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## Article

# Co-movement (sub-)indicator as the measurement of the synchrony of EA and Visegrad Group countries

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*Reference:* Poměnková, Jitka/Klejmová, Eva et. al. (2020). Co-movement (sub-)indicator as the measurement of the synchrony of EA and Visegrad Group countries. In: Ekonomický časopis 68 (3), S. 231 - 251.

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## Co-movement (Sub-)Indicator as the Measurement of the Synchrony of EA and Visegrad Group Countries<sup>1</sup>

Jitka POMENKOVA – Eva KLEJMOVA – Tobias MALACH\*

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### Abstract

*The paper deals with the construction of a co-movement indicator suitable for assessing the synchrony between countries. The indicator is represented as a time series and its construction is based on a reconstruction of a co-spectrum measure from the time-frequency to the time domain. We use the statistically significant part of the power wavelet co-spectrum for pairs of countries. An advantage of the newly proposed co-movement indicator is a possibility to construct sub-indicators which correspond to the predefined frequency range, e.g. business cycle frequencies. In such a way we can obtain a decomposition of the co-movement indicator (covering all frequencies) into, for example, short-run cycles, medium and long business cycles and long-run cycles. The proposed methodology is demonstrated on the US and EA monthly data of industrial production index in 1991 – 2018. A further application is performed on the EA and Visegrad Group Countries with the same data type and time range.*

**Keywords:** co-movement indicator, co-spectrum, globalisation, wavelets

**JEL Classification:** E51, F02, P24

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### Introduction

The globalization of economies has been reflected in many areas of the particular country economy, and thus it is still at the forefront of economic analysts' interests. It can be studied in many contexts, e.g. business cycle (BC) synchronisation; financial globalisation and its effect on the monetary policy; emerging

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<sup>1</sup> The research described in the paper was supported by the Czech Science Foundation via grant n. 17-24309S and by the Czech Ministry of Education in the frame of National Sustainability Program under grant LO1401. For research, the infrastructure of the SIX Center was used.

markets; financial or goods markets; the influence of globalisation on social life, etc. The globalization leads to a transfer of shocks from the economy of one country to other countries, which can significantly influence the evolution of regional as well as world economy. This can be illustrated by the crisis in 2008 which started in the USA, or oil crisis in 1970s. Focusing on the European Area (EA), we can mention other examples: the adoption of Euro currency, leading to the Optimum Currency Area (OCA), and the enlargement of the European Union. Here, one of indicators of a successful integration in the European Economic and Monetary Union is the BC synchronisation (Mundel, 1961), which started a huge amount of analyses of the BC synchrony in various forms.

Naturally, the measuring of globalisation has been reflected in the development of methodological tools. Standard techniques, used for analysing economic time series, such as the regression analysis, correlation, co-integration, or, vector autoregressive modelling, are still popular, in their basic or modified forms, and provide valuable results (Fidrmuc and Korhonen, 2006; Fidrmuc, Ikeda and Iwatsubo, 2012; Benčík, 2011). However, as pointed out by Aguiar-Conraria and Soares (2014), “*economic time series are an aggregation of components operating on different frequencies*”. Thus, the above mentioned methods are not flexible enough to provide an in-depth view into the cyclical character of economic data and do not capture the cyclical structure as a function of time. In the last decade, it is the co-movement wavelet analysis which has become an attractive instrument for measuring synchrony. Even though this methodology has been known in engineering for a long time, it is rather young in economics.

The wavelet methodology helps identify the character of co-movement between countries, and thus understand factors and mechanisms influencing this co-movement. As Aguiar-Conraria and Soares (2014) pointed out, contrary to the time representation of time series, wavelets map the original time series as a function of time and frequency revealing how each periodic component of the time series changes over time. As it is shown in the relevant literature, wavelets can be applied on non-stationary data; they have a very good time-frequency localization; they have flexible settings of parameters, reflecting the character of data; and they enable us to uncover unique complicated patterns over the time (Aloui, Hkhir and Nguyen, 2016; Tiwari, Mutascu and Abulescu, 2016; Berdiev and Chang, 2015; Fidrmuc, Korhonen and Poměnková, 2014; Ftiti, Tiwari and Belanés, 2014; Aguiar-Conraria and Soares, 2014; Boashas, 2016; Walnut, 2013 etc.). The methodology of wavelets also enables us to reconstruct the time-frequency transformed values back into the time domain (Boashas, 2016; Mertins, 1999).

The paper provides the description of synchrony between two time series in the time domain via a co-movement indicator. The indicator is constructed on the basis of time-frequency wavelet co-movement transform of two time series, reflecting both time and frequency character of the data. After the identification of the significant co-movement part via testing, this significant part is reconstructed (i.e. it is inversely transformed) leading to the time representation of the co-movement indicator. This time domain representation contains information about the significant frequency of the co-movement regions, which is an important advantage. Further, the methodology of its construction allows the researcher to select particular frequency regions, e.g. BC frequencies, or to construct the corresponding sub-indicators. The proposed indicator is demonstrated on the industrial production index of the US, EA and Visegrad Group of Countries (V4 Countries) between the years 1991 – 2018. The indicator thus enables us to assess how the co-movement between the US and EA, EA and V4 Countries evolves. The achieved results for the Visegrad Group of Countries and European Union are compared with the results of studies based on a different methodology.

The results confirm the globalisation of the world economy and the transfer of the 2008 crisis into the economies of European countries. Further, it is shown that the synchrony of the EA and V4 Countries is the most striking in medium BC frequencies covering the cycles of the length 1.5 – 4 years. In the case of Hungary, the synchrony is also visible in the short cycles of the length <1.5 years. The obtained co-movement (sub-)indicators can be further regressed with other economic indicators, such as bilateral trade or investment to reveal additional information about interconnections of the countries.

The paper is organized as follows: after the introduction, section 1 provides an overview of the relevant literature, section 0 deals with the methods used in analyses, section 3 describes the data and the settings of methods to be applied. Section 4 presents experiments and results. Section 5 discusses the applicability of the proposed methods on economic research. The paper ends in with the conclusion and the list of references.

## **1. Literature Review**

### **1.1. Globalisation and Measuring of Synchrony**

The following part gives an overview of papers on various methods analysing the globalization and the synchrony of countries. Fidrmuc, Korhonen and Po-měnková (2014) assessed the globalisation between China and G7 countries via wavelets. They proved that the co-movement between the countries differs with

respect to the frequency in which the co-movement is being evaluated. Berdiev and Chang (2015) analysed the synchrony between China, Japan, the US and other Asia Pacific countries via wavelet power spectrum and found out that the strength of BC synchronisation fluctuates across frequencies and over time. Another study in this field is proposed by Aloui, Hkhiri and Nguyen (2016) who used cross wavelet power spectra and wavelet coherence to show that a closer BC synchronisation can lead to a stable and effective monetary union. Tiwari, Mutascu and Abulescu (2016) used a different approach: the rolling wavelet correlation between the stock markets' index returns of the PIIGS countries with the UK and Germany at different frequencies. Their results showed that in low-frequencies the PIIGS stock markets are more correlated with Germany, while in high-frequencies with the UK. The interconnection of financial globalisation and monetary policy effectiveness was studied by Georgiadis and Mehl (2016) who empirically quantified that the financial globalisation has a net impact on monetary policy effectiveness. The relation between the Korean Republic and the European Union, which signed the trade agreement in 2010 (EU-Korea FTA), motivated Michalski (2018) to research the globalisation effect between Visegrad countries (V4: the Czech Republic, Hungary, Poland and the Slovak Republic) and the Republic of Korea. The author identified the impact of South Korean direct investments on trade. He stated that the EU-Korea FTA agreement is an important step in the process of globalisation of economies serving the interests of the most competitive economies. Further, he claimed that V4 Countries are passive players and will continue to remain so as long as they are highly dependent on the effects brought about by foreign direct investments.

Focusing on economic integration of the Central and Eastern Europe, Fidrmuc and Korhonen (2006) reviewed the literature on BC correlation among countries in this area, including V4 Countries. They found out that new member states (especially Hungary, Poland and Slovakia) are highly correlated with euro-area cycles. Furthermore, they found out that the estimated correlations were influenced by the character of individual papers, i.e. frequency of the data, its transform, methodology, country specific. Another author researching the synchrony between EA and V4 Countries was Benčík (2011) who used three time domain approaches, i.e. by calculating cross correlations, by calculating cross correlations from primary impulses, and finally by calculating output gap component correlations from common and country-specific shocks. All three approaches provided similar results, i.e. no synchronisation before 2001, positive synchronisation in 2001 – 2007 and an increasing synchronisation during economic crisis of 2008 – 2009. Further, Štiblarová and Siničáková (2017) assessed the synchrony between V4 and EA via Markov-switching approach. They complemented their results via

Hodrick-Prescott filtered data and revealed that the level of synchrony is medium-to-high. Specifically, Hungary seems to be more synchronised with EA compared to the Slovak Republic.

Another approach for analysing the synchrony via wavelets across the EU-15 and the Euro-12 countries is provided by Aquiraria-Conraria and Soares (2011). Their results for V4 Countries show a high coherency after 2005 and, in the case of Hungary and the Czech Republic, in all frequencies. Additionally, they showed that Slovakia (a country which has already joined the Euro) does not show any strong convergence with Euro-12, while the Czech Republic (a country that have not joined the Euro) does. Further, the assessment of synchrony between V4 and European Union via wavelets is proposed by Hanus and Vacha (2015). They identified an increasing synchrony of V4 after joining the European Union and a high degree of synchronisation in BC frequencies. Bekiros et al. (2015) also focused on the convergence of Eurozone BC. Contrary to the studies working with the linear correlation or causality modelling, they used cross-wavelet coherence of BC amongst the Eurozone and the broad Euro area before and after the financial crisis. They showed that the synchronization varies across different frequency bands and time horizons, and that the coherence increased during the crisis.

This paper follows the idea used in the work of Fidrmuc, Korhonen and Poňková (2014). The authors use wavelet co-spectrum to describe co-movement between two countries and then to regress it with the bilateral trade of these countries. They use the simple average of parts of the co-spectrum in a predefined frequency range, the average being represented as a time series. In this paper, we use an alternative way of how to identify this co-movement as a time series via the inverse transform. While the average can provide a biased estimate of the co-movement indicator, the inverse transform results in the contribution of all spectral components (in the predefined frequency range) to the time series. Therefore, the construction of the indicator in this alternative way respects the character of the data better than the simple average used by Fidrmuc et al. (2014). Furthermore, the use of the proposed indicator can be a very effective instrument for detailed analyses of regression between countries co-movement and other economic time series.

## 1.2. Comparing Methodological Features

We showed in the introduction that there exist several methods which can be used for measuring the synchrony. Each method has certain assumptions which enable an incomplete view on the synchrony. Generally, approaches can be divided into three groups, i.e. the time domain techniques (Fidrmuc and Korhonen, 2006; Benčík, 2011; Štiblářová and Siničáková, 2017; Feldkircher and Korhonen,

2014), the frequency domain techniques (Georgiadis and Mehl, 2016; Fidrmuc, Ikeda and Iwatsubo, 2012; Poměnková and Maršálek, 2012) and the time-frequency domain techniques (Fidrmuc, Korhonen and Poměnková, 2014; Runstler et al., 2018; Aguiar-Conraria and Soares, 2014; Aloui, Hkhiri and Nguyen, 2016 and many others).

As for the time domain techniques, the commonly used methods are correlation techniques. They can, however, lead to results dependent on the length of time series. Thus the resulting vector of correlations is shorter than the time series length, or, both the time series and the rolling correlation vector can have the same length; but then strong edge effects can occur and the estimates of synchrony have large bias at the edge points. This insufficiency was partially solved by applying other methods measuring the synchrony on the basis of grow BC dating. In this case, the results depend on the filtering and dating techniques. Some filters, such as Baxter-King, cause the shortening of the resulting grow cycles, others, e.g. Hodrick-Prescott, produce edge effects or have other limitations. Furthermore, the dating of identified BC give results according to the used methodology, e.g. Canova rules (Canova, 1999), Bry-Boschan algorithm (Bry and Boschan, 1971), Markov-Switching model (Štiblářová and Siničáková, 2017) or concordance index (Grigoras and Stanciu, 2016). None of these approaches is able to identify the frequency structure of the business cycle and measure the synchrony in all frequencies. However, these methods suffer from the fact that the data may contain several hidden periodic components as pointed out by Aguiar-Conraria and Soares (2014).

The frequency domain techniques, such as dynamic correlation can describe the dependence of two time series from the frequency point of view, but they do not provide any explanation with respect to the time. An alternative possibility is a decomposition of the time series into the set of series via a singular value decomposition or Karhunen-Loeve decomposition (Proakis et al., 2002). Then the estimate of proper frequency corresponding to the decomposed time series is difficult because economic time series are not sets of several harmonic components. That is, there is usually a range of frequencies which overlap.

To overcome the limitations described above (both the time or frequency domain methods), we can use time-frequency (TF) approach, e.g. wavelet cross-spectrum which allows the investigation of spectral character of time series with respect to time. Via TF methods, we can analyse how various cyclical components, i.e. long, medium and short cycles of a particular time series evolve through the time (Aloui, Hkhiri and Nguyen, 2016). However, edge effects should be taken into account. In the case of the wavelet transform scalogram/spectrogram, the edge-effect artefacts may potentially occur in the areas delimited by

the cone of influence (COI). The size of this area derives from the type of mother wavelet, scale/frequency resolution and length of the input time series. The edge-effect artefacts arise from the dilation of the wavelet beyond the observation interval and depend on the data character (Torrence and Compo, 1998).

## 2. Methodology

This part presents the methodological background for the algorithm leading to the construction of a co-movement indicator. The algorithm is based on the TF wavelet transform, TF co-movement measure and its significance testing (Walnut, 2013; Boashas, 2016; Mertins, 1999; Torrence and Compo, 1998).

### 2.1. Wavelet Transform

The transform of input time series from the time to the time-scale domain can be done via the Continuous Wavelet Transform (CWT)

$$W_s(a, \tau) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} s(t) \psi\left(\frac{t-\tau}{a}\right) dt, \quad a > 0, \tau \in R \quad (1)$$

where  $s(t)$  is a time series,  $\psi\left(\frac{t-\tau}{a}\right)$  is a scaled version of the mother wavelet,  $\tau$  denotes the time shift, and  $a$  denotes the scale (or frequency) (Walnut, 2013). We can also define its inverse form, i.e. Inverse Continuous Wavelet Transform (ICWT), as

$$s(t) = \text{ICWT}\{W_s(a, \tau)\} = \frac{1}{C_\psi} \int_{-\infty}^0 W_s(a, \tau) \psi\left(\frac{t-\tau}{a}\right) d\tau \frac{da}{a^2}, \quad a > 0, \tau \in R \quad (2)$$

where  $C_\psi < \infty$  comes from the admissibility condition (Boashas, 2016; Mertins, 1999).

### 2.2. Co-movement Measure

Co-movement measures widely used among economist are wavelet cross-spectrum (WCS) or coherence (Boashas, 2016). For two time series  $x(t)$  and  $y(t), t = 1, \dots, n$  the WCS measures the local covariance of these two variables in the scales and can be defined as

$$W_{xy}(a, \tau) = W_x(a, \tau) W_y^*(a, \tau) \quad (3)$$



where  $W_x(a, b)$  and  $W_y(a, b)$  are the coefficients of wavelet transform calculated according to the formula (1). The symbol "\*" denotes complex conjugation (Torrence and Compo, 1998; Aloui, Hkhirri and Nguyen, 2016). Since the WCS is complex, we can define its square absolute value, i.e. power wavelet cross-spectrum (PWCS) (Torrence and Compo, 1998; Mertins, 1999), as

$$PWCS_{xy}(a, \tau) = |W_{xy}(a, \tau)|^2 \quad (4)$$

Another possibility for measuring the co-movement in the TF domain is coherence  $C_{xy}$ , which can be defined as the PWCS normalized to the square of CWT of both time series (Boashas, 2016; Mertins, 1999). Analogously to CWT (eq. (2)), we can calculate the inverse transform for the co-movement measure PWCS denoted as IPWCS (see Mertins, 1999, Ch. 8).

### 2.3. Significance Tests

The initial source of research on the spectrogram testing is provided by Torrence and Compo (1998), followed by Ge (2008; 2013). That is, for two time series  $x(t)$ ,  $y(t)$  and its variances,  $\sigma_x^2$ ,  $\sigma_y^2$  the sampling distribution of its PWCS is

$$\begin{aligned} H_0 : \quad & \text{significant co - movement, if } \frac{|W_{xy}(a, \tau)|^2}{\sigma_x^2 \sigma_y^2} \geq \frac{1}{4} W_2 \\ H_1 : \quad & \text{insignificant co - movement, if } \frac{|W_{xy}(a, \tau)|^2}{\sigma_x^2 \sigma_y^2} < \frac{1}{4} W_2 \end{aligned} \quad (5)$$

where  $W_2$  is the distribution with the probability density function

$$f(z) = 0.5 K_0(z^{1/2}) \quad (6)$$

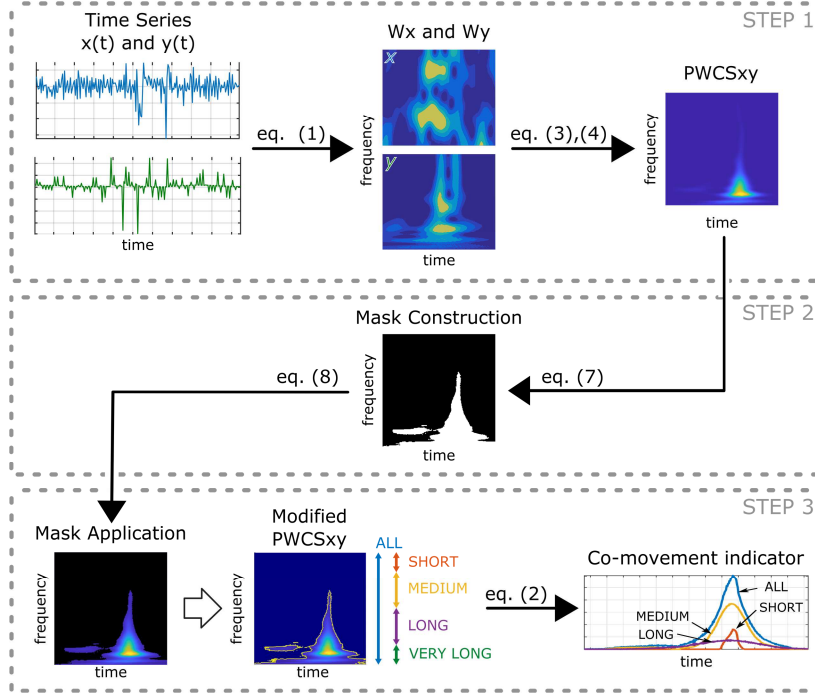
and  $K_0(z^{1/2})$  is the modified Bessel function of order zero for the complex wavelets (Wells, Anderson and Cell, 1962). The significance level  $Z(1 - \alpha)$  for the risk  $\alpha$  can be deduced from  $1 - \alpha$  percentile of the  $W_2$  distribution (Torrence and Compo, 1998; Ge, 2008; 2013).

### 2.4. Algorithm for the Co-movement Indicator Construction

In the following, we propose the diagram (Figure 1) and the description of the algorithm for the construction of the co-movement indicator. Let us have two time series  $x(t)$  and  $y(t), t = 1, \dots, n$ .

Figure 1

## The Diagram of the Co-movement Indicator Construction



Source: Own graphical visualisation of our algorithm for the co-movement indicator construction.

The construction follows these steps:

1. Firstly we estimate the co-movement between the time series according to the equation (4) (e.g. PWCS).
2. Secondly, we do significance testing of PWCS via standard chi-square test. Then we partition the co-movement measure into regions with the significant and insignificant co-movement. That is, we construct the mask

$$M(a, \tau) = \begin{cases} 1(\text{significant co - movement}) & \frac{|W_{xy}(a, \tau)|^2}{\sigma_x^2 \sigma_y^2} \geq \frac{1}{4} W_2 \\ 0(\text{insignificant co - movement}) & \frac{|W_{xy}(a, \tau)|^2}{\sigma_x^2 \sigma_y^2} < \frac{1}{4} W_2 \end{cases} \quad (7)$$

where the threshold  $thr = 0.25Z(1 - \alpha)$  is given by Torrence and Compo (1998).

3. Thirdly, we create a modified PWCS (MPWCS) which contains only the significant part of the PWCS obtained by multiplying PWCS by the mask, i.e.

$$MPWCS(a, \tau) = |W_{xy}(a, \tau)|^2 M(a, \tau) \quad (8)$$

Consequently, we inversely transform this product into the time domain, i.e. we construct the time representation of the co-movement indicator. Or, we inversely transform the pre-defined frequency region (e.g. a subpart of MPWCS in BC frequencies) of this product into the time domain to construct the time representation of the co-movement sub-indicator corresponding to the pre-defined frequency region.

### 3. Data Description

The data set consists of the seasonally adjusted monthly data of industrial production index (IPI) from the OECD database for the US, EA19 (EA) and Visegrad Group of countries (V4), i.e. the Czech Republic (CZ), Hungary (HU), Poland (PL) and the Slovak Republic (SK), between years 1991/M1 – 2018/M5.

#### 3.1. Parameter Setting

As we examine TF selective filtering based on the co-movement between the growth cycles of selected countries, we transform the data by the first order difference (FOD) of the natural logarithmic values. For the time-series co-movement via PWCS calculation, we selected the complex Morlet with the centre frequency  $f_c = 1.5$  and the bandwidth parameter  $f_b = 1$  as the mother wavelet. The setting of the wavelet was motivated by the optimal joint time-frequency concentration and by the common use in economic literature (Aguilar-Conraria and Soares, 2011; Aloui, Hkhirri and Nguyen, 2016; Berdiev and Chang, 2015).

The scales were set to correspond to the equivalent Fourier period (i.e. pseudo-frequencies) of the range of 1 year to 10 years, with 257 individual scales. The conversion of scales to pseudo-frequencies is based on the association of cosine wave of a known frequency with the given wavelet. For further details, see Torrence and Compo (1998). We leave out the COI displaying because, in our case, the pre-filtering using FOD suppresses the edge-effect artefacts (Poměnková, Klejmová and Kučerová, 2019).

The co-movement indicator was constructed for all frequencies (all scales) leading to the time representation of the total co-movement indicator. We also construct the time representation of the co-movement sub-indicators corresponding to the frequency sub-ranges. The division of the frequencies recalculated to the length of cycles was set according to the study of Fidrmuc, Korhonen and Poměnková (2014) and is the following: the short-run cycles (SC) of duration <1.5 years, the medium BC of duration 1.5 – 4 years, the long BC of duration 4 – 8 years and the long-run cycles (LC) of duration >8 years.

## 4. Results

The application was done on the data described in the previous part according to the algorithm described in the methodology (section 2.4). Firstly, we demonstrate the proposed algorithm on the case of co-movement between the US and the EA. Then, we apply this algorithm on the co-movement of the EA and V4 Countries and present the resulting time domain figures for the total co-movement indicator as well as for the sub-indicators corresponding to the pre-defined frequency regions.

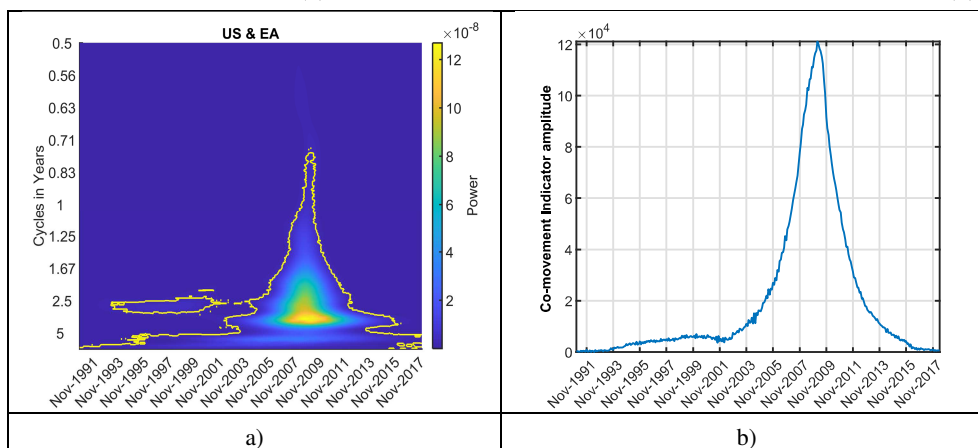
### 4.1. Demonstration of the Co-movement Indicator Construction

The proposed algorithm demonstrated on the case of the US and EA co-movement follows the steps given in section 2.4. Firstly, we estimate PWCS (Figure 2a) of both countries.

Secondly, we construct the mask on the basis of Torrence and Compo (1998) significance testing of PWCS, i.e. we identify significant and insignificant parts of PWCS. Then, we multiply the PWCS (normed by the variances of the US and EA time series) by the mask to obtain time- -frequency co-movement representation. Thirdly, we calculate the inverse transform of the normed PWCS which is an analogy to CWT, and we obtain the time representation of the total co-movement indicator in the time domain (Figure 2b).

Figure 2

PWCS of the US and EA (a) and the Total Co-movement Indicator of the US and EA (b)



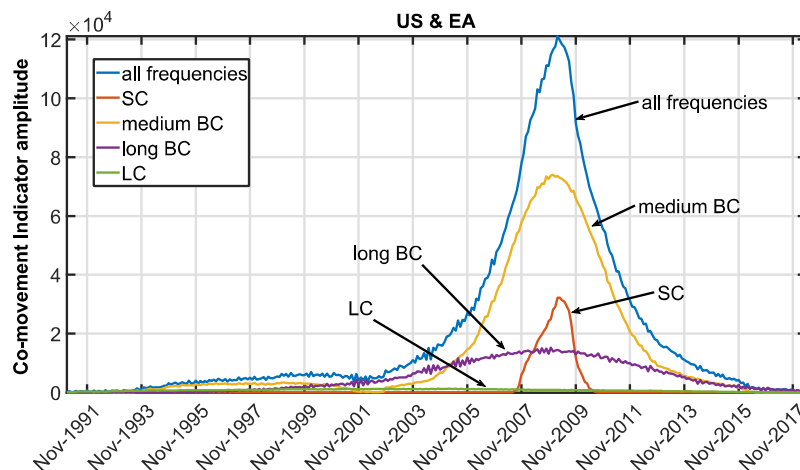
Notes: In the PWCS figure there are: x-axis: time, y-axis: cycles in years, z-axis: values of spectrogram. The figure shows a two-dimensional projection of three-dimensional charts. The intensity of each contour represents the relative importance of different cycles and time. The yellow curves in the figure a) indicate the significant area of PWCS. In the figure b) the co-movement indicator is calculated for all time and frequencies.

Source: Own calculation and visualisation.

Further, for a detailed look into the co-movement indicator behaviour, we split the masked normed PWCS into the frequency sub-regions according to the description given in section 3.1. Similarly, as in the case of total co-movement indicator, we calculate the inverse transform of a frequency sub-region of the masked normed PWCS which leads to the time domain representation of the sub-indicators and corresponds to the pre-defined frequency regions (Figure 3). Note that these sub-indicators are additive and share the same magnitude. Their sum is equal to the total co-movement indicator.

Figure 3

**The Co-movement Indicator for the US and EA in the Frequency Regions: a) the Short-run Cycles of Duration <1.5 years; b) the Medium BC of Duration 1.5 – 4 Years; c) the Long BC of Duration 4 – 8 Years; d) the Long-run Cycles of Duration >8 Years; e) All Frequencies (the total co-movement indicator)**



Source: Own calculation and visualisation.

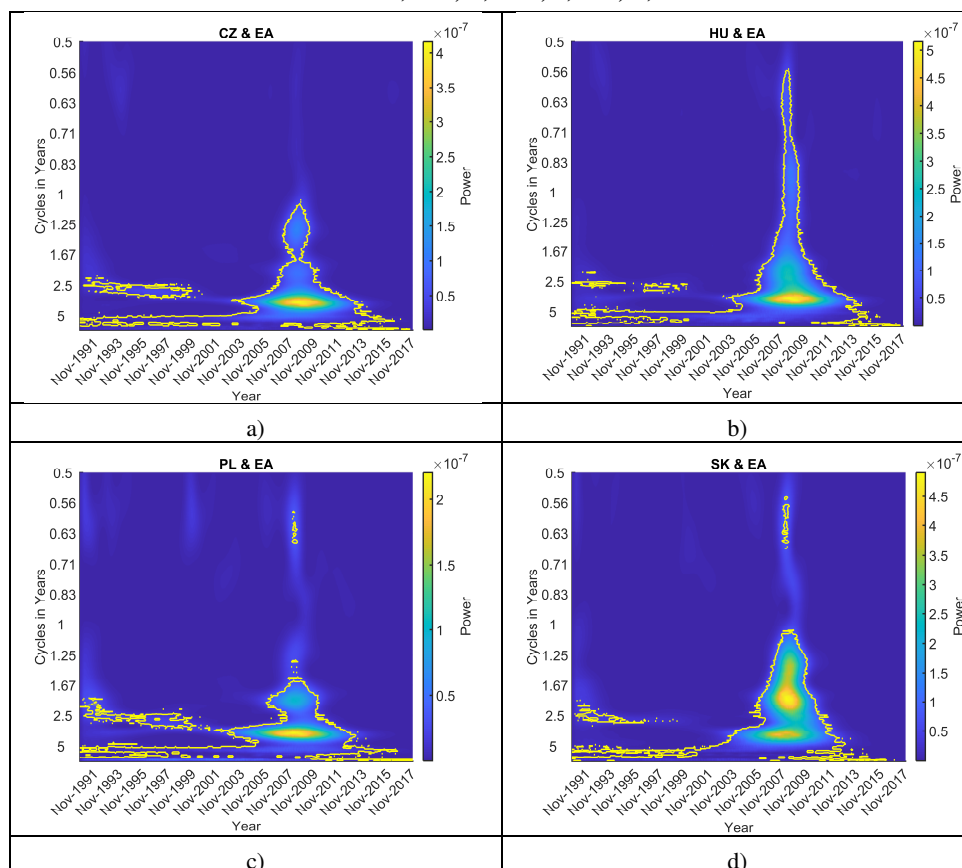
The first overview of the PWCS (Figure 2a) suggests that the most significant event in both countries is the global economic crisis in 2008. Therefore, the shape of the total co-movement indicator (Figure 2b), i.e. the full range of frequencies, is not surprising. We can see an increase of the synchrony in 2002 – 2008, with a peak in 2008, followed by a decrease of the synchrony in 2008 – 2014. The inflexion points determine the period 2006 – 2011 of the concave indicator shape. In this interval, we can find the most energy of the PWCS and thus the strongest co-movement of both countries.

When we focus on the sub-indicators in Figure 3 in the pre-defined frequency sub-ranges, we can see a similarity of the medium BC co-movement indicator shape with the total co-movement indicator shape (i.e. the curve corresponding to all frequencies in Figure 3). Based on this we conclude that in the BC frequencies

of the duration 1.5 – 4 years the economies are the most synchronised. The other sub-indicators, i.e. SC, long BC and LC frequencies, contribute to the overall synchronisation less because in the PWCS there are no other significant areas in the frequency and the time (see PWCS, Figure 2a). The long BC sub-indicator summed with the medium BC indicator cause the total indicator enlargement, while the SC sub-indicator causes the augmentation of the peak in 2008. The LC indicator has the lowest contribution to the total co-movement indicator. In both countries, the EA and the US, the global crisis in 2008 significantly influenced developments in the countries and demonstrated the interdependence, and thus globalization of economies.

Figure 4

PWCS of EA and V4 Countries: a) CZ; b) HU; c) PL; d) SK



Notes: The x-axis represents the time, the y-axis the cycles in years and the z-axis the values of spectrogram. The figures show a two-dimensional projection of three-dimensional charts. The intensity of each contour represents the relative importance of the different periodicities and time. The yellow curves in all figures indicate a significant area of PWCS.

Source: Own calculation and visualisation.

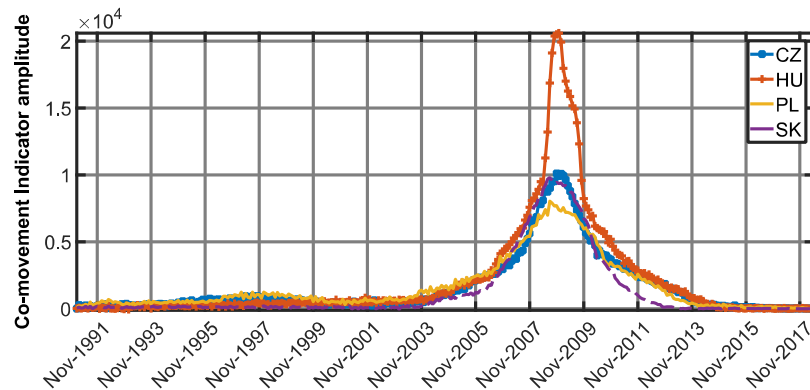
#### 4.2. Application on the EA and V4 Countries

Similarly to the demonstrated case, in the case of the EA and Visegrad countries, first, we look at the PWCS figures (Figure 4a – d). In all four figures, we can identify the most significant co-movement during the global financial crisis, more or less extended across the frequencies, depending on the country. In the case of the EA and the Czech Republic, the Slovak Republic and Poland the significant co-movement prevails in the LC, long BC and medium BC. In the case of the EA and Hungary, the significant co-movement is visible in all frequencies in the 2008 crisis. All these facts are reflected in the co-movement indicator as well as in the sub-indicators.

The total co-movement indicators of the EA and all V4 Countries for all frequencies is shown in Figure 5. We can see a proximity of the co-movement of the Czech Republic, Poland and the Slovak Republic with the EA during the time range; in the case of Poland the indicator achieved a little bit lower value of the peak in the 2008 crisis. In the case of Hungary, the indicator has a similar shape, except for the shape in the time 2007 – 2009 when a significantly higher level of peak was achieved. This fact is explained by the SC co-movement sub-indicator (see the paragraph below).

Figure 5

The Total Co-movement Indicators for V4 Countries and the EA (all frequencies)



Source: Own calculation and visualisation.

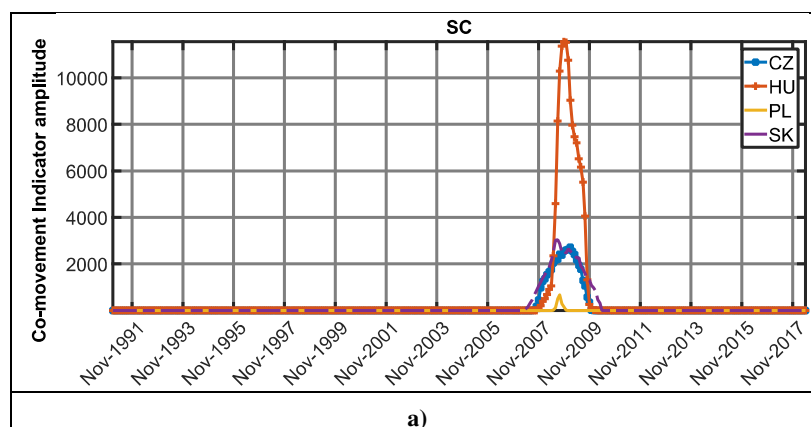
Further, we focus on the sub-indicators. In the case of SC co-movement sub-indicator (Figure 6a) we can see a proximity of the Czech Republic and the Slovak Republic in the shape of the curve. In the case of Poland, the SC sub-indicator does not contribute much to the overall country co-movement, i.e. the EA and Poland do not have co-movement in these frequencies. The opposite

situation occurred for Hungary where the co-movement in the SC plays an important role. As we can see in the PWCS figure (Figure 4), the co-movement with the EA covers all frequencies during the crisis time, which is reflected in the SC sub-indicator.

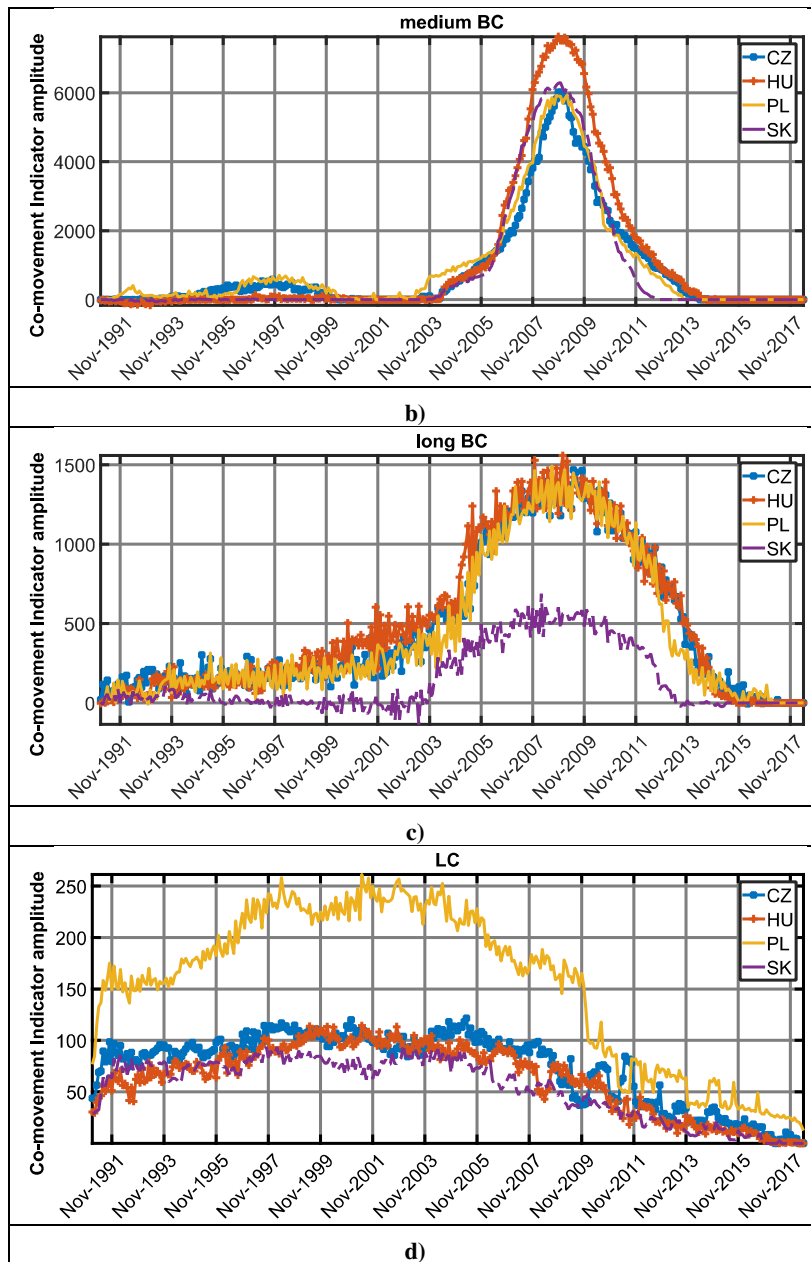
In the medium BC frequencies, similarly to the US and EA case, the Visegrad countries and the EA are the most synchronised and the medium BC co-movement sub-indicator (Figure 6b) contributes most to the total co-movement indicator (with respect to the amplitude of the indicator). Therefore, we can expect a similar reaction of countries to the situation in the EA which primarily reflected the situation in the US. In the case of Hungary we can see a higher position of the peak in 2008. The inflexion points of all medium BC co-movement sub-indicators are determined by the period 2006 – 2010. In this interval there is the highest amplitude (i.e. the most energy) of the PWCSs.

The long BC co-movement sub-indicators (Figure 6c) between the EA and V4 Countries have a similar shape for the Czech Republic, Poland and Hungary. The co-movement sub-indicator for cycles longer than 8 years (Figure 6d), i.e. LC sub-indicators, contributes less to the total co-movement indicator in all countries, and thus to the co-movement of the country and the EA. In the case of the Czech Republic (with respect to the indicator amplitude) we can state that the long BC sub-indicator has the same contribution to the total indicator as the SC sub-indicator.

Figure 6  
Co-movement Indicators for the US and EA in the Frequency Regions: a) the Short-run Cycles of Duration <1.5 Years; b) the Medium BC of Duration 1.5 – 4 Years; c) the Long BC of Duration 4 – 8 years; d) the Long-run Cycles of Duration >8 Years







Source: Own calculation and visualisation.

### 4.3. Discussion on the Achieved Results for V4 Countries and EU Synchrony

Our results confirm the globalisation between economies of the US and EA. Both economies are synchronised mainly in the medium BC frequencies, i.e. the cycles of the length 1.5 – 4 years. This fact, and the interconnection of both

economies, support the increase of the countries co-movement in the area of  $\pm 6$  years around the global economic crisis in 2008.

Focusing on the V4 Countries and EU, we found out that the synchrony of V4 Countries is the most significant in the medium BC frequencies (i.e. the cycles of the length 1.5 – 4 years). Further, this synchrony increased after 2004 (i.e. after joining the European Union) and achieved its peak during the global-financial crisis between 2008 – 2009. This demonstrates that the globalisation of economies was transferred into small open economies such as the V4 Countries. We also identified certain specifics for Hungary, i.e. a similar significance of the synchrony for the short cycles of the length  $< 1.5$  years. In the case of Slovakia, we identified a low level of synchrony in the long business cycles (i.e. the cycles of the length 4 – 8 years) compared to the other V4 Countries; Hungary, Poland and the Czech Republic achieved similar levels. Since we work with grow business cycles (FOD transform of the data removing long-term trend as it is common in analysis of BC synchrony), the cycles of the length  $> 8$  years do not contribute to the synchrony of countries in the same way as the long BC, medium BC or short cycles. That is, we do not investigate the long-term trend co-movement of the countries.

Focusing on the studies which do not use the TF approach, we can conclude the following. Our results are close to those presented by Fidrmuc and Korhonen (2006), i.e. V4 Countries synchrony was stimulated by the EU entrance in 2004. Comparing our results with Štiblářová and Siničáková (2017), we are in agreement that Hungary seems to be more synchronised than the rest of the V4 group. Our detailed TF analysis additionally revealed that this difference can be caused by a higher level of synchrony in the short-run cycles. Further, our results are the same as shown in the study of Benčík (2011). He concluded that V4 Countries were not synchronised with the EU before 2001, but were positively synchronised in 2001 – 2007. He also identified an increasing synchronisation during the economic crisis of 2008 – 2009.

As for the studies based on the wavelet analysis, our approach provided results confirming the findings mentioned in Literature Review.

## **5. Further Use of the Co-movement (Sub-)indicator**

As mentioned in the introduction, the co-movement analysis continues to be a promising method which is employed by a number of economic analysts. If we focus on the Central and Eastern European countries (CEECs), where V4 Countries belong, we can mention the work of Fidrmuc and Korhonen (2006) who provided a meta-analysis of BC correlation between CEECs. They showed that

many papers proved that a driving force for the analysis of synchronisation and globalization of the countries was the OCA and the satisfaction of Maastricht criteria. Their paper concluded that CEECs countries, including V4 Countries, are highly correlated with the EA cycles. The results presented in this paper confirm the previous findings and extend them via constructing co-movement indicator and sub-indicators.

Specifically, the methodology proposed in this paper can directly expand the findings of Rana, Cheng and Chia (2012) who focused on the analysis of relationships between trade intensities and the synchronisation of BC in East Asia and Europe. They found out that the major factor in the regions co-movement is intra-industry trade. Our methodology can identify which areas of the cyclical movements of intra-industry trade and BC are the most important.

Similarly, the results of this paper can improve the work of Fidrmuc, Korhonen and Poměnková (2014) by modifying the calculation of the depending factor. The authors use the simple average of parts of the co-spectrum in a predefined frequency range, the average being represented as a time series. We use an alternative way of how to identify this co-movement as a time series via the inverse transform. While the average can provide a biased estimate of the co-movement indicator, the inverse transform results in the contribution of all spectral components (in the predefined frequency range) to the time series. Therefore, the construction of the indicator in this alternative way respects the character of the data better than the simple average.

Further, Štiblářová and Siničáková (2017) analysed the synchrony of V4 Countries with the Euro Area via detecting turning points and the calculation of concordance index, i.e. via measuring the coincidence of the BC phases in the time domain. The authors found out the medium-to-high level of synchronisation with some differences in an amplitude and evolution in time. In the case of Hungary the authors found out that Hungary is more synchronised with the EA compared to the Slovak Republic. The authors also analysed the disaggregated data of industrial production to obtain a more precise look at the co-movement of the countries. Considering this conclusion, the methodology proposed in our paper can be a plausible instrument which can reveal the coincidence of BC phases in specific frequency ranges.

Finally, an interesting study on the impact of globalisation on V4 Countries was presented by Michalski (2018). He analysed the interconnection between the Republic of Korea (ROK) and V4 Countries via selected short- and mid-term effects in trade in goods on the basic statistics. The author identified the impact of South Korean direct investments on trade deficit of the V4 with ROK, and the structure of bilateral trade.

This conclusion can be supported by the proposed co-movement indicator analysis used in the regression with the countries bilateral trades or other indicators, such as investments.

As we can see, our proposed methodology can bring additional valuable insights into econometric analyses as it can reveal additional information supporting economic hypotheses about the globalisation of economics in the world or in a regional context.

## Conclusion

The presented paper focused on the description of the synchrony between countries via a co-movement (sub-)indicator. It proposed a new method of constructing the indicator: designed as the reconstruction of the significant power wavelet co-spectrum from the time-frequency to the time domain. There are two advantages of the indicator: i) contrary to the previously used time domain metrics, the co-movement indicator is a vector represented in the time domain without any loss of observations; ii) the methodology of co-movement indicator allows the construction of the total co-movement indicator as well as the construction of the sub-indicator which corresponds to the predefined frequency range (e.g. BC frequencies). Further, based on the second advantage, we can divide PWCS into frequency sub-regions and then construct sub-indicators if this is in the scope of research.

The proposed methodology is demonstrated on the case of the US and EA. We found out that both subjects are synchronised mainly in medium BC frequencies covering cycles of the length 1.5 – 4 years. Further, we could see an increase of the countries co-movement in the area of  $\pm 6$  years around the global economic crisis 2008. Further, we measured the synchrony between the EA and Visegrad Group of Countries. Similarly, as in the case between the US and EA, the synchrony of EA and Visegrad Group of Countries is the most important in medium BC frequencies. Only in the case of Hungary, we additionally identified the same importance of short cycles of the length of less than 1.5 years. That is, the crisis in Hungary was reflected in a wider range of frequencies than in the Czech Republic, Poland and the Slovak Republic. The results confirm the fact of globalisation of economies and the transfer of important shocks, such as the 2008 global economic crisis into European countries.

The obtained co-movement (sub-)indicators can be further used in regression analyses for the research of the relation of economic indicators, such as bilateral trade or investment. They can reveal more information about economic interconnections and influencing factors.

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