	0.0696	0.0212	0.6296
			0.0149	0.1018	0.0404	1.1953
			0.0235	0.1105	0.0806	2.0723

* indicates the significancy at 95% confidence level. 95% confidence intervals: 0.4279-0.5592, Scale ratio : 2

As demonstrated in Table 2, when we take the confidence intervals of Weron (2002) into account, there is no long memory sign in the log returns of the Nikkei225 and HangSeng indexes. In spite of that, the *H* values of the SSE and STI indexes, 0.5919 and 0.5903, respectively, demonstrate the long memory property in the log returns.

Unlike the log returns, the findings of the absolute returns exhibit high persistency in the volatility of all of the indexes, and indicate the existence of long memory property in the volatility. In order to evaluate the success of the estimation models, we presented root mean square errors (henceforth, RMSE) statistics in conjunction with the Hurst exponent and the confidence intervals. RMSE can be stated as the goodness-of-fit measure of the models. It shows the discrepancy between the observed variables and their estimated values (Ratkowsky, 2004).

In the plots below we presented the third output p of Weron (2011). Figure 1 and Figure 2 exhibit the path of 32 p values which are the average standard deviations of the detrended walk for all the divisors. The Hurst exponent in the DFA model is simply the slope of the p values with respect to the divisors (d) on a log-log scale. The increasing slope of the plots means larger Hurst exponent value in the related country.

Figure 1. The behavior of the p statistic versus divisors (d) for log-returns

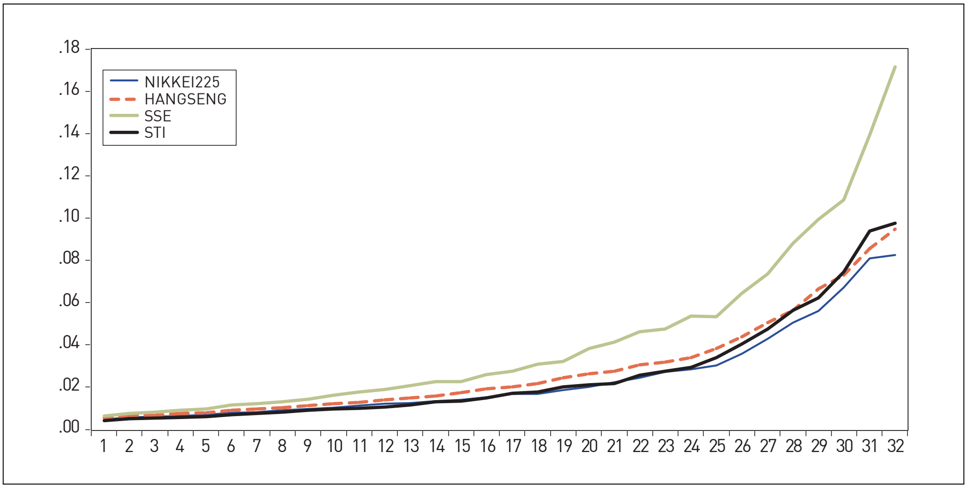
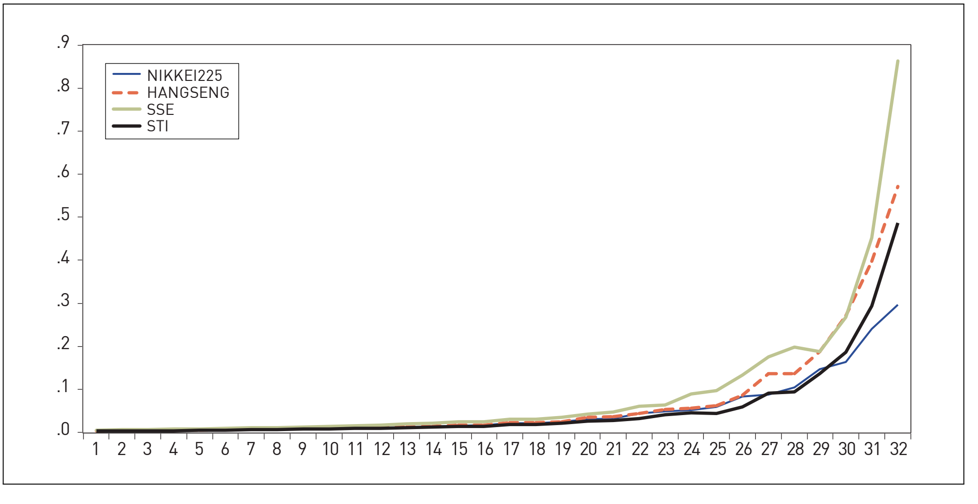


Figure 2. The behavior of the p statistic versus divisors (d) for absolute returns



3. Smith's Modified GPH Test

When the d notation shows the integration level of a series, this situation is generally demonstrated by $I(d)$. While, in general, $I(0)$ and $I(1)$ situations have been analysed in literature, after Hosking (1981) showed that d can have fractional values, it became a measure of the long memory and the persistency of the autocorrelation structure of the financial time series.

Table 3. GPH and Smith's (2005) Modified GPH Estimates of Log Returns

		GPH	Modified GPH				
			k=1	*k=2*	k=3	*k=4*	*k=5*
Nikkei 225	*Plug-in selection of J*	-0.0083 (0.0305)	0.0819 (0.0714)	0.0345 (0.0539)	0.0320 (0.0461)	0.0239 (0.0412)	0.0170 (0.0377)
	$J=\sqrt{T}$	-0.0142 (0.0816)	0.0099 (0.2505)	0.0483 (0.1876)	0.0644 (0.1655)	0.0742 (0.1544)	0.0806 (0.1479)
HangSeng	*Plug-in selection of J*	0.0323 (0.0305)	0.0608 (0.0714)	0.0500 (0.0539)	0.0382 (0.0461)	0.0160 (0.0412)	-0.0045 (0.0377)
	$J=\sqrt{T}$	-0.0040 (0.0816)	-0.0285 (0.2505)	0.0033 (0.1876)	0.0071 (0.1655)	0.0066 (0.1544)	0.0057 (0.1479)
SSE	*Plug-in selection of J*	0.0134 (0.0723)	-0.0698 (0.1928)	-0.0465 (0.1380)	-0.0147 (-0.014)	0.0146 (0.0146)	0.0454 (0.0923)
	$J=\sqrt{T}$	0.0153 (0.0816)	-0.0864 (0.2505)	-0.0541 (0.1876)	-0.0494 (0.1655)	-0.0451 (0.1544)	-0.0409 (0.1479)
STI	*Plug-in selection of J*	0.1354** (0.0598)	-0.1147 (0.1536)	0.0138 (0.1118)	0.0164 (0.0941)	0.0750 (0.0833)	0.0702 (0.0758)
	$J=\sqrt{T}$	-0.0305 (0.0816)	-0.3450 (0.2505)	-0.1765 (0.1876)	-0.1185 (0.1655)	-0.0895 (0.1544)	-0.0726 (0.1479)

* and ** indicate the significancy at 95% and 99% confidence level, respectively. Standard errors are within the parenthesis.

Similar to the Hurst exponent H which shows long memory in the interval between 0.5 and 1, Hosking (1981) procured the same situation for d in the interval between 0 and 0.5. In addition to different kinds of methods for the modeling of long memory in conditional mean and conditional variance, by modifying Geweke and Porter-Hudak's (1983) classic GPH model Smith (2005) introduced a new model which considers level shifts. As stated by Smith (2005), if the d statistic is larger in the modified model than in the classic GPH, this can be attributed to level shifts. Smith (2005) used different values for k, and periodograms J, in the d modeling. He also stated that the average bias can be minimized for k=3.16, so he offered to use k=3 in the estimation process. Following Smith (2005), we used different k values in the modeling process of long memory. According to the findings of k=3, there is no long memory property in the log returns of the Nikkei225, HangSeng, SSE and STI data for the 95 percent confidence interval.

Table 4. GPH and Smith's (2005) Modified GPH Estimates of Absolute Returns

		GPH	Modified GPH				
			k=1	*k=2*	*k=3*	*k=4*	*k=5*
Nikkei 225	Plug-in selection of J	0.3291** 0.0822	-0.1367 (0.2261)	0.08323 (0.1595)	0.1158 (0.1327)	0.176 (0.1166)	0.2777** (0.1056)
	$J=\sqrt{T}$	0.3227** 0.0816	-0.0986 (0.2505)	0.0416 (0.1876)	0.08932 (0.1655)	0.1139 (0.1544)	0.1287 (0.1479)
HangSeng	Plug-in selection of J	0.3456** 0.03052	0.5393** (0.0714)	0.4564** (0.05393)	0.4311** (0.04612)	0.4286** (0.04122)	0.4243** (0.03771)
	$J=\sqrt{T}$	0.5326** 0.0816	0.2977 (0.2505)	0.3993* (0.1876)	0.4296** (0.1655)	0.4442** (0.1544)	0.4526** (0.1479)
SSE	Plug-in selection of J	0.3661** 0.03052	0.4282** (0.0714)	0.3759** (0.05393)	0.3645** (0.04612)	0.3412** (0.04122)	0.3651** (0.03771)
	$J=\sqrt{T}$	0.4979** 0.0816	0.6127** (0.2505)	0.5884** (0.1876)	0.5754** (0.1655)	0.5675** (0.1544)	0.5633** (0.1479)
STI	Plug-in selection of J	0.3305** 0.03052	0.4808** (0.0714)	0.4319** (0.05393)	0.3942** (0.04612)	0.3718** (0.04122)	0.3643** (0.03771)
	$J=\sqrt{T}$	0.4492** 0.0816	0.6737** (0.2505)	0.6045** (0.1876)	0.5691** (0.1655)	0.5496** (0.1544)	0.5378** (0.1479)

* and ** indicate the significancy at 95% and 99% confidence level, respectively. Standard errors are within the parenthesis.

Regarding absolute returns, although there is no long memory property in the volatility of the Nikkei225 index, the d values of the other three indexes' absolute returns are statistically significant at the 95 percent confidence interval. On the other hand, for the fixed J selection of the k=3, the d value of the modified model is smaller than the classic model in the HangSeng index. It suggests that the long memory may arise from the level shifts in the HangSeng index volatility. However, we see that both models' d values are statistically significant. So, regardless of whether or not there are level shifts, the HangSeng index volatility has the long memory property.

In summary, according to the findings of Smith's (2005) modified GPH model, while there is no long memory in the log returns of all indexes except for the Nikkei225 index, there is long memory in the volatility of the HangSeng, SSE and STI series. These results contrast with the findings of the DFA in terms of log returns for SSE and STI, and in terms of absolute returns for the Nikkei225 index. Because, according to the results of DFA test, log returns of SSE and STI indexes have long memory properties with values of 0.5919 and 0.5903, respectively. As a nonparametric method, DFA analyzes the long memory features without taking the structural breaks into account. Although there is ample literature which displays the robustness of the model, the concern about the existince of structural breaks in the time series directed us to the modified GPH model of Smith (2005). It is likely that long memory signs in the DFA regarding the SSE and the STI dataset arise from the spurious effects of the structural breaks. Similar results can be seen in Smith (2005) or Yalama and Celik (2013) as well. We can see that Smith's (2005) modified GPH test results do not confirm the conclusions obtained from the DFA. In addition to the log returns, the absolute return results of the Nikkei225 index in the DFA analysis show the existence of long memory in the volatility, while there is no evidence in Smith's (2005) modified GPH model. At that point, as the Smith's (2005) modified GPH model takes level shifts into account, we accept that there is no long memory in the log returns of any indexes, or in the volatility of the Nikkei225 index.

4. Fractal Dimensions

As noted by Peters (1996), there is an obvious relationship between the fractal dimension (D) and the Hurst exponent (H). While the Hurst exponent measures the long memory property, the fractal dimension considers the long memory from a different angle measuring the jaggedness of the series. We can present the relationship of D and H as follows:

$$D + H = 2 \tag{13}$$

In the random walk situation, the H value will be equal to 0.5, and therefore D=1.5. When the D value gets closer to 1, the time series displays a trend property and will resemble a line. If the D value gets closer to 2, the time series will have a more jagged structure and be far from the trend.

Table 5. Fractal Dimensions of Index Values

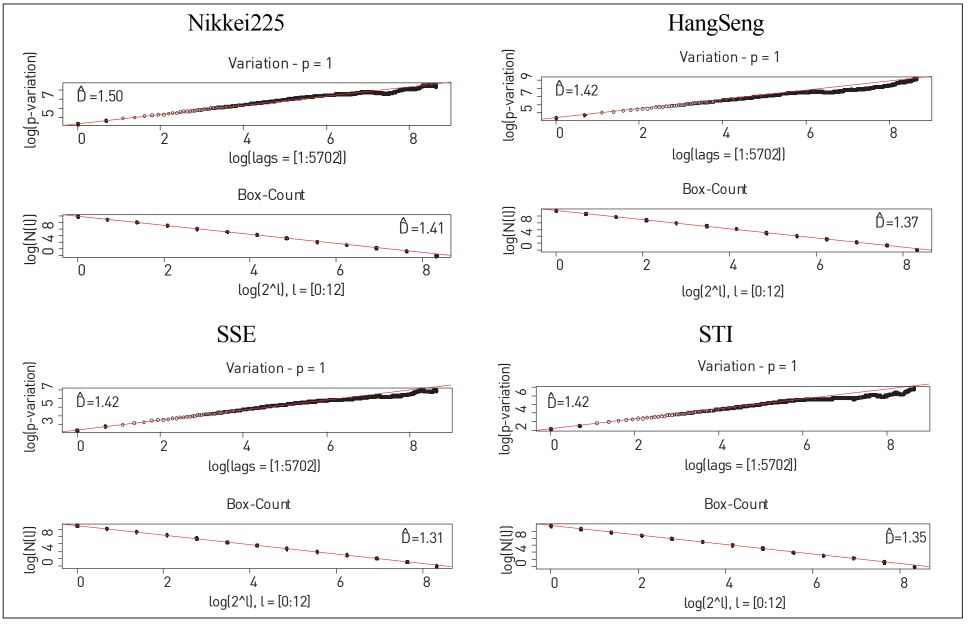
	Nikkei	HangSeng	SSE	STI
Box-Counting	1.41	1.37	1.31	1.35
Variation (p=1)	1.50	1.42	1.42	1.42

According to the Box-Counting statistics, all index series have long memory properties. However, if we consider the Variation (p=1) results suggested by Gneiting et al. (2012), we see that there is no long memory property in the Nikkei225 series.

It is worth stating that these fractal dimension tests were conducted via the raw price series. Consequently, using the fractal dimension methods, we present a different approximation to the long memory issue. Combining the information obtained from the long memory and the fractal dimension tests, we can say that while there is an obvious fractal feature in the SSE and STI series, we cannot see any fractal property in the Nikkei225 index. As for the HangSeng series, results are very complicated for this index.

The graphs below show the log-log regression results for the fractal dimension estimator.

Figure 3. Log-log Regression Plots of the Fractal Dimension Estimator

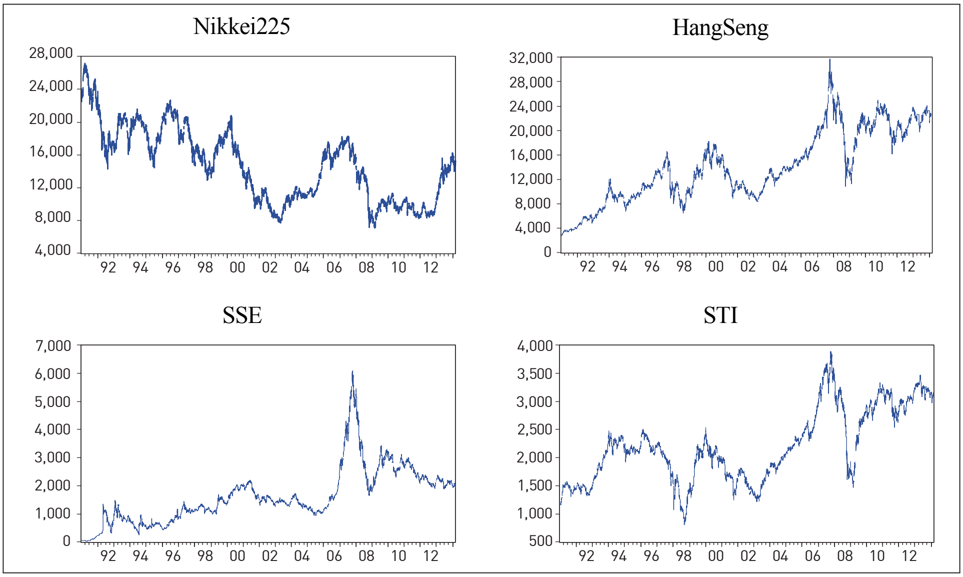


Regarding the fractal dimension, the plots seen in the Figure 3 above demonstrate the lines obtained via the regressing of N(ε) and ln $\ln\frac{1}{\varepsilon}$ values (see Equation 9) that is used in the fractal dimension calculation. Thus, the slope of this linear line yields fractal dimension D.

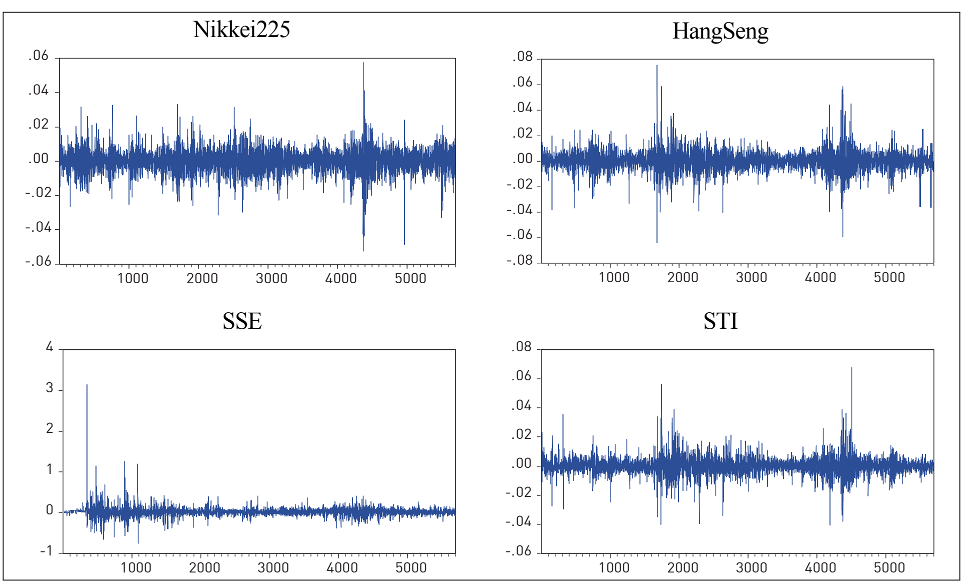
V. Conclusion

According to the fractal market hypothesis, financial markets are considered as dynamic systems with nonlinear, predictable and non-integer fractal dimensions. As stated by Deboeck (1994), this hypothesis allows the variation of the dimension of probability distribution of returns in the range of 1-2 with integer and non-integer values. As for the Efficient Market Hypothesis, it assumes that the dimension of the probability distribution of returns is always equal to 2. In light of this information, in this study we analyzed the fractality features of the leading Asian stock market indexes via two measurements of the fractality long memory analysis and fractal dimension tests. Long memory analyses were conducted via both non-parametric and semi-parametric methods and so we obtained self-similarity parameter H and fractional differencing parameter d for all stock index returns. We also performed fractional dimension analysis for raw price data in order to examine the fractality of the stock index thoroughly and in detail. Using an extended period of time, ranging from 01/04/1991 to 03/10/2014, we consciously kept the crisis periods within the time interval of the study. The potential level shift effects of the crisis periods were considered via the modified GPH model that takes structural breaks into account. According to the results of DFA analysis for log-returns, only the SSE and the STI indexes presented long memory properties. However, when we take structural breaks into account via the modified GPH model it was seen that neither the SSE and STI nor the Nikkei225 and HangSeng indexes possess long memory properties. We concluded that these spurious effects in the log-returns may have arisen from the heavy crisis periods that the Asian markets were exposed to in 1998 and also the mortgage crisis of 2008. Regarding volatility, the results demonstrated that modified GPH and DFA methods gave different results about the persistency of volatility in the Nikkei225 index returns. Both the modified GPH and DFA models, on the other hand, showed that the HangSeng, SSE and STI volatilities have long memory properties. We suggest that the confusing results of the Nikkei225 may originate from the structural breaks in the volatility, likewise the findings concerning the log-returns. Hence, we accepted the results of modified GPH method since we have also seen that modified GPH and fractal dimension findings are consistent with each other. The Variation (p=1) method showed that the Nikkei225 index price's fractal dimension value, which characterizes the local memory in the series, is 1.5, in which case price movements display random behaviors. On the other hand, according to the Box-Counting method, the fractal dimension value closest to 1.5 belongs to the Nikkei225 index, as well. In conclusion, we have seen that although there is a slight sign of fractality in the HangSeng, SSE and STI indexes, we could not find any evidence about the fractality of the Nikkei225 index. These findings revealed that the Nikkei225 index has more efficient market properties compared to the HangSeng, SSE and STI indexes.

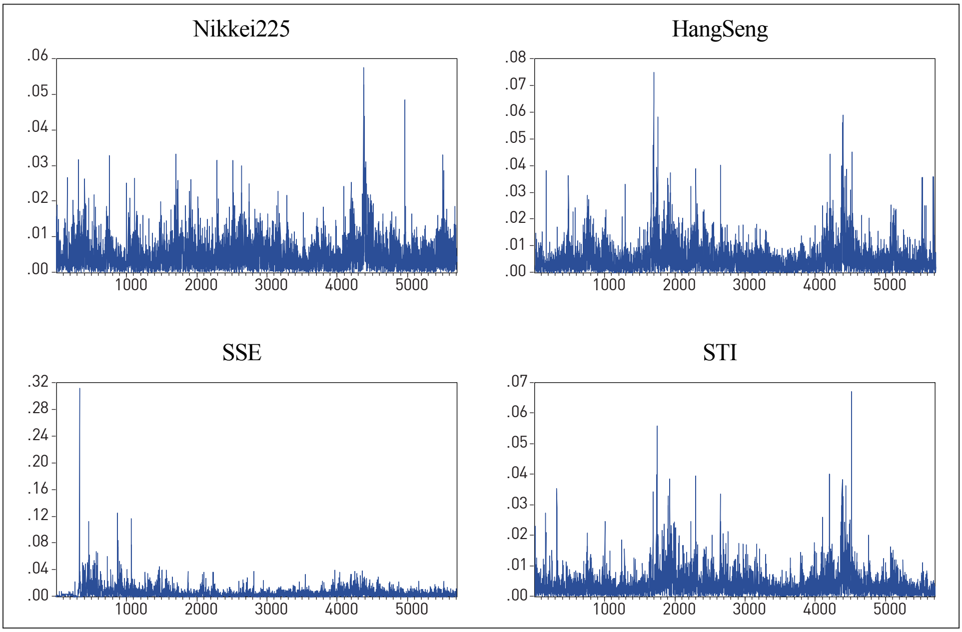
Appendix 1. Price Series of the Stock Indexes



Appendix 2. Log returns of the Stock Indexes



Appendix 3. Absolute Returns of the Stock Indexes



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About the Author

Samet Günay is currently Assistant Professor of Finance at Istanbul Arel University, Turkey in the Finance and Banking Program. He has been at National University of Singapore as visiting PhD student in 2013 and at Indiana State University, USA, in 2014 as visiting scholar. His major is in finance and research interest focuses on fractals and financial econometrics. He received his BS from the Uludag University, MA from the Anadolu University and Ph.D. from Istanbul University.

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