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Relative Efficiency of Component GARCH-EVT Approach in Managing Intraday Market Risk

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The purpose of this study is to estimate intraday Value-at-Risk (*VaR*) and Expected Shortfall (*ES*) of high frequency stock price indices taken from select markets of the world. The stylized properties indicate that the return series exhibit skewed and leptokurtic distributions, volatility clustering, periodicity of volatility and long memory process in volatility, all of which together suggest the usage of Component GARCH- EVT combined approach on periodicity adjusted return series to forecast accurate intraday *VaR* and *ES*. Hence the study estimates intraday *VaR* and *ES* using Component GARCH-EVT combined approach with different innovation distributions such as normal, student-*t* and skewed student-*t* and compares its relative accuracy with the benchmark GARCH-EVT model with different distributions. The Component GARCH-EVT models in general perform better than GARCH-EVT models and the model with skewed student-*t* innovations forecasts more accurately. The study is useful for market participants involved in frequent intraday trading in such markets. (JEL: G10, G15, G17, G19)

Keywords: deseasonalized; intraday; value at risk; expected shortfall; component GARCH; EVT

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I. Introduction

In the last two decades, Value-at-Risk (*VaR*) has become a widely used tool in risk management of financial institutions and regulators. A *VaR* model measures market risk by determining how much the value of a portfolio could decline with $\alpha\%$ probability over a certain time horizon τ, as a result of changes in market prices or rates. Another related and useful measure of risk is the Expected Shortfall (*ES*) which is defined as the expected size of a loss that exceeds *VaR*. Where *VaR* addresses the question: "How bad can things get?", the *ES* addresses the question: "If things go bad, what is the expected loss?"

Much effort has been made on developing increasingly sophisticated risk models of *VaR* type for daily data and/or longer horizon, but the issue of intraday market risk measurement has been less explored. With increased access to intraday financial data bases and advanced computing power, it has now become possible to address the question of how to define practical risk measures for investors or market makers on an intraday basis. For active market participants such as high-frequency traders, day traders or market makers, trading risk should be evaluated on shorter-than-daily intervals since the horizon of their investments is generally less than a day. For example, day traders may liquidate any open positions at closing, in order to preempt any adverse overnight moves resulting in large gap openings. Brokers must also be able to calculate trading limits as fast as clients place their orders. Significant intraday variations in asset prices affect the margins a client has to deposit with a clearing firm, and this should be taken into consideration while designing an appropriate model to estimate the margins. Often banks also use intraday risk analysis for internal control of their trading desk.

Sometimes high frequency traders are engaged in algorithm trading which often causes high intraday price oscillations. Hence, regulators worldwide are currently discussing whether intraday / high-frequency trading should require regulatory intervention. Intraday / high-frequency trading can be regulated by imposing capital requirements. Such capital requirements would be based on intraday risk measures such as intraday *VaR* and *ES*. Hence, to address the need for intraday risk management for regulators, high frequency traders and market makers operating on an intraday basis, the study proposes a method of computing intraday *VaR* and *ES* using high frequency data.

The most commonly used *VaR* models assume that the probability distribution of the daily / intraday financial asset return is normal, an assumption that is far from reality. Many of the asset returns exhibit significant amount of excess kurtosis. This means that the probability distributions of these returns have “fat tails” so that extreme outcomes happen much more frequently than that would be predicted by the normal distribution assumption. Thus the present study applies extreme value theory (EVT) to calculate intraday *VaR* and *ES* that allows the user to choose more generalized fat tailed distributions for the high frequency stock market returns.

Classical EVT approach assumes that the observations are independent and identically distributed (*iid*), which is far from reality. Hence, following the two stage approach of McNeil and Frey (2000), the study first uses a generalized autoregressive conditional heteroscedasticity (GARCH) type model in stage one with a view to filtering the return series to obtain (nearly) *iid* residuals and then in stage two, it applies the EVT framework to the standardized residuals. This GARCH-EVT combined approach is termed as conditional EVT model. The advantage of this GARCH-EVT combination lies in its ability to capture conditional heteroscedasticity in the data through the GARCH framework, while at the same time modelling the extreme tail behaviour through the EVT method.

Following McNeil and Frey (2000), many researchers use GARCH-EVT approach along with other traditional models on different data sets and find that this approach performs better than other competing models for *VaR* estimations (Bali and Neftci, 2003; Bystrom, 2004; Fernandez, 2005; Cotter, 2007; Ghorbel and Trabelsi, 2008; Marimoutou et al., 2009; and Karmakar and Shukla, 2015). In all their studies, the GARCH-EVT model is applied on daily data to forecast daily *VaR*. There are few studies which have used the sophisticated GARCH-EVT model on high frequency data and observed that the sophisticated model performs better than traditional model in forecasting intraday *VaR* (Ergun and Jun, 2010, and Chavez-Demoulin and McGill, 2012). While there are voluminous studies on *VaR* forecasting based on GARCH as well as sophisticated EVT models, the literature on *ES* forecasting is limited to a few studies (Embrechts et al., 2005; Watanabe, 2012; and Karmakar and Paul, 2016).

While estimating *VaR* and *ES* based on the GARCH-EVT approach, researchers in the above studies at times refine the mean and variance equations of the flexible GARCH model to incorporate the empirical stylized pattern in volatility. Some regularity in financial series, however, seems difficult to reproduce with a mere refinement of the mean and variance equations of the GARCH model. One such regularity in return series is long memory behaviour of volatility which has attracted much attention of the researchers to estimate volatility accurately. To capture the long memory property, Engle and Lee (1999) introduce a Component GARCH model where the conditional variance is decomposed into a permanent component or trend and a transitory component. The model allows the classical GARCH (single component) as a particular case and thus provides an encompassing framework to investigate the necessity of introducing two components in the volatility process. Using two stock indices, namely the S&P 500 and the NIKKEI 225, Engle and Lee (1999) show the superiority of this new specification over the classical GARCH model and their findings are well supported in subsequent studies for a number of stock indices (Tauchen, 2001; Deo et al., 2006; Christoffersen et al., 2008).

Since accurate estimation of volatility is one of the essential exercises in *VaR* and *ES* prediction and given the superiority of Component GARCH model in estimating volatility, Component GARCH-EVT combination has been used here for predicting *VaR* and *ES*. To the best of our knowledge, the effectiveness of long memory Component GARCH-EVT framework in forecasting *VaR* and *ES* has not yet been investigated. This study computes intraday *VaR* and *ES* using high frequency data from three different stock market environments across India, the UK and the US applying Component GARCH-EVT combination so that both long memory behavior and occurrence of extreme events in intraday financial data series can be well accommodated. Further, it also computes intraday *VaR* and *ES* on the same data set following simple GARCH-EVT model as a benchmark model. Then the accuracy of the proposed Component GARCH-EVT approach for intraday *VaR* and *ES* estimation with the benchmark GARCH-EVT model has been compared.

The rest of the paper is organised as follows. Section II presents a brief overview of EVT, describes the estimation of *VaR* and *ES* and then explains McNeil and Frey’s (2000) two stage approach called Conditional EVT to estimate and forecast dynamic intraday *VaR* and *ES*. Section III focuses on the data used in the study. Section IV presents the empirical findings. Finally, section V concludes the study.

II. Modelling the tails of stock return distributions

In the following subsections, the study provides an overview of theoretical framework of EVT, describe *VaR* and *ES* and explain how conditional EVT is applied to *VaR* and *ES*.

A. Extreme Value Theory

EVT focuses on statistical modelling of extreme observations of a random variable. It computes tail related measures considering rare and extreme events. There are two popular methods of determining extremes in data over a certain time horizon. The first approach, named as block maxima (BM), divides the time horizon into certain blocks or periods and takes into account the maximum of each block or period.

However, this method is not appropriate for financial time series since financial returns exhibit volatility clustering. As BM method considers only maximum return in each period, a considerable number of relevant data points may be excluded from the analysis. The second approach, known as the peak over threshold (POT) method, utilizes data more efficiently above a given threshold. Therefore, the POT method has become the method of choice in financial applications. Since the study is dealing with financial returns, the POT method has been used.

The study considers here a sequence of *n iid* random variables $X(x_1, x_2, \dots x_n)$ that represents the residuals of the intraday return series. The excess distribution F(*x*), which is the probability that *X* exceeds a fixed threshold *u*, can be estimated using a generalized Parato distribution (GPD) fitted by the maximum likelihood method. The tail estimator is as follows:

$$F(x) = 1 - \frac{k}{n}\left[1 + \xi\frac{(x-u)}{\Psi}\right]^{-\frac{1}{\xi}}, \text{ for } X > u \tag{1}$$

where ξ is the shape parameter, Ψ is the scale parameter, *n* is the total number of observations, and *k* is the number of observations above the threshold *u*. For a given probability, $q > F(u)$, the tail quantile can be obtained by inverting the tail estimation formula above to get (See, Embrechts et al., 1997)

$$x_q = u + \frac{\psi}{\xi}\left[\left(\frac{1-q}{k/n}\right)^{-\xi} - 1\right] \tag{2}$$

B. Estimation of VaR and ES

As referred in the introduction, two important measures of market risk are the *VaR*, and the *ES* which are mathematically defined as follows: Suppose a random variable X with continuous distribution function F models the return distribution of a risky financial portfolio over the specified time horizon. For a given probability q, *VaR* can be defined as the qth quantile of the distribution F

$$VaR_q = F^{-1}(1-q) \tag{3}$$

where F^{-1} is the so-called quantile function defined as the inverse of the distribution function F. As *VaR* is exactly the same extreme quantile defined earlier by Eq. (2), it can be estimated by

$$V\hat{a}R_q = x_q = u + \frac{\psi}{\xi}\left[\left(\frac{1-q}{k/n}\right)^{-\xi} - 1\right] \tag{4}$$

The *ES* for risk X at given probability level q is formally defined as

$$ES_q = E\left(X \mid X > VaR_q\right) \tag{5}$$

The *ES* is estimated by the following equation

$$ES_q = \frac{VaR_q}{1-\xi} + \frac{\psi - \xi u}{1-\xi} \tag{6}$$

C. Conditional EVT applied to VaR and ES

As mentioned earlier the EVT approach described above cannot be applied directly to the return series which are non-*iid*. Hence following the two stage approach of McNeil and Frey (2000), the study first uses a GARCH type model to filter the return series and then it applies the EVT to the GARCH residuals. This approach is known as Conditional EVT.

The Conditional EVT approach is implemented as follows:

1. Fit any GARCH model to the return data by quasi-maximum likelihood. That is, maximize the log-likelihood function of the sample

assuming a distribution of innovations.

2. Consider the standardized residuals computed in Stage 1 to be realizations of a white noise process, and estimate the tails of innovations using EVT. Next, compute the quantiles of innovations for different values of q.

It has been assumed that the dynamics of conditional mean returns can be represented by the following AR(1) model,

$$r_1 = a_0 + a_1 r_{t-1} + \varepsilon_t$$
$$= \mu_t + \varepsilon_t = \mu_t + \sqrt{h_t} Z_t \quad (7)$$

where $\mu_t = a_0 + a_1 r_{t-1}, a_0, a_1$ are parameters, r_{t-1} are lagged returns, ε_t are residuals or the innovations of the process, z_t is the standardized residual which is defined by $\varepsilon_t / \sqrt{h_t}$ and h_t is conditional variance of ε_t. The study also assumes that the conditional variance h_t follows two different GARCH models: one is the benchmark GARCH(1, 1) model which is a popular way of modelling volatility and the other is the long memory Component GARCH (1,1) process as an alternative to the benchmark GARCH (1,1) model. Both the models are briefly explained below:

GARCH (1,1) model

Bollerslev (1986) proposes a generalized autoregressive conditional heteroscedasticity, GARCH (1,1) model:

$$h_t = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 h_{t-1} \quad (8)$$

where $\omega > 0$, $\alpha_1 \geq 0$ and $\beta_1 \geq 0$. The GARCH (1,1) process defined above is stationary when $\alpha_1 + \beta_1 < 1$.

Component GARCH (1,1) model

The above GARCH model comprises only one component which cannot capture the long term dependence structure in the volatility of the financial time series. In order to capture such volatility persistence, Engle and Lee (1999) re-write the GARCH (1,1) model in Eq. (8) as –

$$h_t = \sigma^2 + \alpha_1 \left(\varepsilon_{t-1}^2 - \sigma^2 \right) + \beta_1 \left(h_{t-1} - \sigma^2 \right) \quad (9)$$

where it is assumed that $\alpha_1 + \beta_1 < 1$ so that the model becomes stationary and the multi-step forecasts get converged upon the unconditional variance, $\sigma^2 = \omega / (1 - \alpha_1 - \beta_1)$.

The Component GARCH model then extends the GARCH model by allowing the long-run volatility to be time-varying

$$\begin{aligned} h_1 &= q_t + \alpha_1 \left(\varepsilon_{t-1}^2 - q_{t-1} \right) + \beta_1 \left(h_{t-1} - q_{t-1} \right) \\ q_t &= \omega + \rho q_{t-1} + \varphi \left(\varepsilon_{t-1}^2 - h_{t-1} \right) \end{aligned} \tag{10}$$

In this specification, two autoregressive components contribute separately to the overall conditional variance at time *t*. While, one component arrests the long-run effect of an innovation, the second component captures the short-run transitory impact of a variance innovation. Here the long-run effect is described by ρ and the short-run transitory impact is governed by $\alpha_1 + \beta_1$.

The original GARCH model assumes the innovations ε_t to be Gaussian. However, the assumption turns out to be inadequate as the empirical evidence rejects the idea that financial returns are normally distributed. Hence its replacement with a fat-tailed, possibly skewed distribution was a natural and quite effective extension. Thus while fitting GARCH (1, 1) and Component GARCH (1, 1) defined above, three different assumptions for the innovation distributions, ε_t, in Eq. (7): the normal; the Student *t* (in short, *t*); and the generalized asymmetric *t* (in short, skewed *t*) have been utilized.

Standardized residuals or innovations can now be computed for all three distributions separately, as

$$Z_t = \frac{r_t - \mu_t}{\sqrt{h_t}} \tag{11}$$

If the standardized residuals are *iid* and the fitted model is well-specified, stage 1 is ended by estimating the conditional mean (μ_{t+1}) and variance (h_{t+1}) for interval *t*+1 by using standard 1-step ahead forecasts.

In stage 2, the study applies the EVT tools kit to the standardized residuals (Z_t) and estimate the $V\hat{a}R_q$ and ES_q quantiles defined by Eqs. (4) and (6), respectively. An estimate of the Conditional *VaR* is

$$VaR_q^{t+1} = \hat{\mu}_{t+1} + \sqrt{\hat{h}_{t+1}} V\hat{a}R_q, \tag{12}$$

and an estimate of the Conditional *ES* is

$$ES_q^{t+1} = \hat{\mu}_{t+1} + \sqrt{\hat{h}_{t+1}} ES_q, \tag{13}$$

where $\hat{\mu}_{t+1}$ is the one-step ahead conditional mean and $\hat{h}_{t+1}$ is the one-step ahead conditional variance, and $V\hat{a}R_q$ and ES_q are given by Eq. (4) and (6) respectively, applied to the negative standardized residuals.

III. Data, properties and the stylized facts

The data set used in this study includes three high frequency stock price indices, one each taken from India, the UK and the US. The names of respective indices along with their corresponding short forms are reported against the three countries in first three columns of table 1. The price indices data have been extracted from Bloomberg system for a period of approximately four and half years from January 01, 2010 to June 10, 2014 at 5 minute interval. It is noted that stock trading hours vary from country to country. The specific trading periods during which the price records are available are reported for different countries in column 4 of table 1.

Following Andersen and Bollerslev (1998), the study chooses the 5-minute frequency data of each of the three indices to avoid microstructure error of the market. For each series it obtains 5 minute continuously compounded returns for each interval *n* on day *t* by $r_{t,n} = \log\left(P_{t,n} / P_{t,n-1}\right)$, where $P_{t,n}$ is the closing price for interval *n* on day *t* and $P_{t,\,n-1}$ is the opening price for interval *n* on day *t*. The last three columns of table 1 respectively, show number of sample observations in the return series per day, number of trading days and total number of sample observations finally after removing certain figures from the raw price set for each country. Let us see how the study has arrived at the final number of observations for India. Consistent with the literature, overnight return is excluded. Again consistent with the literature, days which do not have 76 five-minute intervals are also excluded, which finally leaves us with $r_{t,n}$ where $t = 1, \ldots, 1097$, $n = 1, \ldots, 76$, for a total of 83,372 observations for India. Following the same procedure the study has finalized the numbers of sample observations in the return series for the rest of the two countries.

TABLE 1. Name of countries and the corresponding indices along with other relevant information

Country	Name of Indices	Short forms of Indices	Trading period (local time) during which price points are available	Obs./Day	No. of Days	Total Obs.
India	S&P CNX Nifty Index	Nifty	9:10 hrs – 15:30 hrs	76	1,097	83,372
UK	FTSE 100 Index	FTSE	8:00 hrs – 16:30 hrs	102	1,098	111,996
US	S&P 500 Index	SPX	9:30 hrs – 16:00 hrs	78	1,107	86,346

Note: The last three columns of table 1 respectively, show number of sample observations in the return series per day, number of trading days and total number of sample observations finally after removing certain figures from the raw price set for each country. Let us see how the study has arrived at the final number of observations for India. Consistent with the literature, overnight return is excluded. Again consistent with the literature, days which do not have 76 five-minute intervals are also excluded, which finally leaves us with $r_{t,n}$ $t = 1, \ldots, 1097$, $n = 1, \ldots, 76$, for a total of 83,372 observations for India. Following the same procedure the final numbers of sample observations in the return series for the rest of the other countries have been computed.

Table 2 presents the summary statistics of the distribution of returns for each series. The mean values of the series are all approximately zero. The sample skewness, in all three countries, is negative which suggests that the negative shocks are more frequent than the positive ones. The excess-kurtosis estimate is very high in all data series which means that return distributions are leptokurtic, with much heavier tails than the normal distribution. The non-normality of the distribution is also confirmed by the high Jarque-Bera statistics. On the basis of Ljung-Box Q statistic, the hypothesis that all correlation coefficients up to 16 lags are jointly zero is rejected for all countries. Therefore, it can be concluded that return series in each country present some linear dependence in returns. In addition, the statistically significant serial correlations in squared returns [Q^2 (16)] imply that there are non-linear dependences in all return series. This indicates volatility clustering. Therefore, the summary statistics reported in table 2 demonstrate the defining characteristics of stock returns of three countries: occasional extreme movements, volatility clustering and fat-tailed distributions. While the presence of volatility clustering needs an appropriate GARCH model to filter the return series of each market separately, the existence of occasional extreme movements and fat-tailed distribution further motivate the exploration of Conditional EVT to estimate intraday *VaR* and *ES*.

One stylized property of high-frequency returns, which has been documented in many studies, is that most intraday equity return volatilities exhibit strong periodicity (e.g. Andersen and Bollerslev, 1997; Aradhyula and Ergün, 2004; Bollerslev and Ghysels, 1996; Goodhart and O'Hara, 1997; Martens et al., 2002). Volatility is typically higher at the opening and towards the close of trade and lower during midday. To examine this intraday periodicity, the study has estimated ACF of the absolute returns for each return series up to 30 days and plotted the same in a graph.[1] It appears from the graph that the apparent U-shaped periodicity recurs every day for all three series. This intraday periodicity in volatility observed in the data series can corrupt the estimates of traditional time-series models (e.g., GARCH-type models) as demonstrated by Andersen and Bollerslev (1997) and Martens et al. (2002). Therefore, to prevent distortion of the results, the intraday

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1. To examine this intraday periodicity, the study has estimated ACF of the absolute returns for each return series up to 30 days and plotted the same in a graph which is not shown in this paper.

TABLE 2. Descriptive Statistics of original returns (Full sample from January 01, 2010 to June 10, 2014)

	Total Obs.	Mean	Std. Dev.	Skewness	Kurtosis	Jarque-Bera	Q(16)	Q^2 (16)
	83,372	–7.59E–06	0.001	–0.500	17.441	1059522	97.40	8312.35
India						(0.000)	(0.000)	(0.000)
	111,996	–3.49E–07	0.001	–0.051	14.112	928893	117.04	46521.66
UK						(0.000)	(0.000)	(0.000)
	86,346	2.67E–06	0.001	–0.363	25.879	2411458	100.92	35691.53
US						(0.000)	(0.000)	(0.000)

Note: The table reports summary statistics for the 5-minute interval stock market returns (r_t) of the three countries. The *p* values are given in the parentheses indicate that the L-Jung box Q and Q^2 statistics are significant at better than 1% levels.

seasonality must be taken out prior to estimating any model. Hence, first the seasonality has been removed from the raw return series and then the deseasonalized filtered return series has been used to estimate the traditional time series models. Following a method proposed by Taylor and Xu (1997) and subsequently used by others, our study explains the deseasonalized filtered return as the *n*-th intraday return divided by an estimated seasonality term,

$$\tilde{r}_{t,n} = r_{t,n} / S_{t,n} \quad (n = 1, 2, ..., N), \tag{14}$$

where $r_{t,n}$ is the *n*th intraday return on day *t* and $S_{t,n}$ is the respective seasonality term, for *N* intraday periods. The seasonality term includes averaging the squared returns for each intraday period, i.e.:

$$\hat{S}^2_{t,n} = \frac{1}{T}\sum_{t=1}^{T} r^2_{t,n} \quad (n = 1, 2\ , . . . , N) \tag{15}$$

where *T* is the total number of days in the sample. This method seems to be quite effective as it almost removes the U-shaped pattern from all series which are evident from estimated ACF of the absolute deseasonalized returns series plotted in the same graph mentioned above. Now the deseasonalized return series can safely be used for estimating an appropriate GARCH model to measure volatility.

IV. Empirical Findings

The empirical study has been split into two parts: an in-sample study where the sample data are used for model estimation, and an out-of-sample study to compare the accuracy of long memory Component GARCH-EVT approach in forecasting intraday *VaR* and *ES* with simple GARCH-EVT framework under different distribution innovations. To do this, the full data sample in each market is divided into an in-sample period from January 01, 2010 to December 31, 2012 on which models are based and an out-of-sample period from January 01, 2013 to June 10, 2014 over which forecasting performance of *VaR* and *ES* is measured. All relevant information related to in-sample and out-of-sample period are reported in table 3.

TABLE 3. Relevant information of in-sample and out-of-sample periods

	In-Sample			Out-of-Sample		
	Obs./Day	No. of Days	Total Obs.	Obs./Day	No. of Days	Total Obs.
India	76	742	56,392	76	355	26,980
UK	102	738	75,276	102	360	36,720
US	78	749	58,422	78	358	27,924

Note: Three columns of both in-sample and out-of-sample periods, show number of sample observations in the return series per day, number of trading days and total number of sample observations finally after removing certain figures from the raw price set for each country. If number of days and total observations for both in-sample and out-of-sample periods are added, it will match with the number of days and total observations for the total period shown in table 1.

A. In-sample evidence

The first step is to fit GARCH and Component GARCH models in order to capture the conditional volatility of in-sample intraday deseasonalized return series. The study has applied the AR (1) – GARCH (1,1) and AR(1) – Component GARCH (1,1) specifications on different deseasonalized return series for each country. It has estimated the conditional mean series $(\hat{\mu}_{t-n+1},.....,\hat{\mu}_t)$ and standard deviation series $\left(\sqrt{\hat{h}_{t-n+1}},.....,\sqrt{\hat{h}_t}\right)$ for AR (1, 1)-GARCH (1, 1) and AR (1, 1)-Component GARCH (1, 1) models, and calculated standardized residuals as

$$z_{t-n+1} = \frac{r_{t-n+1} - \hat{\mu}_{t-n+1}}{\sqrt{\hat{h}_{t-n+1}}},............, z_t = \frac{r_t - \hat{\mu}_t}{\sqrt{\hat{h}_t}} \tag{16}$$

The standardized residuals are found to be close to *iid*.

As mentioned earlier, the study would employ the POT method using GPD for tail estimation of the standardized residual series. The first step in this modelling is to choose the threshold for identifying the relevant tail region. However, this choice is subject to a trade-off between variance and bias. By increasing the number of observations for the series of maxima (a lower threshold), some observations from the centre of the distribution are introduced in the series, and the tail index is more precise but biased (i.e., there is less variance). On the other hand, choosing a high threshold reduces the bias but makes the estimator more volatile (i.e., there are fewer observations). Thus the threshold estimation becomes more of an art than a science in balancing this trade-off between bias and variance.

There is no unique choice of the threshold level. A number of diagnostic techniques exist for this purpose including graphical bootstrap methods (see Embrechts et al., 1997; Reiss and Thomas, 1997). To optimize this trade-off between variance and bias inefficiency, the study performs a Monte Carlo simulation study. Return time-series are simulated from a known distribution for which the tail index can be computed. For each time series, the tail index value is estimated for different threshold levels. The choice of the optimal value is based on the bias and the mean squared error (*MSE*) criteria which allow one to take into account the trade-off between bias and inefficiency. The procedure is detailed in appendix.

Typically the threshold is chosen subjectively by looking at certain plots such as the Mean Excess plot or the Hill plot, which are standard practices in EVT. Here the study determines the threshold using the Mean Excess plot which is a plot of Mean Excess Function (MEF). From the MEF, the threshold can be selected on the criterion of linearity in MEF plot. While choosing threshold level subjectively from MEF plot, it had been made sure that the number of exceedances does not fall beyond a range in which bias and *MSE* are minimized as explained in the appendix.

Based on the MEF plots[2] the study has chosen the thresholds which along with their related statistics are reported in table 4. The value of threshold ranges from 0.969 to 1.294 and the number of exceedances, *k* (the number of points above the threshold) is found to vary from 5521 to 9878 which is large enough to facilitate a good estimation. In each case, the resulting exceedances *k* total roughly 10% of the sample, which is consistent with percentages reported by McNeil and Frey (2000).

The optimal value of shape (ξ) and scale (ψ) parameters are estimated and reported in table 4. In all eighteen cases (3 markets × 2 models × 3 distributions), the ξ estimate is positive suggesting that the left tail of the distribution of standardized residuals is characterized by heavy tailedness. The table further documents the EVT tail quantiles: $V\hat{a}R_q$ and ES_q for each country which are obtained from Eqs. (4) and (6) respectively using the values of *n, u, k,* ξ and ψ of the respective series at the specified end tail of α%.

After estimating the parameters, the study calculates the robust Conditional *VaR* and *ES* estimates based on Eqs. (12) and (13) respectively, where it multiplies the GARCH volatilities with quantiles and finally adds the conditional means.

Because intraday seasonality has been taken into account, the intraday forecasts of conditional mean ($\hat{\mu}_{t+1}$) and variances ($\hat{h}_{t+1}$) are calculated based on deseasonalized filtered returns. So to compute VaR_q^{t+1} and ES_q^{t+1} for the original returns, it requires to re-include the seasonality component to the intraday forecasts of condition mean and variance based on the deseasonalized filtered returns. To do so, $\hat{\mu}_{t+1}$ and $\hat{h}_{t+1}$ are multiplied by the appropriate seasonal term $\hat{S}_{t,n}$ and its square $\hat{S}_{t,n}^2$ respectively, i.e.

2. The MEF of each of 18 negative return series (3 markets × 2 models × 3 distributions) has been plotted separately, but not shown in this paper. To get the mean excess function of negative returns (left tail) the study has transformed the residual series z_t in to $-z_t$ then the results for the minimum can be directly deduced from those of maximum.

TABLE 4. Parameter estimates for the AR-GARCH-EVT called Conditional EVT model (In-sample from January 01, 2010 to December 31, 2012)

	Dist	u	k	k/n (%)	ξ	ψ	*VaR* Quantile 0.95	0.99	0.995	*ES* Quantile 0.95	0.99	0.995
A. GPD fit of standardized residuals obtained from AR(1)-GARCH(1,1)												
India	N	1.225	5530	9.81	0.08** (6.261)	0.58** (53.740)	1.623	2.668	3.162	2.288	3.422	3.960
	t	1.082	7212	12.79	0.08** (6.642)	0.58** (61.380)	1.642	2.703	3.201	2.313	3.460	3.998
	Skewed t	1.011	7810	13.85	0.07** (6.530)	0.58** (64.674)	1.633	2.698	3.194	2.305	3.449	3.982
UK	N	1.294	6548	8.70	0.06** (4.742)	0.57** (56.810)	1.614	2.606	3.064	2.296	3.406	3.914
	t	1.103	8948	11.89	0.04** (3.811)	0.59** (67.461)	1.620	2.629	3.084	2.317	3.445	3.196
	Skewed t	1.037	9878	13.12	0.03** (3.471)	0.59** (71.300)	1.618	2.629	3.083	2.304	3.439	3.969
US	N	1.212	5872	10.05	0.05** (3.911)	0.58** (58.131)	1.625	2.635	3.096	2.239	3.294	3.781
	t	1.243	5640	9.65	0.05** (4.042)	0.59** (53.190)	1.635	2.655	3.122	2.253	3.304	3.777
	Skewed t	1.252	5521	9.45	0.06** (4.043)	0.58** (52.381)	1.631	2.651	3.119	2.251	3.298	3.767

(*Continued*)

TABLE 4. (Continued)

	Dist	u	k	k/n (%)	ξ	ψ	*VaR* Quantile 0.95	*VaR* Quantile 0.99	*VaR* Quantile 0.995	*ES* Quantile 0.95	*ES* Quantile 0.99	*ES* Quantile 0.995
B. GPD fit of standardized residuals obtained from AR(1)-Component GARCH(1,1)												
	N	0.993	7898	14.00	0.05** (5.402)	0.59** (66.931)	1.637	2.686	3.167	2.251	3.293	3.760
India	*t*	1.065	7458	13.23	0.06** (6.431)	0.01** (63.531)	1.652	2.707	3.196	2.267	3.318	3.789
	Skewed *t*	1.125	6651	11.79	0.07** (6.159)	0.58** (59.286)	1.639	2.693	3.185	2.264	3.314	3.785
	N	1.086	9211	12.24	0.03** (3.481)	0.59** (69.275)	1.621	2.627	3.078	2.260	3.324	3.809
UK	*t*	1.059	9771	12.98	0.04** (3.610)	0.59** (71.271)	1.633	2.646	3.101	2.276	3.354	3.848
	Skewed *t*	1.066	9579	12.73	0.03** (3.538)	0.59** (70.582)	1.629	2.643	3.098	2.272	3.352	3.847
	N	0.969	8162	13.97	0.02* (1.920)	0.61** (68.644)	1.640	2.659	3.108	2.276	3.315	3.772
US	*t*	1.011	8056	13.79	0.03** (2.470)	0.61** (65.890)	1.656	2.690	3.148	2.302	3.362	3.833
	Skewed *t*	1.015	8150	13.95	0.03** (2.491)	0.61** (65.892)	1.652	2.686	3.145	2.298	3.359	3.831

(*Continued*)

TABLE 4. (Continued)

Note: Panel A and Panel B of table 6 report in-sample ML estimates of the GPD for AR-GARCH-EVT and AR-Component GARCH-EVT models respectively with normal (*N*), student-*t* (*t*) and skewed student-*t* (Skewed *t*) distributions governing the error terms. u = threshold level, k = No. of exceedances, k/n = percentage of exceedances where n = total no. of observations. The *t* statistics are given in the parenthesis in columns 6 and 7, while asterisks demonstrate the level of statistical significance. The asterisks (*) and (**) denote significance at 5% and 1% level, respectively.

$$\tilde{\mu}_{t+1} = (\hat{\mu}_{t+1})(\hat{S}_{t,n}) \tag{17}$$

$$\tilde{h}_{t+1} = (\hat{h}_{t+1})(\hat{S}^2_{t,n}) \tag{18}$$

where, $\tilde{\mu}_{t+1}$ and $\tilde{h}_{t,n}$ are the transformed forecast of conditional mean and variance, respectively for the original returns, and $\hat{S}_{t,n}$ is estimated by the method described in the previous section. Thus an estimate of the *VaR* and *ES* for the original returns is

$$VaR_q^{t+1} = \tilde{\mu}_{t+1} + \sqrt{\tilde{h}_{t+1}} V\hat{a}R_q \tag{19}$$

$$\text{and} \quad ES_q^{t+1} = \tilde{\mu}_{t+1} + \sqrt{\tilde{h}_{t+1}} ES_q \tag{20}$$

B. Out-of-sample evidence

So far, we have discussed how one can fit the Conditional EVT model to in-sample data. In practice, however, a risk manager is probably more interested in how well he or she can predict future extreme movements than in accurately modelling the past. To compare the accuracy of Component GARCH-EVT model with the benchmark GARCH-EVT model in forecasting intraday *VaR* and *ES*, the study has performed back testing of each method on out-of-sample return series first at the entire sample and then at sub-sample.

Backtesting (entire sample)

For backtesting, the following procedure is used. On the first day of the out-of-sample period, the most recent n returns are used to estimate the parameters for each model. The magnitude of *n* is set to be equal to the length of the in-sample period, That is, *n* = 56,392 for Indian market, *n* = 75,276 for the UK market and *n* = 58,422 for the US market as reported in table 3. From the parameter estimates, the next interval *VaR* is computed. Now, keeping the size of the window *n* fixed, the estimation procedure is rolled forward and the next interval *VaR* is calculated. The main advantage of this rolling window technique is that it captures dynamic time-varying characteristics of data in different time periods. As documented by McNeil and Frey (2000) and Gencay et al. (2003), within the backtest period, it is difficult to choose the best parameterization every time in out-of-sample period, so it is assumed that the models selected to the in-sample return series are adequate on each rolling window of the respective series. A similar restriction is also there for the GPD modelling. In a longer back testing period, it is less feasible to examine the fitted model carefully at each interval and to select a new value of k (the number of exceedances above the threshold u) for the tail estimator each time. For this reason the study always sets k so that the 90^{th} percentile of the innovation distribution is estimated by historical simulation, as suggested by McNeil and Frey (2000). Thus on each interval $t \in T$ it fits a new AR (1) – GARCH (1,1) and AR (1) – Component GARCH (1,1) with normal, *t* and skewed *t* innovations and determines new GPD tail estimates for each market. The intraday *VaR* and *ES* for the original returns are forecasted based on Eqs. (19) and (20) respectively, which have re-included the intraday periodicity component.

Backtesting of VaR

Various methods and tests have been suggested for measuring *VaR* model accuracy. Here, the study first uses different Likelihood ratio tests and then applies more powerful Dynamic Quantile (DQ) test.

To assess the forecasting performance of the *VaR* methods, the study has used the likelihood ratio tests for unconditional coverage, independence and conditional coverage, which are proposed by Christoffersen (1998). Based on the likelihood ratio tests the relative performance of each model is evaluated in terms of the violation. A violation is said to take place whenever r_{t+1}, the log-natural return in $t+1\rangle VaR_q^t$. The test of unconditional coverage checks whether or not the overall number of violations is statistically acceptable. The test of independence aims at verifying possible clustering of violations over time. The test of conditional coverage checks in which respect the time series of *VaR* violations does not satisfy the correct conditional coverage. Briefly, the tests can be implemented in a convenient likelihood framework and are distributed asymptotically chi-squared. Readers are referred to Christoffersen (2003, Chap. 8) for technical details on the test statistics.

Panels A, B and C of table 5 show statistics of unconditional, independence and conditional coverage tests, respectively for the different models at p=95%, p= 99% and p=99.5%. As a quick reference guide, the absence of ‘asterisks’ in the tables indicates that the

TABLE 5. Statistical tests of Likelihood Ratios (entire sample)

	GARCH-EVT			Component GARCH-EVT		
	N	*t*	Skewed *t*	*N*	*t*	Skewed *t*
A. Test of Unconditional Coverage						
α = 5%						
India	7.665**	4.463*	4.226*	1.387	0.614	0.489
UK	12.187**	6.557*	7.595**	2.600	0.589	0.552
US	6.117*	0.601	0.785	0.690	0.006	0.006
α = 1%						
India	5.313*	0.952	0.952	2.573	1.353	1.353
UK	6.129*	3.939*	4.157*	1.933	1.933	1.786
US	3.431	0.038	0.038	0.191	1.247	0.998
α = 0.5%						
India	4.146*	1.280	1.490	1.717	1.086	1.086
UK	1.751	0.320	0.320	0.071	0.071	0.071
US	1.182	0.003	0.003	0.019	0.041	0.082
Number of Violetions	6	3	3	0	0	0
B. Test of Independence						
α = 5%						
India	17.853**	29.459**	30.400**	8.152**	15.699**	16.291**
UK	30.054**	35.169**	36.329**	18.147**	24.434**	24.316**
US	12.362**	23.777**	25.428**	16.945**	23.911**	23.911**
α = 1%						
India	16.652**	20.243**	20.243**	6.767**	14.240**	17.380**
UK	4.521*	4.055*	4.105*	3.526	3.526	3.480
US	2.621	6.875**	6.875**	5.042*	3.506	3.412
α = 0.5%						
India	6.233*	23.372**	23.592**	5.529*	13.327**	13.327**
UK	1.441	3.353	3.353	1.052	1.052	1.052
US	5.134*	7.580**	7.580**	4.331*	4.063*	3.998*
Number of Violations	7	8	8	7	6	6

(*Continued*)

TABLE 5. (Continued)

	GARCH-EVT			Component GARCH-EVT		
	N	*t*	Skewed *t*	*N*	*t*	Skewed *t*
C. Test of Conditional Coverage						
α = 5%						
India	25.518**	33.921**	34.626**	9.539**	16.313**	16.780**
UK	42.241**	41.726**	43.924**	20.747**	25.023**	24.869**
US	18.480**	24.378**	26.213**	17.635**	23.917**	23.917**
α = 1%						
India	21.965**	21.195**	21.195**	9.340**	15.593**	18.733**
UK	10.650**	7.994*	8.263*	5.459	5.459	5.267
US	6.052*	6.913*	6.913*	5.233	4.753	4.410
α = 0.5%						
India	10.379**	24.652**	25.082**	7.246*	14.413**	14.413**
UK	3.192	3.673	3.673	1.123	1.123	1.123
US	6.315*	7.583*	7.583*	4.350	4.104	4.080
Number of Violations	8	8	8	4	4	4

Note: The table presents statistical tests of unconditional coverage (uc), test of independence (ind) and test of conditional coverage (cc) of the intraday VaR forecasts under each competing approach in panel A, B and C respectively considering entire sample data. The test is asymptotically distributed as χ^2 with d.f. one for the first two tests and with d.f. two for the last test. The asterisks (*) and (**) denote significance at 5% and 1% level, respectively.

difference between theoretical and empirical violation ratios is not statistically significant. For the unconditional coverage test reported in panel A, out of 9 cases (3 markets × 3 quantiles) analyzed, the GARCH-EVT with normal innovation fails 6 times, the GARCH-EVT with *t* innovation fails 3 times and the GARCH-EVT with skewed *t* innovation fails 3 times. But the Component GARCH-EVT combination does not fail on a single occasion. In terms of LR statistic the Component GARCH-EVT with skewed *t* innovation appear to perform the best as in majority of the cases the LR value is minimum. For the independence test reported in panel B, Component GARCH-EVT with *t* and skewed *t* innovations perform equally better than other models. For the conditional coverage test reported in panel C, the Component GARCH-EVT models again outperform the GARCH-EVT combinations.

TABLE 6. Statistical test of Dynamic Quantiles (entire sample)

	GARCH-EVT			Component GARCH-EVT		
	N	*t*	Skewed *t*	*N*	*t*	Skewed *t*
α = 5%						
India	65.351**	58.266**	59.006**	21.219**	22.856**	21.537**
UK	77.650**	73.838**	74.914**	32.372**	31.312**	31.195**
US	67.831**	68.371**	71.918**	54.695**	48.357**	51.187**
α = 1%						
India	79.671**	70.079**	70.071**	21.750**	26.815**	21.059**
UK	13.276*	12.865*	12.795*	10.988	9.329	9.092
US	39.358**	35.578**	35.553**	28.543**	20.148**	20.028**
α = 0.5%						
India	107.962**	115.24**	116.594**	36.431**	39.333**	39.320**
UK	7.702	8.184	8.176	6.312	5.532	5.517
US	24.679*	27.003**	27.095**	13.726*	11.210	11.043
Number of Violations	8	8	8	7	6	6

Note: The table presents statistical test of dynamic quantiles (DQ) of the intraday *VaR* forecasts under each competing approach considering entire sample data. The test is extended up to four lags and is asymptotically distributed as with d.f. six. The asterisks (*) and (**) denote significance at 5% and 1% level, respectively.

In the Christoffersen's (1998) LR test applied above, only the first lag is considered, however, as for high frequency data used in the present study, a few further lags are worth to be tested. Engle and Manganelli (2004) proposed an alternative joint test of independence and unconditional coverage, known as dynamic quantile (DQ) test in which more than one lags can be considered. The proposed test statistic follows a χ^2 distribution, where degrees of freedom is equivalent to the number of vector of instruments of X_t. Readers are referred to Engle and Manganelli (2004) for technical details on the test statistics.

Table 6 presents our backtesting results of DQ test for the different models at p=95%, p= 99% and p=99.5%. As a quick reference guide, the absence of 'asterisks' in the table indicates that the difference between theoretical and empirical violation ratios is not statistically significant. It appears from the table that out of 9 cases (3 markets × 3 quantiles) analyzed, the null hypothesis is rejected 8 times under each of the GARCH-EVT models with normal, *t* and skewed *t* innovations. The number of rejections is less for the Component GARCH-EVT models. Like LR test shown earlier, here too the Component GARCH-EVT with skewed *t* innovation appear to perform the best as in majority of the cases its DQ value is minimum.

So far the study has done the backtesting analysis of the different *VaR* models using various tests. The Component GARCH-EVT models in general perform better than GARCH-EVT models in estimating and forecasting *VaR*. Within the Component GARCH-EVT models, although no model with a particular innovation consistently dominates others, the model with skewed *t* innovations performs relatively better than the models with other innovations.

Backtesting of ES

To backtest the estimated ES_q value, the study uses the measure proposed by Embrechts et al (2005).

The Embrechts et al. (2005) measure is given by

$$E = (|E_1| + |E_2|)/2 \tag{21}$$

where $E_1 = \frac{1}{c}\sum_{t\in\kappa}\phi_t$, $E_2 = \frac{1}{d}\sum_{t\in\eta}\phi_t$, $\phi_t = r_{t+1} - ES_q^{t+1}$, c is the number of intervals for which a violation of *VaR*, i.e., $r_{t+1} < V\hat{a}R_q^{t+1}$ occurs and κ is the set of intervals for which it happens, *d* is the number of periods for which ϕ_t is less than the empirical quantile and η is the set of periods for which it happens. A good estimation of *ES* will lead to a low value of *E*. Readers are referred to Embrechts et al (2005) for further technical details on the test statistics.

Table 7 reports backtesting results of *ES* for different models and shows the value of the measure (E) for 95%, 99% and 99.5% quantiles. The minimum value of the same is marked with 'asterisks' for each of the cases. It appears from the table that out of 9 cases (3 markets × 3 quantiles) analyzed, the Component GARCH-EVT model with skewed *t* innovation has achieved the minimum value of *E* five times and both the models: Component GARCH-EVT with normal and Component GARCH-EVT with *t* have achieved the minimum value of *E* twice. Interestingly, not in a single case the GARCH-EVT model with any of the innovations has achieved the minimum value of *E*. Thus the Component GARCH-EVT models in general perform better than the GARCH- EVT models in estimating *ES*, and within the Component GARCH-EVT models, the model with skewed *t* innovations performs the best.

TABLE 7. Statistical tests of Expected Shortfall (entire sample)

	GARCH-EVT			Component GARCH-EVT		
	N	*t*	Skewed *t*	*N*	*t*	Skewed *t*
α = 5%						
India	0.019	0.017	0.017	0.011	0.003	0.003*
UK	0.004	0.007	0.007	0.007	0.001	0.001*
US	0.023	0.017	0.017	0.008*	0.014	0.014
α = 1%						
India	0.051	0.046	0.045	0.015	0.011*	0.012
UK	0.051	0.050	0.051	0.037	0.035	0.034*
US	0.036	0.038	0.039	0.025*	0.033	0.033
α = 0.5%						
India	0.129	0.129	0.129	0.069	0.063	0.063*
UK	0.068	0.068	0.068	0.064	0.061*	0.061
US	0.087	0.086	0.084	0.067	0.060	0.060*
Min Value Occurences	0	0	0	2	2	5

Note: The table presents Embrechts et. al. measure (scaled up by x10^3) of the intraday *ES* forecasts under each competing approach considering entire sample data. The presence of (*) represents the minimum value (checked up to five point after decimal) of the measure among the approaches for each stock index under a given confidence level.

Backtesting (sub-sample)

So far, the study has examined the relative efficiency of Component GARCG-EVT models in forecasting intraday *VaR* and *ES* at the entire sample. It would also be interesting to see how sensitive are the results to different sample sizes? How do they react in sub-samples? If the entire sample is divided in two halves, i.e., the first half and the second half, would the Component GARCH-EVT models be still better than the GARCH- EVT models? In order to address these questions, the study has done the backtesting of each model on the sub-sample data of the first half and second half, separately.[3] While doing backtesting, it has

3. The first half covers the period from January 01, 2010 to February 15, 2012 and the second half covers the period from February 16, 2012 to June 10, 2014. Next, the study splits each half into two parts: an in-sample study where the sample data are used for model estimation and an out-of-sample study to compare the forecasting accuracy. The first half for each market comprises an in-sample period from January 01, 2010 to June 30, 2011 and an out-of-sample period from July 01, 2011 to February 15, 2012. Similarly, for second half, in-sample estimation has been done for the period of February 16, 2012 to August 31, 2013 and out-of-sample forecast has been performed on September 01, 2013 to June 10, 2014.

TABLE 8. Statistical tests of Likelihood Ratios (sub-sample)

	First Half					
	GARCH-EVT			Component GARCH-EVT		
	N	*t*	Skewed *t*	*N*	*t*	Skewed *t*
A. Test of Unconditional Coverage						
α = 5%						
India	16.101**	12.093**	13.039**	10.032**	2.943	2.479
UK	4.356*	2.139	2.139	0.329	0.003	0.000
US	1.040	0.599	0.599	0.039	0.009	0.018
α = 1%						
India	0.222	0.008	0.008	0.570	0.001	0.001
UK	0.985	1.773	1.773	2.802	3.205	2.438
US	0.756	1.794	1.794	1.794	2.959	2.959
α = 0.5%						
India	3.960**	0.038	0.004	0.217	0.004	0.004
UK	0.119	0.338	0.338	0.338	1.394	1.394
US	0.082	0.300	0.300	1.164	0.893	0.167
	Second Half					
α = 5%						
India	4.269*	4.160*	4.312*	1.834	1.243	1.243
UK	2.038	1.972	1.046	0.236	0.023	0.014
US	0.656	0.856	0.928	1.918	1.518	1.248
α = 1%						
India	0.181	0.355	0.355	0.122	0.069	0.069
UK	1.026	2.548	0.745	0.119	0.119	0.115
US	0.155	0.041	0.041	0.041	0.138	0.108
α = 0.5%						
India	0.381	0.243	0.136	0.551	0.243	0.243
UK	0.037	9.312**	0.104	0.276	0.276	0.392
US	1.166	0.499	0.340	0.047	0.340	0.340

(*Continued*)

TABLE 8. (Continued)

	First Half					
	GARCH-EVT			Component GARCH-EVT		
	N	*t*	Skewed *t*	*N*	*t*	Skewed *t*
B. Test of Independence						
α = 5%						
India	14.145**	14.994**	15.506**	6.709**	10.647**	10.329**
UK	5.070*	5.827*	5.827*	0.677	1.520	1.504
US	4.340*	0.008	0.008	5.913*	0.006	0.003
α = 1%						
India	15.063**	17.684**	17.684**	8.273**	13.988**	14.843**
UK	0.364	0.456	0.456	0.560	0.771	0.707
US	6.002**	0.002	0.002	9.456**	0.015	0.015
α = 0.5%						
India	1.961	25.388**	25.795**	4.813*	3.715	3.715
UK	0.695	0.657	0.657	0.657	0.566	0.566
US	1.102	1.203	1.203	2.352	1.368	1.326
	Second Half					
α = 5%						
India	7.005**	17.456**	18.136**	4.172*	9.630**	11.761**
UK	15.603**	48.364**	20.837**	11.205**	2.893	3.012
US	3.944*	3.773	3.682	5.432*	3.222	2.555
α = 1%						
India	5.404*	18.032**	18.032**	3.786	16.366**	16.366**
UK	4.588*	14.105**	4.433*	6.223*	3.223	3.217
US	6.647**	3.489	3.489	3.132	3.285	3.184
α = 0.5%						
India	14.063**	13.734**	13.496**	1.110	2.268	2.268
UK	12.878**	13.745**	5.818*	2.432	2.432	2.373
US	4.195*	2.357	2.249	0.886	1.011	1.011

(*Continued*)

TABLE 8. (Continued)

	First Half					
	GARCH-EVT			Component GARCH-EVT		
	N	*t*	Skewed *t*	*N*	*t*	Skewed *t*
C. Test of Conditional Coverage						
α = 5%						
India	30.246**	27.087**	28.546**	16.741**	13.591**	12.808**
UK	9.427**	7.966*	7.966*	1.006	1.523	1.504
US	5.380	0.607	0.607	5.952	0.014	0.021
α = 1%						
India	15.285**	17.692**	17.692**	8.843*	13.989**	14.844**
UK	1.349	2.229	2.229	3.362	3.976	3.145
US	6.758*	1.796	1.796	11.250**	2.974	2.974
α = 0.5%						
India	5.921	25.426**	25.799**	5.030	3.719	3.719
UK	0.814	0.995	0.995	0.995	1.960	1.960
US	1.184	1.503	1.503	3.515	2.260	1.493
Number of Violations	12	9	9	9	4	4
	Second Half					
α = 5%						
India	11.274*	21.616**	22.448**	6.006*	10.873**	13.004**
UK	17.641**	50.336**	21.884**	11.441**	2.916	3.047
US	4.600	4.629	4.611	7.349*	4.739	3.803
α = 1%						
India	5.585	18.387**	18.387**	3.908	16.435**	16.435**
UK	5.614	16.654**	5.178	6.342*	3.342	3.332
US	6.802*	3.530	3.530	3.173	3.423	3.292
α = 0.5%						
India	14.444**	13.977**	13.632**	1.661	2.511	2.511
UK	12.915**	23.058**	5.921	2.708	2.708	2.764
US	5.362	2.856	2.589	0.933	1.351	1.351
Number of Violations	15	14	11	7	4	4

Note: The table presents statistical tests of unconditional coverage (uc), test of independence (ind) and test of conditional coverage (cc) of the intraday *VaR* forecasts under each competing approach in panel A, B and C respectively considering sub-sample data. The out-of-sample forecast of first half is performed for the period starting from July 01, 2011 to February 15, 2012 and that of second half has been performed on September 01, 2013 to June 10, 2014. The test is asymptotically distributed as χ^2 with d.f. one for the first two tests and with d.f. two for the last test. The asterisks (*) and (**) denote significance at 5% and 1% level, respectively.

TABLE 9. Statistical tests of Dynamic Quantiles (sub-sample)

	First Half					
	GARCH-EVT			Component GARCH-EVT		
	N	*t*	Skewed *t*	*N*	*t*	Skewed *t*
α = 5%						
India	31.376**	29.292**	31.559**	19.875**	22.575**	23.060**
UK	24.968**	26.521**	25.484**	10.531	12.198	12.359
US	21.005**	22.338**	22.347**	14.633*	12.538	11.681
α = 1%						
India	37.221**	47.834**	47.837**	18.346**	32.739**	33.329**
UK	7.269	8.872	8.891	7.506	11.102	10.317
US	8.281	9.284	9.294	12.633*	11.104	11.125
α = 0.5%						
India	16.408*	21.853**	21.854**	15.304*	16.312*	10.306
UK	4.286	4.877	4.895	5.577	5.584	5.603
US	4.722	5.875	5.880	5.385	5.057	5.275
Violations	5	5	5	5	3	2
	Second Half					
α = 5%						
India	19.483**	24.462**	25.402**	9.014	15.837*	13.263*
UK	21.705**	133.867**	29.603**	15.392*	11.942	10.115
US	14.818*	20.976**	21.345**	17.146**	10.117	12.432
α = 1%						
India	27.347**	65.910**	65.064**	11.528	45.526**	45.508**
UK	19.684**	95.472**	8.863	13.869*	11.746	10.157
US	14.991*	17.643**	17.770**	9.047	10.290	9.263
α = 0.5%						
India	13.591*	73.166**	71.722**	4.517	19.719**	19.728**
UK	17.317**	125.262**	22.922**	15.471*	11.317	10.879
US	17.785**	19.066**	18.242**	6.532	12.334	12.010
Violations	9	9	8	4	3	3

Note: The table presents statistical test of dynamic quantiles (DQ) of the intraday *VaR* forecasts under each competing approach considering sub-sample data. The out-of-sample forecast of first half is performed for the period starting from July 01, 2011 to February 15, 2012 and that of second half has been performed on September 01, 2013 to June 10, 2014. The test is extended up to four lags and is asymptotically distributed as χ^2 with d.f. six. The asterisks (*) and (**) denote significance at 5% and 1% level, respectively.

TABLE 10. Statistical tests of Expected Shortfall (sub-sample)

	First Half					
	GARCH-EVT			Component GARCH-EVT		
	N	*t*	Skewed *t*	*N*	*t*	Skewed *t*
α = 5%						
India	0.033	0.031	0.031	0.028	0.029	0.028*
UK	0.053	0.047	0.047	0.041*	0.047	0.045
US	0.057	0.202	0.058	0.055	0.043	0.042*
α = 1%						
India	0.187	0.139	0.134	0.176	0.134*	0.134
UK	0.010	0.020	0.019	0.035	0.010	0.009*
US	0.137	0.190	0.134	0.115	0.103	0.102*
α = 0.5%						
India	0.351	0.270	0.270	0.348	0.244	0.242*
UK	0.047	0.041	0.040	0.049	0.040	0.039*
US	0.194	0.182	0.184	0.161	0.104*	0.110
Violations	0	0	0	1	2	6
	Second Half					
α = 5%						
India	0.043	0.047	0.049	0.037*	0.045	0.044
UK	0.028	0.058	0.022	0.022	0.021	0.021*
US	0.024	0.023	0.023	0.024	0.024	0.023*
α = 1%						
India	0.089	0.086	0.085*	0.133	0.133	0.133
UK	0.060	0.209	0.064	0.072	0.054*	0.055
US	0.032	0.033	0.032	0.029	0.025	0.023*
α = 0.5%						
India	0.273	0.262	0.254	0.315	0.296	0.245*
UK	0.081	0.325	0.080	0.088	0.092	0.079*
US	0.017*	0.022	0.026	0.031	0.020	0.019
Violations	1	0	1	1	1	5

Note: The table presents Embrechts et. al. measure (scaled up by x103) of the intraday ES forecasts under each competing approach considering sub-sample data. The out-of-sample forecast of first half is performed for the period starting from July 01, 2011 to February 15, 2012 and that of second half has been performed on September 01, 2013 to June 10, 2014. The presence of (*) represents the minimum value (checked up to five point after decimal) of the measure among the approaches for each stock index under a given confidence level.

followed the same procedure used for the entire sample.

Table 8 shows statistics of unconditional, independence and conditional coverage tests for the two different sub-periods, separately. It appears from the table that here again the Component GARCH-EVT in general performs better than GARCH-EVT in both the sub-samples. In the first half, out of 27 cases (3 markets × 3 quantiles × 3 tests) analyzed, the GARCH-EVT with normal innovation fails 12 times, and both the GARCH-EVT with *t* and skewed *t* innovations fails 9 times. But the Component GARCH-EVT with normal innovation fails 9 times and both the Component GARCH-EVT with *t* and skewed *t* innovations fail only 4 times. More or less similar results are observed in the second half.

Table 9 presents backtesting results of DQ test. Here again the Component GARCH-EVT model in general performs better than the GARCH-EVT model in both the sub-samples. While in first half the Component GARCH-EVT with skewed *t* performs best, in second half both the Component GARCH-EVT with *t* and skewed *t* innovations perform equally better than the other models.

The backtesting results of *ES* reported in table 10 also suggest that the Component GARCH-EVT model in general performs better than GARCH-EVT in both the sub-samples. The Component GARCH-EVT with skewed *t* has achieved the minimum value of *E* for maximum number of times in both the places (i.e, 6 times in first half and 5 times in the second half).

Thus while doing the backtesting at sub-samples, we have got the same results as observed at the entire sample. The Component GARCH-EVT models in general perform better than the GARCH-EVT models in estimating *VaR* and *ES*. And within the Component GARCH-EVT models, the model with skewed *t* innovations relatively performs better than the models with other innovations.

V. Conclusion

The study estimates intraday *VaR* and *ES* based on Component GARCH-EVT approach and compare its forecasting accuracy with the benchmark GARCH-EVT model. The data set used in the study includes the 5 minute price indices of three stock markets across India, the UK and the US. The preliminary analysis of the data shows that the 5 minute returns series are all leptokurtic, slightly negatively skewed and have a zero mean. Moreover, there are linear dependence in returns and the series exhibit high volatility and volatility clustering. The findings suggest the exploration of the GARCH-EVT to forecast *VaR* and *ES*. One stylized property of the high frequency return series is that the series displays strong periodicity patterns in intraday volatility. Since the GARCH-type models can be corrupted by intraday periodic patterns, the study uses the deseasonalized filtered returns, instead of raw returns, to estimate the volatility model. However, the *VaR* and the *ES* are later computed for the original returns by re-including the intraday periodicity component. It has estimated intraday *VaR* and *ES* using Component GARCH-EVT combined approach with different innovations such as normal, *t* and skewed *t* and examined its relative accuracy with the benchmark GARCH-EVT model with different innovations. To measure the accuracy of different models, the study has done backtesting on out-of-sample return series first at the entire sample and then at the sub-samples. The finding is same for the entire sample and the two sub-samples. The Component GARCH-EVT models in general perform better than the GARCH-EVT models in estimating *VaR* and *ES*. And within the Component GARCH-EVT models, the model with skewed *t* innovations relatively performs better than the models with other innovations. The apparent superiority of this combination with skewed *t* innovation to forecast *VaR* and *ES* should come as no surprise. The Component GARCH model can better estimate the volatility than the simple GARCH model and the Component GARCH with skewed *t* innovation combined with EVT explicitly models the heavy tails and the skewness of the return series. The study is useful for market participants (such as intraday traders and market makers) involved in frequent intraday trading in such equity markets.

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Appendix. Threshold Selection for the GPD approach

To investigate the issue of threshold choice (i.e. choice of *k*), a small simulation study has been performed following McNeil and Frey (2000). Random samples have been generated separately from each of the three distributions (normal, *t* and skewed *t*) where sample size corresponds to the window length which has been used for in-sample estimation. The degrees of freedom are calculated from the moments of model residuals. Now, the quantiles are estimated from the series with various values of *k* using GPD. The study restricts attention to values of

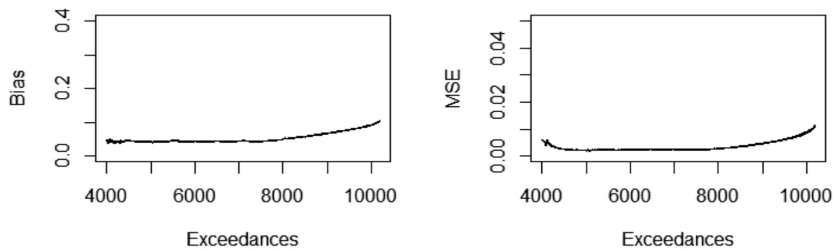

FIGURE 1.1.— Normal

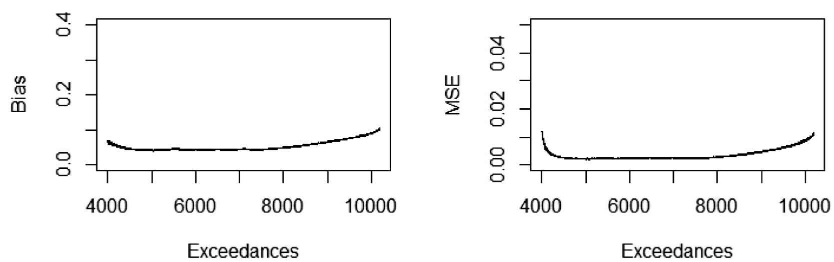

FIGURE 1.2.— *t*

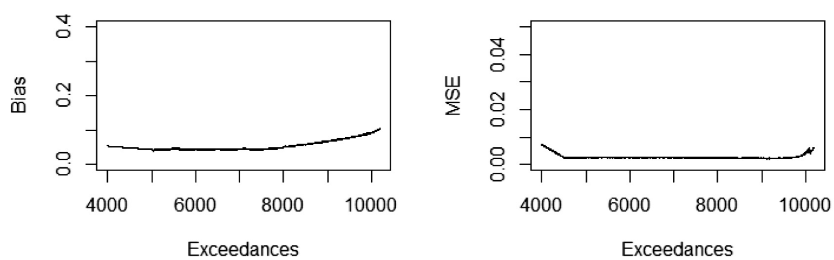

FIGURE 1.3.— Skewed *t*

k such that k > window length*(*1-q*), so that the target quantile is beyond the threshold. For each stock index returns, the study estimates bias and mean squared errors (*MSE*) using Monte Carlo estimates based on 1000 independent samples. For example, it estimates bias and *MSE* $\left(\hat{z}_{q,k}\right)$ by

$$Bias = \frac{1}{1000}\sum_{j=1}^{1000}\hat{z}_{q,k}^{(j)} - z_q$$

$$MSE\left(\hat{z}_{q,k}\right) = \frac{1}{1000}\sum_{j=1}^{1000}\left(\hat{z}_{q,k}{}^{(j)} - z_q\right)^2$$

where, $\hat{z}_{q,k}{}^{(j)}$ represents the quantile estimate obtained from the j-th sample and z_q is the theoretical quantile estimates of the respective distribution.

The study has calculated the bias and *MSE* of GPD estimator of the 95-th percentile against k for all the time series but for brevity the results are reported only for India. The results for the three distributions (with 5 degrees of freedom calculated based on residual series of India for *t* and skewed *t*) are depicted in figures 1.1, 1.2 and 1.3 respectively.

From the figures it appears that minimum of bias and *MSE* for all three distributions could be achieved when number of exceedances (k) varies from 4500 to 8000 (roughly, 8% to 15%). Therefore, while choosing threshold level subjectively from MEF plot, it has been made sure that the number of exceedances does not fall beyond this limit.

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