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Climate Risks and Stock Market Volatility Over a Century in an Emerging Market Economy: The Case of South Africa

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Climate Risks and Stock Market Volatility Over a Century in an Emerging Market Economy: The Case of South Africa

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Abstract

Because climate change broadcasts a large aggregate risk to the overall macroeconomy and the global financial system, we investigate how a temperature anomaly and/or its volatility affect the accuracy of forecasts of stock returns volatility. To this end, we do not only apply the classical GARCH and GARCHX models, but rather we apply newly proposed model-free prediction methods, and use GARCH-NoVaS and GARCHX-NoVaS model to compute volatility predictions. These two models are based on a normalizing and variance-stabilizing transformation (NoVaS transformation) and are guided by a so-called model-free prediction principle. Applying the new models to data for South Africa, we find that climate-related information is helpful in forecasting stock returns volatility. Moreover, the novel model-free prediction method can incorporate such exogenous information better than classical methods. Our findings have important implications for academics, investors and policymakers.

Keywords: Climate risks; Volatility forecasting; Model-free prediction; GARCH and GARCHX; South Africa

JEL Codes: C32; C53; C63; Q54

1 Introduction

Rietz (1988), and later Barro (2006), have proposed models of rare disasters to explain the equity premium puzzle, initially identified by Mehra and Prescott (1985). More recently, Wachter (2013) and Tsai and Wachter (2015) have extended this line of research by studying models in which aggregate consumption follows a normal distribution with low volatility most of the time, but a far out-in-the-left-tail realization of consumption can occur with some probability, creating disaster risk. The disaster risk not only substantially raises the equity premium, but the time-variation in its probability produces high stock-market volatility. Moreover, in another recent contribution, Sundaesan (2023), builds on the literature on inattention to develop a model in which rare disaster risk enhances uncertainty, as well as its, persistence. In this model, agents decide on whether and how to prepare for different future states of the world by collecting information, but they also optimally ignore events that are sufficiently unlikely, implying that the realization of such events does not resolve, but rather increases uncertainty. As a result, when agents have dispersed beliefs, uncertainty catalyzes uncertainty and generates endogenous persistence.

The traditional present discounted value model of asset prices (Shiller et al., 1981; Shiller, 1981) implies that asset price volatility depends on the variability of cash flows and the discount factor. Because an uncertain economic environment will tend to affect the volatility of future cash flows (Bernanke, 1983) and the discount factor (Schwert, 1989), one can hypothesize a positive predictive relationship between uncertainty, originating from rare disaster events, and stock market volatility. In other words, well-established theoretical channels exist that warrant a detailed empirical analysis of the link between rare disaster risk and stock market volatility. We lay out in this research the results of such an empirical analysis.

Our objective is to forecast stock returns volatility of an important emerging market economy, namely South Africa, using the informational content of rare disaster risk, over the monthly period from 1910:02 to 2023:02. In line with the burgeoning literature on climate finance, we use changes in temperature anomaly and its volatility as an empirical proxy of the theoretical concept of rare disaster risk, as in the advanced financial market movements-based research by Balvers et al. (2017), Donadelli et al. (2020), Balcilar et al. (2023), Bonato et al. (2023b), and Salisu et al. (2023), given that climate change poses a large aggregate risk to the overall macroeconomy and the global financial system due to the associated occurrences of rare

disasters (Giglio et al., 2021; Stroebel and Wurgler, 2021; van Benthem et al., 2022). To this end, we study data that span more than a century because climate change is a slow-moving process and its effects have tended to aggravate over time as economies have become more and more industrialized.

In this regard, our choice of South Africa as a case study of an emerging market economy is motivated not only by the availability of stock market data spanning over a century, but also because, as stressed by Mensi et al. (2014, 2016), a standalone analysis of the South African stock market is warranted due to its high degree of sophistication. In addition, South Africa is one of the largest exporters of strategic commodities like, coal, chrome, diamond, gold, ilmenite, iron ore, manganese, palladium, platinum, rutile, vanadium, vermiculite, and zirconium. While being a major global commodity exporter, and given the dominance of the mining industry (which contributes roughly 8% of its Gross Domestic Product (GDP) as per the Facts & Figures Pocketbook 2022 of the Minerals Council South Africa)¹, the South African economy is run primarily on fossil fuel (coal)-generated energy, so that the country ranks as fourteenth and first in terms of carbon dioxide emissions in the world and Africa, respectively (Statista, 2023). Moreover, because South Africa is a semi-arid country, a global average temperature increase of 1.5 °C would correspond to a 3 °C of average local temperature and, thereby, raise the likelihood of risk of extreme weather events in the country.²

Evidently, the prominence of climate-related disaster risk for South Africa, and the potential influence of such risk on its stock market volatility, is indeed a pertinent issue, as appropriate modeling and out-of-sample prediction of stock market volatility is important due to several reasons (as outlined, for example, by Poon and Granger (2003), Rapach et al. (2008)). Firstly, modern finance theory implies that volatility is a key input to investment decisions and portfolio choices. Secondly, volatility is a key input to standard pricing formulas for derivative securities. For example, in order to price an option, one needs reliable estimates of the volatility of the underlying asset. Thirdly, financial risk management, according to the Basle Accord as established as early as 1996, requires modeling and forecasting of volatility as a compulsory input to risk-management models used by financial institutions around the world. Last but not least, stock market volatility, as was evident during the Global Financial Crisis and the recent COVID-19 pandemic, can have severe repercussions on the economy as a whole via its effect on real economic activity and public confidence. Forecasts of stock market volatility, thus, can serve as a measure for the vulnerability of the overall financial system and the whole economy and, thereby, can help policy makers design appropriate preventive policies.

Not surprisingly, the academic literature on stock market volatility of South Africa, in terms of econometric methods, primarily involving variations of the Generalized Autoregressive Conditional Heteroskedasticity (GARCH)-family, and predictors being considered, is quite large, to say the least. A comprehensive review of this literature is beyond the scope and objective of this paper. We refer interested readers to the works of Moolman and Du Toit (2005), Mangani (2008), Samouilhan and Shannon (2008), Babikir et al. (2012), Chinzara (2011), Mandimika and Chinzara (2012), Afuecheta et al. (2016), Sigauke et al. (2016), Cakan and Gupta (2017), Cheteni (2017), Naik et al. (2018), Muzindutsi et al. (2020), Dwarika et al. (2021), Salisu and Gupta (2022), Kaseke et al. (2022), Gupta et al. (2023), and the references cited therein.³ However, despite the wide variation of econometric methods that researchers have used and the plethora of predictors that they have considered, no research has yet been done on the role of climate risks in forecasting South African stock returns volatility. In light of the fact that changes in temperature and its volatility can have strong general equilibrium effects (Donadelli et al., 2017, 2021a,b, 2022), these predictors are likely to encompass the information contained in a wide array of macroeconomic and financial (and even behavioral (Sheng et al., 2022)) predictors that have been used in earlier research to forecast stock price volatility of South Africa, with the added advantage that data on changes in temperature and its volatility is available in a consistent manner for over 110 years.

Having said this, like Salisu and Gupta (2022), we control for the role of fundamentals- and sentiments-based information via the West Texas Intermediate (WTI) oil and precious metals (gold and silver) prices. The oil price (or its returns) is a good proxy of macroeconomic and financial predictors because of its potential to move stock prices through its impact on changes in expected cash flows and/or the discount rate, output, monetary and fiscal policy, and macroeconomic and financial uncertainties (Degiannakis et al., 2018; Smyth and Narayan, 2018). Gold, in turn, serves the dual roles of a consumption good as jewelry, and investors regard it as a “safe haven”, i.e., investors consider it valuable in times of severe financial turmoil. In contrast, silver is a precious metal with similar uses as gold in consumption, but lacks the status of a “safe haven”. It follows that the ratio of gold-to-silver prices, because it should be largely unaffected by consumption shocks, reveals variation in risk, with this price ratio rising when investor sentiment is weak and/or investors become more risk averse (Salisu et al., 2022).⁴ In the context of research on the

¹<https://www.mineralscouncil.org.za/special-features/1345-facts-figures-pocketbook-2022>.

²See report from Boston Consulting Group at: <https://www.bcg.com/publications/2022/how-south-african-mining-can-address-climate-change-challenges>.

³In terms of the international literature on modeling and predictability of stock market volatility, see Ben Nasr et al. (2010, 2014), Bhowmik and Wang (2020), Muguto and Muzindutsi (2022), Salisu and Gupta (2022), Segnon et al. (2023) for detailed reviews.

⁴The idea emanates from the gold-to-platinum price ratio proposed by Huang and Kilic (2019) to capture global risk, given that data on platinum prices only stretch back to 1968.

predictability of South African stock market volatility, several types of primarily univariate GARCH models, and when exogenous predictor(s) are added, GARCH-X models have been utilized. While we also implement these models studied in earlier research, we go beyond earlier literature in that we also use the recently developed model-free method, NoVaS, which applies normalizing and variance-stabilizing transformation (NoVaS transformation) to perform volatility predictions. The NoVaS method builds on the model-free prediction principle, first proposed by Politis (2003), which, in turn, has been shown to outperform a wide array of models from the GARCH-class in terms of volatility forecasting (see, for example, Gulay and Emec (2018), Wu and Karmakar (2021, 2023b)). Motivated by the superior performance of the newly developed model-free GARCH-NoVaS model, Wu and Karmakar (2023a) extended this framework to a model that renders it possible to incorporate exogenous predictors, which we then use to study the role of climate risks, as captured by changes in temperature anomaly and/or its volatility, over and above oil returns and the gold-to-silver price ratio, in forecasting stock returns volatility of South Africa. In other words, our paper not only makes an important empirical contribution while analyzing the role of climate change in shaping the risk profile of a mining-intensive emerging market economy for the first time, but does so by using recent methodological advances made in the context of volatility forecasting, and by using the recent focus on the NoVaS approach.

We organize the remainder of this research as follows. In Section 2, we describe the forecasting models we use in our empirical research. In Section 3, we outline our model evaluation criteria. After discussing the data in Section 4, we report our empirical results in Section 5. In Section 6, we conclude and discuss the implications of our findings.

2 Forecasting models

2.1 Classical models

The classic GARCH(1,1) model as proposed by Bollerslev (1986) can be described as follows:

$$\begin{aligned} Y_t &= \sigma_t W_t, \\ \sigma_t^2 &= a + a_1 Y_{t-1}^2 + b_1 \sigma_{t-1}^2. \end{aligned} \quad (1)$$

where $a \geq 0$, $a_1 > 0$, $b_1 > 0$, and $W_t \sim i.i.d. N(0, 1)$. After including a vector of exogenous covariates, $\mathbf{X} = (X_1, \dots, X_m)$, we can wrap the exogenous covariates into the prediction process by turning the GARCH(1,1) model into the following GARCHX(1,1,1) model:

$$\begin{aligned} Y_t &= \sigma_t W_t, \\ \sigma_t^2 &= a + a_1 Y_{t-1}^2 + b_1 \sigma_{t-1}^2 + \mathbf{c}^T \mathbf{X}_{t-1}, \end{aligned} \quad (2)$$

where \mathbf{X}_{t-1} represents $(X_{1,t-1}, \dots, X_{m,t-1})$ and \mathbf{c} are the coefficients of these exogenous variables to be estimated (see Francq et al. (2019) for an in-depth discussion on the properties of such GARCHX(1,1,1) model). In order to implement a moving-window out-of-sample prediction experiment with classical methods, we first need to estimate the GARCH(1,1) and GARCHX(1,1,1) models⁵, and then we compute predictions iteratively (see Section 3 for details).

2.2 NoVaS-type models

We next present two model-free prediction methods which have been developed recently – the GARCH-NoVaS and GARCHX-NoVaS models. These models are guided by the model-free prediction principle and rely on the normalizing and variance-stabilizing transformation (NoVaS transformation) to do predictions.

2.2.1 GARCH-NoVaS model

We first introduce the GARCH-NoVaS model, which is built on Eq. (1). We focus on the parsimonious GARCH-NoVaS model proposed by Wu and Karmakar (2023b). The corresponding transformation and inverse transformation functions can be written as follows:

$$W_t = \frac{Y_t}{\sqrt{\alpha s_{t-1}^2 + \sum_{i=1}^q \tilde{c}_i Y_{t-i}^2}}; \quad Y_t = \sqrt{W_t^2 (\alpha s_{t-1}^2 + \sum_{i=1}^q \tilde{c}_i Y_{t-i}^2)}, \quad (3)$$

where α is a constant that plays a similar role as the constant parameter, a , in Eq. (1); s_{t-1}^2 is the sample variance of $\{Y_1, \dots, Y_{T-1}\}$; the parameter q is a large enough constant (we use 20 in our empirical research), and $\{\tilde{c}_1, \dots, \tilde{c}_q\}$ represents $\{a_1, a_1 b_1, a_1 b_1^2, \dots, a_1 b_1^{q-1}\}$ scaled by multiplying with a scalar $\frac{1-\alpha}{\sum_{j=1}^q a_1 b_1^{j-1}}$.

⁵For estimation of the GARCH and GARCHX models, we use the *fGarch* (Wuertz et al., 2013) and *garchx* packages (Sucarrat, 2020) for the R language and environment for statistical computing (R Core Team, 2023).

In short, the model-free prediction principle is about a distribution-match problem. Assuming that we have observed one series $\{Y_1, \dots, Y_T\}$, we transform this series to another series $\{\epsilon_1, \dots, \epsilon_T\}$ with *i.i.d.* components (chosen as standard normal in this paper) through an invertible transformation function H_T . Because the prediction of *i.i.d.* components is a trivial matter given a L_1 (MSE) or L_2 (MAE) loss criterion, we can obtain the optimal predictor of ϵ_{T+1} first and then transform it back to the prediction of Y_{T+1} with the inverse function H_T^{-1} . As for the GARCH-NoVaS model, we have ready-made transformation functions H_T and H_T^{-1} as shown in Eq. (3). Thus, our goal is to determine the coefficients $\{\tilde{c}_1, \dots, \tilde{c}_q\}$ such that Eq. (3) are indeed appropriate transformation functions. We decompose this problem into two parts: (1) Variance stabilization, which is used to get unity variance; (2) Normalization, which is to create *i.i.d.* components. Under the fact that the transformed series from the financial log-returns is usually uncorrelated, the transformation from the original series to the *i.i.d.* series can be guaranteed by integrating these two parts. Due to the rescaling manipulations, $\alpha + \sum_{i=1}^q \tilde{c}_i = 1$, which serves to satisfy the requirement of variance stabilization. The optimal combination of α, a_1, b_1 is selected by minimizing $|KURT(W_t) - 3|$ to satisfy the normalizing requirement; here $KURT(W_t)$ is the kurtosis of the transformed series $\{W_t\}$. Empirically, $\{W_t\}$ is usually symmetrical, thus the kurtosis can be a simple metric to describe the distance between the distribution of $\{W_t\}$ and the standard normal distribution. Also, normalizing the marginal distribution is sufficient in our analysis.

After having determined the coefficients of this transformation function, we can apply the model-free prediction idea to set up our forecasting experiment. For example, if we consider the one-step-ahead prediction with observed $\{Y_1, \dots, Y_T\}$, we can first represent Y_{T+1} by W_{T+1} and \mathcal{F}_T which is the sigma-field of observed $\{Y_1, \dots, Y_T\}$, i.e.,

$$Y_{T+1} = \sqrt{W_{T+1}^2 (\alpha s_T^2 + \sum_{i=1}^q \tilde{c}_i Y_{T+1-i}^2)} = f_{GA}(W_{T+1}, \mathcal{F}_T), \quad (4)$$

where we use f_{GA} to denote that the above representation is derived from the GARCH-NoVaS method. the ideal case is that we know F_W which is the distribution of W_t and then we can approximate the distribution of Y_{T+1} by simulating W_{T+1} from F_W . Similarly for multi-step ahead predictions, we can represent Y_{T+h} by $\{W_{T+1}, \dots, W_{T+h}\}$ and \mathcal{F}_T as $Y_{T+h} = f_{GA}(W_{T+1}, \dots, W_{T+h}, \mathcal{F}_T)$. If F_W is known, we can still simulate the vector $\{W_{T+1}, \dots, W_{T+h}\}$ from F_W and approximate the distribution of Y_{T+h} . However, we can only capture the distribution of W_t by \hat{F}_W which is the empirical distribution of the transformed series in practice. Therefore, we have to replace the simulation technique with the bootstrap, i.e., we bootstrap M (taken as 2000 in this paper) sets of $\{W_{T+1,m}^*, \dots, W_{T+h,m}^*\}_{m=1}^M$ from \hat{F}_W . Then, we can approximate the optimal predictor of Y_{T+h} as follows:

$$\begin{aligned} L_1 \text{ optimal predictor: Median of } & \{f_{GA}(W_{T+1,m}^*, \dots, W_{T+h,m}^*, \mathcal{F}_T); m = 1, \dots, M\}; \\ L_2 \text{ optimal predictor: } & \frac{1}{M} \sum_{m=1}^M f_{GA}(W_{T+1,m}^*, \dots, W_{T+h,m}^*, \mathcal{F}_T). \end{aligned} \quad (5)$$

Moreover, by the continuing mapping theorem, we can further approximate the optimal prediction of $g(Y_{T+h})$ for any continuous function $g(\cdot)$.

2.2.2 GARCHX-NoVaS model

Recently, [Wu and Karmakar \(2023a\)](#) have extended the GARCH-NoVaS model to include exogenous variables, that is, they have developed a so-called GARCHX-NoVaS model via similar steps to find the transformation function of the GARCH-NoVaS model. In order to simplify the notation, we consider the case of only one exogenous covariate, X_t . The case of multiple exogenous covariates can be analyzed in an analogous way. In line with the GARCH-NoVaS transformation, we write the transformation function H_T corresponding with the GARCHX-NoVaS method as follows:

$$W_t = \frac{Y_t}{\sqrt{\alpha s_{t-1,Y}^2 + \beta s_{t-1,X}^2 + \sum_{i=1}^p a_1 b_1^{i-1} Y_{t-i}^2 + \sum_{i=1}^p c_1 b_1^{i-1} X_{t-i}^2}}, \quad (6)$$

where $s_{t-1,Y}^2$ and $s_{t-1,X}^2$ are the sample variance of $\{Y_1, \dots, Y_{t-1}\}$ and $\{X_1, \dots, X_{t-1}\}$, respectively. We set $p = q$ in our empirical research. Guided by the model-free prediction principle, the plan is to optimize the coefficients according to the variance stabilization and normalization requirement so as to get a qualified transformed series and its corresponding empirical distribution, \hat{F}_W . Also, we can express Y_{T+h} as

$$Y_{T+h} = f_{GAX}(W_{T+1}, \dots, W_{T+h}, \mathcal{F}_T, \mathcal{F}_{X,T+h}), \quad (7)$$

where $\mathcal{F}_{X,T+h}$ is the sigma-field of $\{X_1, \dots, X_{T+h}\}$ (we should notice that we assume that we know the future exogenous variables). Thus, the multi-step-ahead predictions of the GARCHX-NoVaS method can be computed by applying the same bootstrap approach as described explained in [Section 2.2.1](#). See [Wu and Karmakar \(2023a\)](#) for more details on the development of the GARCHX-NoVaS model.

3 Model evaluation

In order to evaluate the prediction performance of the different models, we consider two measures: (1) the sum of squared prediction errors (SSPE), with this statistic aiming to compare the prediction performance in an absolute way, and, (2) the CW test statistic proposed by [Clark and West \(2007\)](#), which, in turn, can be used to compare the forecasting performance of two nested models, i.e., to test whether a parsimonious null model and a larger model have equal predictive accuracy.

In order to define a suitable SSPE metric for long-term predictions ($h > 1$), we consider the below time-aggregated predictions as studied by [Wu and Karmakar \(2021\)](#) to measure the forecasting performance of the different models at an overall level (for other applications of this approach, see [Chudý et al. \(2020\)](#); [Karmakar et al. \(2022\)](#)):

$$\overline{Y}_{T,h}^2 = \sum_{k=1}^h (\widehat{Y}_{T+k}^2/h)^2, \quad (8)$$

where $\overline{Y}_{T,h}^2$ is the h -step ahead time-aggregated volatility prediction for $\{T+1, \dots, T+h\}$. In order to fully exhaust the dataset (which consists of a total of N observations), we further focus on moving-window out-of-sample predictions, i.e., we use $\{Y_1, \dots, Y_T\}$ to predict $\{Y_{T+1}^2, \dots, Y_{T+h}^2\}$, then we use $\{Y_2, \dots, Y_{T+1}\}$ to predict $\{Y_{T+2}^2, \dots, Y_{T+h+1}^2\}$, and so on until we reach the end of the sample (that is, until we use $\{Y_{N-T+h+1}, \dots, Y_{N-h}\}$ to predict $\{Y_{N-h+1}^2, \dots, Y_N^2\}$). Here, T denotes the moving-window size, which we fix at values between 240 and 500 in our empirical study. Thus, we can define the SSPE with the time-aggregated metric as below:

$$P = \sum_{l=T}^{N-h} (\overline{Y}_{l,h}^2 - \sum_{k=1}^h (Y_{l+k}^2/h))^2, \quad (9)$$

where $\overline{Y}_{l,h}^2$ denotes the time-aggregated prediction for each moving-window forecasting and $\sum_{k=1}^h (Y_{l+k}^2/h)$ denotes the corresponding realized average squared returns.

In addition to this numerical comparison, we consider the CW test proposed by [Clark and West \(2007\)](#) to verify whether the parsimonious null model and the nested model have equal predictive accuracy. For further details on the CW test, especially its application in the context of the type of analysis we consider in our empirical research, we refer a reader to the research by [Wu and Karmakar \(2023a\)](#) and [Clark and West \(2007\)](#).

4 Data

Our aim is to predict the volatility of the Johannesburg Stock Exchange (JSE) All Share Index (ALSI), i.e., JSE-ALSI, with the raw data of the index obtained from Global Financial Data (GFD).⁶ We convert the raw data to log-returns in percentages. The data for the controls of fundamentals- and sentiments-based information, i.e., the WTI oil, gold, and silver prices, are used to generate the log-returns (OR) of the oil price, and the ratio of the gold-to-silver prices (GS). The corresponding raw data were obtained from GFD and Macrotrends.⁷

The temperature anomaly (relative to a historical mean over 1991-2020) data for South Africa, upon specifying its coordinates, i.e., stretching latitudinally from 22°S to 35°S and longitudinally from 17 °E to 33°E, is available from the National Oceanic and Atmospheric Administration (NOAA).⁸ We work with the first difference of the temperature anomaly, and also apply the GARCH or NoVaS models to obtain the corresponding conditional volatilities of the temperature anomaly series to be used as an additional measure of climate risks. In particular, to capture climate risks, we compute the month-on-month change of the temperature anomaly, i.e., DTA, as well as the year-on-year change, i.e., DYTA, to avoid any concerns regarding seasonal effects.

Before analyzing the different forecasting models, we first check the properties of the log-returns, DTA, and DYTA to see whether the series indeed are heteroskedastic. We plot the three time series in [Fig. 1](#).

Eyeballing [Fig. 1](#), a volatility clustering phenomenon is quite obvious. In order to statistically validate this phenomenon, we apply the McLeod and Li (ML) test ([McLeod and Li, 1983](#)). The null hypothesis of the ML test is that there is no autoregressive conditional heteroskedasticity among the lags considered. The test, when applied to our data, produces p -values of near zero for all lags considered until the maximum (i.e., 31) allowed by the function “McLeod.Li.test” in *R* from the *TSA* package ([Chan et al., 2022](#)), with details of the results available upon request from the authors. Hence, we detect strong evidence of heteroskedasticity in the variables of our concern.

⁶<https://globalfinancialdata.com/>.

⁷<https://www.macrotrends.net/>.

⁸See: <https://www.ncei.noaa.gov/access/monitoring/climate-at-a-glance/global/time-series>.

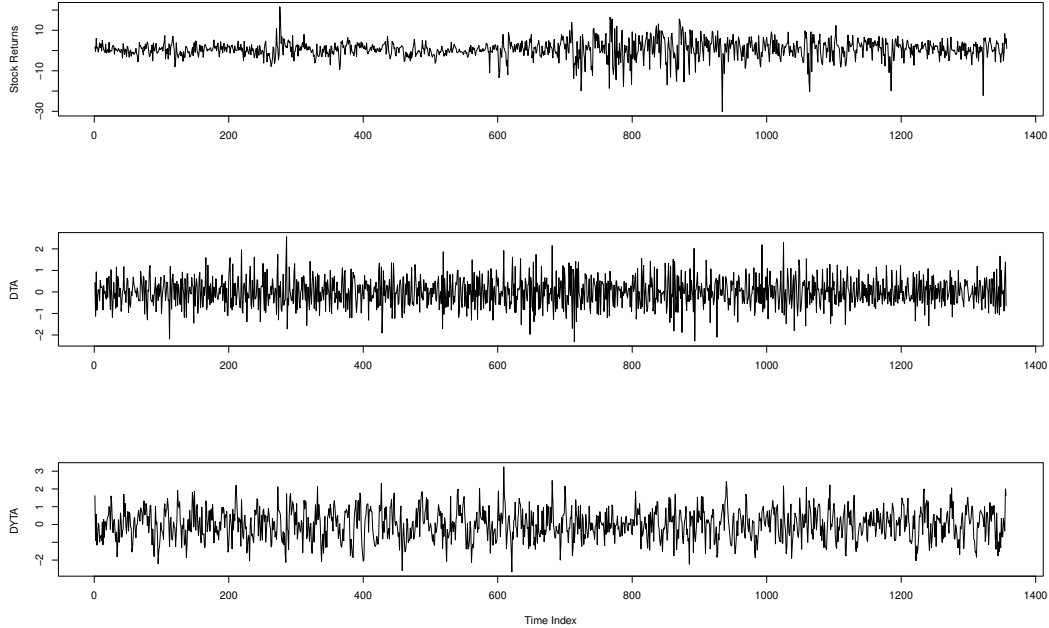


Figure 1: Plots of stock log-returns, DTA, and DYTA

5 Empirical results

In order to study the role played by oil log-returns, the ratio of the gold-to-silver prices, and climate risks, we include the different exogenous covariates in our forecasting models step by step and then distinguish four types of models:

- Stage-1 model: We apply the GARCH and GARCH-NoVaS models to compute predictions. These two models are the benchmark for classical and model-free type methods.
- Stage-2 model: We add OR and GS to the model. This results in GARCHX and GARCHX-NoVaS models with two covariates.
- Stage-3 model: We take DTA or DYTA data into account based on Stage 2 models. Meanwhile, we keep including OR and GS as exogenous variables.
- Stage-4 model: We estimate the volatilities of DTA and DYTA by means of GARCH or NoVaS models and then use the estimates as additional covariates. In order to simplify notation, we denote the volatility of DTA/DYTA estimated by a GARCH model as DTAV1/DYTAV1, while we use DTAV2/DYTAV2 to denote the volatility of DTA/DYTA as estimated by means of a NoVaS model.

In order to fully exhaust the dataset, we consider moving-window out-of-sample predictions, i.e., we make predictions based on a sliding window with 240 or 500 observations. For the prediction horizon, we consider 1-, 3-, 6-, and 12-step-ahead horizons.

We report our empirical results in [Tables 1 to 3](#). We summarize in [Table 1](#) the results of a comparison of the Stage-1 model and the Stages 2-4 models, where the GARCH model is the benchmark model. In [Table 2](#), we document the results of a comparison between the Stage-2 model and the Stage-3 models. We use the GARCHX-Stage-2 model as the benchmark model. Similarly, we summarize in [Table 3](#) the performance of the Stage-3 model relative to the Stage-4 models. In order to simplify the presentations of the SSPE, which is computed according to [Eq. \(9\)](#), we divide the SSPE of the GARCH model by the SSPE of the other models and denote this ratio as the Ratio of Squared Errors between those models and the benchmark, that is, we use this ratio to measure the relative performance of different models.

Table 1: Stage-1 comparisons

Prediction Step	Ratio of Squared Errors				P-value of CW-test			
	1	3	6	12	1	3	6	12
Moving window size 500 :								
GARCH (Benchmark)	1.000	1.000	1.000	1.000				
GARCH-NoVaS	0.998	0.957	0.883	0.746				
GARCHX-2	1.000	1.004	0.998	0.987	0.087	0.036	0.014	0.003
GARCHX-NoVaS-2	0.995	0.959	0.881	0.732	0.061	0.032	0.000	0.000
Moving window size 240 :								
GARCH (Benchmark)	1.000	1.000	1.000	1.000				
GARCH-NoVaS	1.070	1.025	0.908	0.684				
GARCHX-2	1.023	1.060	1.092	1.059	0.449	0.401	0.349	0.030
GARCHX-NoVaS-2	0.990	0.934	0.842	0.644	0.041	0.000	0.000	0.000

Note: GARCHX-2 and GARCHX-NoVaS-2 are Stage-2 models where the OR and GS information is involved in the prediction process based on the parsimonious model GARCH or NoVaS.

Table 2: Stage-2 comparisons

Prediction Step	Ratio of Squared Errors				P-value of CW-test			
	1	3	6	12	1	3	6	12
Moving window size 500 :								
GARCHX-2 (Benchmark)	1.000	1.000	1.000	1.000				
GARCHX-NoVaS-2	0.994	0.956	0.882	0.742				
GARCHX-3-DTA	1.000	1.000	1.005	0.996	0.386	0.309	0.986	0.102
GARCHX-NoVaS-3-DTA	1.004	0.947	0.878	0.732	0.894	0.001	0.013	0.000
GARCHX-3-DYTA	1.000	1.001	1.003	1.004	0.940	0.917	0.943	0.806
GARCHX-NoVaS-3-DYTA	0.997	0.955	0.882	0.734	0.525	0.143	0.187	0.001
Moving window size 240 :								
GARCHX-2 (Benchmark)	1.000	1.000	1.000	1.000				
GARCHX-NoVaS-2	0.967	0.882	0.771	0.608				
GARCHX-3-DTA	1.002	1.003	1.006	0.996	0.837	0.787	0.929	0.154
GARCHX-NoVaS-3-DTA	0.968	0.887	0.772	0.602	0.234	0.473	0.240	0.042
GARCHX-3-DYTA	1.004	1.009	1.009	1.010	0.994	0.998	0.976	0.847
GARCHX-NoVaS-3-DYTA	0.963	0.888	0.770	0.601	0.070	0.723	0.192	0.023

Note: the Stage-3 model takes DTA/DYTA into account, e.g., GARCHX-3-DTA represents the Stage-3 GARCHX model with OR, GS and DTA exogenous covariates.

Table 3: Stage-3 comparisons

Prediction Step	Ratio of Squared Errors				P-value of CW-test			
	1	3	6	12	1	3	6	12
Moving window size 500 :								
GARCHX-3-DTA (Benchmark)	1.000	1.000	1.000	1.000				
GARCHX-NoVaS-3-DTA	1.004	0.948	0.874	0.734				
GARCHX-4-DTAV1	1.001	1.000	0.995	0.992	0.576	0.298	0.037	0.014
GARCHX-NoVaS-4-DTAV1	1.001	0.960	0.881	0.749	0.156	0.991	0.721	0.960
GARCHX-4-DTAV2	1.000	0.999	0.993	0.990	0.419	0.103	0.001	0.002
GARCHX-NoVaS-4-DTAV2	0.999	0.962	0.876	0.743	0.081	0.993	0.404	0.808
Moving window size 240 :								
GARCHX-3-DYTA (Benchmark)	1.000	1.000	1.000	1.000				
GARCHX-NoVaS-3-DYTA	0.997	0.953	0.879	0.731				
GARCHX-4-DYTAV1	1.000	1.000	0.998	0.999	0.974	0.371	0.144	0.357
GARCHX-NoVaS-4-DYTAV1	1.005	0.957	0.885	0.740	0.961	0.647	0.797	0.801
GARCHX-4-DYTAV2	1.000	0.999	1.002	0.997	0.891	0.205	0.823	0.133
GARCHX-NoVaS-4-DYTAV2	1.003	0.957	0.887	0.740	0.926	0.581	0.882	0.808
Moving window size 240 :								
GARCHX-3-DTA (Benchmark)	1.000	1.000	1.000	1.000				
GARCHX-NoVaS-3-DTA	0.966	0.884	0.766	0.603				
GARCHX-4-DTAV1	1.003	1.003	0.999	1.015	0.789	0.811	0.269	0.923
GARCHX-NoVaS-4-DTAV1	0.973	0.880	0.756	0.585	0.762	0.091	0.008	0.005
GARCHX-4-DTAV2	1.001	1.000	1.001	1.003	0.700	0.481	0.477	0.516
GARCHX-NoVaS-4-DTAV2	0.977	0.882	0.761	0.585	0.941	0.176	0.066	0.008
Moving window size 240 :								
GARCHX-3-DYTA (Benchmark)	1.000	1.000	1.000	1.000				
GARCHX-NoVaS-3-DYTA	0.960	0.880	0.763	0.593				
GARCHX-4-DYTAV1	1.000	1.001	0.997	0.994	0.879	0.691	0.069	0.184
GARCHX-NoVaS-4-DYTAV1	0.965	0.867	0.752	0.562	0.777	0.002	0.053	0.001
GARCHX-4-DYTAV2	1.001	1.001	1.004	1.005	0.945	0.864	0.861	0.640
GARCHX-NoVaS-4-DYTAV2	0.967	0.872	0.752	0.580	0.853	0.012	0.026	0.020

Note: the Stage-4 model further considers the volatility of DTA and DYTA by taking Stage-3 models as the parsimonious ones. We use DTAV1 and DTAV2 to represent the volatility of DTA estimated by GARCH and NoVaS models, respectively. We can explain the meanings of DYTAV1 and DYTAV2 similarly. For example, GARCHX-4-DYTAV1 represents the Stage-4 GARCHX model with OR, GS, DYTA and volatility of DYTA estimated by GARCH.

The following results emerge from our forecasting experiment:

- *The effects of OR and GS:* The role of fundamentals- and sentiments-based information is revealed by the comparison of the Stage- 1 and 2 models in [Table 1](#). Taking the GARCH model as the benchmark, the Stage-2 GARCH model performs better when we use the SSPE statistic to evaluate 6 and 12 steps-ahead predictions (moving window of size 500). The results of the CW test corroborate that the MSPE of the GARCH-Stage-2 model is significantly smaller in a statistical sense than that of the benchmark model. However, for the moving window with 240 observations, the benchmark model beats the Stage-2 GARCH model. One reason may be that the sample size is not large enough to get a satisfactory estimation of the GARCH-X model. However, OR and GS are also statistically beneficial to the predictions when we study the NoVaS method. Moreover, this improvement can also be observed for the 240-moving-window.
- *The effects of DTA/DYTA:* The results that we report in [Table 2](#) show that, for GARCH-type models, with a 500- or 240-moving-window, the improvement in SSPE brought about by including DTA or DYTA in the models is negligible. Actually, the Stage-2 GARCH model outperforms the Stage-3 GARCH model, irrespective of whether we study DTA or DYTA, for 1, 3 and 6 steps-ahead predictions. The corresponding CW tests are not significant. The NoVaS-type models, however, can utilize climate information to yield more accurate forecasts. For example, the GARCHX-NoVaS-3-DTA model is better than the corresponding Stage-2 NoVaS model when we use a 500-moving-window. The corresponding CW test also implies that we can reject the null hypothesis. However, the gain in forecast accuracy is hardly visible for predictions based on a 240-moving-window, but it is still statistically significant at a significance level 0.05. According to our results, DTA is more useful when the moving window size is 500, and DYTA is more useful for a 240-moving-windows.
- *The effects of volatilities of DTA/DYTA:* According to [Table 3](#), the volatility of DTA and DYTA is almost useless to improve the forecast accuracy of the GARCHX models, and almost all CW tests when applied to the corresponding Stage-3 and -4 models cannot reject the null hypothesis. Interestingly, the NoVaS-type models produce some forecasting benefits after including the volatility of DTA or DYTA, especially for long prediction horizons and a short moving window. For two types of volatility, DTAV1 and DYTAV2, the forecasts are slightly more accurate than their counterparts estimated by the NoVaS model.
- *The effects of applying model-free NoVaS prediction technique:* It is evident from [Tables 1 to 3](#) that the NoVaS-type models are much better than the corresponding GARCH models for all 4 stages. More importantly, when we add climate risks to the NoVaS model, we observe that forecasting performance improves. The classical GARCH model, however, fails to take advantage of the information embedded in these covariates. All in all, the combination of the temperate anomaly and its volatility captured by a GARCH model gives the best model (Stage-4 NoVaS) due to its large MSE accuracy and robustness.

6 Conclusion

We have studied, using a dataset that covers more than a century, the contribution of climate risks to the accuracy of forecasts of stock returns volatility based on data for South Africa, an important emerging market economy. We have measured climate risks by studying temperature anomaly and/or its volatility. Our findings show that climate risks do have predictive value for stock market volatility, where the novel model-free prediction method (GARCHX-NoVaS) can incorporate the information embedded in climate data better than classical methods, as witnessed by the result that the NoVaS models that include climate information achieve a stronger improvement of forecast accuracy than GARCH-type models, and the fact that the NoVaS model with the volatility of changes in temperature anomaly estimated by the GARCH approach is the best model in terms of the forecast evaluation criterion and its robustness.

As outlined in the introduction, appropriate modeling and accurate forecasting of volatility based on factors (predictors), has ample implications for portfolio selection, the pricing of derivative securities and risk management, making it a metric of paramount importance to not only investors, but also policymakers. Hence, our findings indicate that the local climate risks can assist in terms of the abovementioned pertinent issues in South Africa, over the above information contained in (proxies of) fundamentals and sentiments. Academically speaking, we provide empirical confirmation of the theoretical predictions that link rare disaster risks, modeled through weather patterns, with stock returns volatility, in an emerging market-setting. In this regard, from a statistical perspective, we also show the role of a model-free approach in appropriately capturing and predicting volatility.

As part of future research, it would be interesting to extend our work to other emerging economies, conditional on the availability of long spans of historical data, as well as to the currency markets of South Africa, and other fossil-fuel exporters, following [Bonato et al. \(2023a\)](#).

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