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Book

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Forecasting Volatility of Commodity, Currency, and Stock Markets: Evidence from Markov Switching Multifractal Models

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Abstract

This paper adopts a bivariate Markov switching multifractal (MSM) model to reexamine co-movement in stochastic volatility between commodity, foreign exchange (FX) and stock markets. After the 2007-2008 global financial crisis understanding volatility linkages and the correlation structure between these markets becomes very important for risk analysts, portfolio managers, traders, and governments. Using daily data on stock indices and FX rates from developed and emerging countries and a range of commodities such crude oil, natural gas, aluminum, copper, gold, silver, platinum, wheat, corn, soybean and soybean oil we find evidence of (re)correlation between commodity, FX and stock markets. The bivariate MSM model compares favorably to a bivariate DCC-GARCH and univariate MSM model, especially at short (1, 5 and 10 days) forecasting horizons. Furthermore, we discuss its implications for risk and portfolio management.

Keywords: Multifractal processes, Volatility co-movement, Commodity returns, Foreign exchange returns, Stock returns

JEL classification: C53; C58; G15

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1 Introduction

The rapid financialization of the commodity markets after the 2007-2008 global financial crisis coupled with the increased financial integration between markets has led to an increase in dependence between foreign exchange (FX), stock and commodity markets over the past decade, see, e.g., Fry-McKibbin and McKinnon (2023), Wang and Cheung (2023), Delatte and Lopez (2013), Mensi et al. (2013), Du et al. (2011), Creti et al. (2013), Sadorsky (2014), Ding et al. (2021), Dai et al. (2020), Ali et al. (2020), Nguyen et al. (2020), and Uddin et al. (2020). Recently, Wang and Cheung (2023) find that the strongness of the link between commodity prices, financial assets and exchange rates in the post 2007-2008 global financial crisis period depends on the degree of financialization different commodities experience. As result, a better understanding of the relationship between FX, stock and commodity markets becomes very important due its implications for volatility forecasting, risk management, asset allocation, and monetary policy, see, e.g., Fry-McKibbin and McKinnon (2023), Wen and Wang (2021).

The literature has put forward different theoretical frameworks[2] to explain the link between commodity prices and other financial assets. While Papers by Chen et al. (2010) and Rossi (2012) argues the existence of a structural link between future commodity prices and exchange rates through the terms of trade and income channel, other works have established financial linkages across commodity and other financial markets, see, e.g., Tang and Xiong (2012). Cheng and Xiong (2014) explain how financialization has transformed the commodity markets through economic mechanisms such as storage, risk sharing and information discovery that facilitate the functioning of the commodity markets. As consequence, these transformations increase the interdependence between commodity markets and other financial markets. In fact, storage reduces fluctuations in commodity prices due its stabilizing forces on demand and supply shocks Deaton and Laroque (1996). In the standard storage theory of Kaldor (1939), Brennan (1958), and Telser (1958), the commodity producers have the option to adjust their production levels through inventories, and thus, the marginal benefit of storage is defined to be the convenience yield. This suggests that the future spreads between the futures and spot prices are impacted by the levels of nominal interest rates because they are the main determinants of the financing cost of the carry trade. Frankel (2006) and Gruber and Vigfusson (2018) show how monetary policy affects the commodity price volatility through this interest-rate channel.

Cheng and Xiong (2014) argue that financialization through risk sharing and information discovery channels has deepened and strengthened the linkage between commodity prices and other financial assets. Their argumentation is based on the hedging pressure theory of Keynes (1923), Hicks (1939) and Hirshleifer (1988). According to the theory financialization reduces hedging pressure, improves risk sharing, and facilitates volatility spillover to commodity markets,

[2]We refer the reader to Adekoya and Oloyide (2021) for a recent review on the linkages among different markets.

see Tang and Xiong (2012). In order to hedge commodity price risks, commodity producers that are on the short side of the futures markets have to offer the financial investors or speculators positive risk premia that give them the incentive to take the long side of futures markets. By taking the long side, financial investors or speculators act as providers of liquidity to hedgers, however, due to their time-varying risk appetites, they can become consumers of liquidity from hedgers whenever market conditions change. As result, the dual role of the financial investors causes volatility spillover from other financial markets to commodity markets. Furthermore, it is well-documented that futures markets play a crucial role in information discovery during periods of informational frictions in the global supply, demand and inventory of commodities. Singleton (2014) shows that the information discovery process can influence the expected returns of commodity futures due to the presence of heterogeneous financial investors.

In this paper our goal is to shed light on the implications of the relationship between commodity, stock and FX markets for forecasting accurate volatility in a multivariate framework. Forecasting volatility in asset markets is not only critical for portfolio selection, risk management and the pricing of derivatives, but is also of high importance for policymakers in designing monetary and fiscal policies as volatility shocks represent uncertainty (Liu et al. (2020); Liu and Gupta (2022)). Since the seminal papers by Engle (1982) various uni- and multivariate volatility models have been put forward and successfully used to produce accurate volatility forecasts in stock, commodity, FX markets, see, e.g., Zaharieva et al. (2020) for a recent review on multivariate stochastic volatility (SV) models, Bauwens et al. (2012) and Francq and Zakoïan (2019) for a review on multivariate GARCH models. Recent developments in modeling and forecast volatility in commodity and financial markets are the adaptation of the multifractal processes to finance by Mandelbrot (1974), Calvet and Fisher (2001, 2004)[3]. While the academic literature on modeling the linkage between stock, FX and commodity markets and forecasting volatility using the DCC-GARCH model of Engle (2002) is vast, see, e.g., Ashfaq et al. (2019), Kumar and Anandarao (2019) among others, it remains on this particular subject using the multivariate multifractal processes somewhat sparse. Our contribution is to close this gap in the literature.

To this end, we consider a new class of volatility models that are designed via the multifractal processes. In fact, the theory of multifractal measures was originally proposed by Mandelbrot (1974) to model turbulent flows and subsequently has received a huge attention in modeling and forecasting financial market volatility in the recent years due to certain similarities of volatility to fluid turbulence. The main reasons for this increased interest in applying multifractal processes for forecasting volatility are two folds: (i) the multifractal model provides a simple uniform framework for long-memory and fat tails in the volatility process and structural breaks through regime switching, (ii) it

[3]see Lux and Segnon (2018) for an excellent review on uni- and multivariate multifractal volatility models and their applications in finance.

has been showed to be robust and capable of producing more accurate volatility forecasts than the traditional (MS-)GARCH models, see Calvet and Fisher (2004), Lux (2008), among others.

We illustrate the performance of our bivariate multifractal model against the bivariate DCC-GARCH and univariate GARCH models using daily data on stock indices and FX rates from eight developed and emerging countries (Malaysia, Australia, Russia, South Africa, Norway, Mexico, Canada and Brazil) and a range of commodities such as crude oil, natural gas, Aluminum, copper, Gold, silver, Platinum, Wheat, corn, soybean, soybean oil. The rationale behind the selection of these particular countries stems from their significant roles as major exporters in these commodity markets. The economic and financial landscapes of these nations are deeply intertwined with the global movements of these commodities, making their currencies and stock markets highly responsive to changes in commodity prices. For example, Canada, Norway and Russia are influential in the global energy sector, especially in the markets for oil and natural gas. Australia's economic fortunes are strongly linked to the export of metals such as aluminum and gold, with South Africa being a prominent exporter of platinum and gold. In the same vein, Brazil is a key player in the market for soybeans.

Our out-of-sample results indicate that the bivariate multifractal model produces more accurate volatility forecasts and outperforms the DCC-GARCH and univariate multifractal models at short- and long forecasting horizons and across all three markets. This suggests that the bivariate multifractal framework is more appropriate to model volatility and dynamic conditional correlations between commodity, stock and foreign exchange markets.

The rest of the paper is organized as follows. Section 2 presents the data sets used in our empirical analysis. The bivariate multifractal model is presented in Section 3. Section 4 presents and discusses the empirical results and finally, Section 5 concludes.

2 Multifractal models

Most financial market models are based on the additive structure of asset returns dynamics, and models with multiplicative operations have been introduced under the heading of multifractal models. Mandelbrot et al. (1997) introduces the multifractal model of asset returns (MMAR), and the multifractal model assumes that returns r_t follow a compound process, in which an incremental fractional Brownian motion is subordinate to the cumulative distribution function of a multifractal measure. However, the practical applicability of MMAR suffers from the non-causal nature of the time transformation and non-stationarity due to the inherent restriction to a bounded interval. These limitations have been overcome by the development of an iterative version of the multifractal models, including the Markov-switching multifractal model (MSM), see Calvet and Fisher (2004) and Lux (2008). MSM models asset returns as:

$$r_t = \sigma \left(\prod_{i=1}^{k} M_t^{(i)} \right)^{1/2} \cdot \epsilon_t, \tag{1}$$

the instantaneous volatility is determined by the product of k volatility components or multipliers $M_t^{(1)}$, $M_t^{(2)}$ ..., $M_t^{(k)}$, with a constant scale parameter σ. In addition, M_t can be drawn from either a discrete distribution, e.g., a binomial distribution in Calvet and Fisher (2004), or a continuous distribution, e.g., lognormal distribution in Lux (2008). Each volatility component is updated at time t with probability γ_i depending on its rank within the hierarchy of multipliers or remains unchanged with probability $1 - \gamma_i$. Calvet and Fisher (2004) specify the transition probabilities as:

$$\gamma_i = 1 - (1 - \gamma_1)^{(b^{i-1})}, \tag{2}$$

with parameters $\gamma_1 \in (0, 1)$ and $b \in (1, \infty)$; In contrast, without introducing additional parameters, Lux (2008) proposes $\gamma_i = 2^{(k-i)}$. Both specifications guarantee convergence of the discrete-time multifractal process to a limiting continuous-time version with random renewals of the multipliers. In this approach, asset returns volatilities are conceived as hierarchical multiplicative processes with heterogeneous components at different lifetimes.

This novel approach preserves the hierarchical structure of MMAR, but dispenses with its restriction to a bounded interval. While this model is asymptotically "well-behaved" (i.e., it shares all the convenient properties of Markov-switching processes), it is still capable of capturing some important properties of financial markets time series data, namely, volatility clustering and the power-law behaviour of the autocovariance function of absolute moments:

$$Cov(|r_t|^q, |r_{t+\tau}|^q) \propto \tau^{2d(q)-1}. \tag{3}$$

2.1 Bivariate multifractal models

In order to study the interactions and comovements among financial assets, multifractal models can be easily extended to multivatiate setting without imposing much restrictions. For a bivariate models, Calvet and Fisher (2006) assume that instantaneous volatility is composed of heterogenous frequencies, and model the bivariate asset returns r_t as:

$$r_t = \sigma \otimes [g(M_t)]^{1/2} \otimes \epsilon_t. \tag{4}$$

Here, r_t, σ, and u_t are all bivariate vectors: $r_t = \begin{bmatrix} r_{1,t} \\ r_{2,t} \end{bmatrix}$, $\sigma = \begin{bmatrix} \sigma_1 \\ \sigma_2 \end{bmatrix}$, $\epsilon_t = \begin{bmatrix} \epsilon_{1,t} \\ \epsilon_{2,t} \end{bmatrix}$, and $\otimes$ denotes element by element multiplication. σ is the vector of constant scale parameters (the unconditional standard deviation); ϵ_t is a 2×1

vector whose elements follow a bivariate standard normal distribution, with an unknown correlation parameter ρ. $g(M_t)$ is the vector of the products of multifractal volatility components, i.e. $g(M_t) = \begin{bmatrix} g(M_{1,t}) \\ g(M_{2,t}) \end{bmatrix}$, with each element defined as in the univariate case:

$$g(M_{j,t}) = \prod_{i=1}^{k} M_{j,t}^{(i)}, \tag{5}$$

as the product of volatility components for asset j, and the bivariate volatility components at frequency i of series $j = 1, 2$:

$$M_t^{(i)} = \begin{bmatrix} M_{1,t}^{(i)} \\ M_{2,t}^{(i)} \end{bmatrix}. \tag{6}$$

$M_t^{(i)}$ are drawn from the bivariate binomial distribution $M = (M_1,\ M_2)'$, with M_1 taking values $m_1 \in (1, 2)$ and $2 - m_1$, and M_2 taking values $m_2 \in (1, 2)$ and $2 - m_2$. While the framework by Calvet et al., (2006) allow for variation of the correlation ρ_m between components M_1 and M_2, they report that a correlation ρ_m equal to one is never rejected in their empirical applications. We therefore restrict this parameter to unity to economize on the number of parameters to be estimated.

In addition, whether or not a volatility component (new arrival) being updated for the individual multifractal processes is governed by the transition probabilities, we use $\gamma_i = 2^{(k-i)}$ as in Lux (2008). The correlation of arrivals between the two series is characterized by a parameter $\lambda \in [0, 1]$, i.e., the probability of a new arrival at hierarchy level i for one time series given a new arrival in the other time series is $(1 - \lambda)\gamma_i + \lambda$. New arrivals are independent if $\lambda = 0$ and simultaneous if $\lambda = 1$.

2.2 Two-stage estimation

The dynamics of MSM with a binomial distribution multipliers can be considered as a special case of a Markov-switching process, therefore its likelihood function can be derived by determining the exact form of each possible component in the transition matrix since the state spaces are finite. Let r_t be the set of joint return observations $\{r_{j,t}\}$ for $j = 1, 2$, and $t = 1, 2 \ldots T$. The likelihood function can be explicitly evaluated as below:

$$\begin{aligned} f(r_1, \cdots, r_T; \Theta) &= \prod_{t=1}^{T} \left[f(r_t | M_t = m^i) \cdot \sum_{i=1}^{(2^n)^k} P(M_t = m^i | r_1, \cdots, r_{t-1}) \right] \\ &= \prod_{t=1}^{T} f(r_t | M_t = m^i) \cdot (\pi_{t-1} A). \end{aligned} \tag{7}$$

Θ is a set of parameters to be estimated, n is the number of assets. There are three elements within the likelihood function above, namely, (1) the density of the innovation r_t conditional on M_t, which is $f(r_t|M_t = m^i) = \frac{F_N\{r_t \div [\sigma \otimes \eta^{1/2}]\}}{\sigma \otimes \eta^{1/2}}$; $F_N\{\cdot\}$ denotes the bivariate standard normal density function, $\div$ represents element-by-element division, and $\eta = g(M_t)$; (2) a vector of conditional probability of $\pi_{t-1} = (\pi_{t-1}^1, \ldots, \pi_{t-1}^{(2^n)^k})$, and $\pi_t^i = P(M_t = m^i|r_1, \cdots, r_t)$; (3) the transition matrix A contains components A_{ij} which are $P(M_{t+1} = m^j|M_t = m^i)$. Note that $i, j = \{\ 1,\ 2\ \ldots\ (2^n)^k\}$ and it indicates the transition matrix A has the dimension of $4^k \times 4^k$ for a bivariate model.

Since the large degree of heterogeneity of volatility trajectories that can be modelled with a relatively large number of k is one of most attractive features of the multifractal approach, therefore, with a larger number of k, the numerous multiplications with the transition matrix A within an optimization step pose computational constraints on this straight forward approach of Eq. (7). One can easily see the computational complexity of evaluating the $4^k \times 4^k$ elements of the transition matrix at each time-step for the maximum likelihood estimation. In practice, it hardly works for the number of multipliers larger than 5, i.e., $k > 5$, due to the capacity of current personal computers. We therefore adopt a two-stage procedure proposed by Calvet and Fisher (2006), which combines an maximum likelihood (ML) estimator for the first group of parameters $\{m_{1,i}, \sigma_i\}$ with an simulation based ML estimator for the second group $\{\rho$ and $\lambda\}$. The second stage is to maximise the simulation based likelihood through the particle filter approach.

As particle filters treat the discrete support generated by the particles as the 'true' filtering density, this allows us to produce an approximation to the prediction probability density $P(M_{t+1} = m_t^i|r_t)$, by using the discrete support of the number B of particles, and then the one-step-ahead conditional probability is

$$\pi_{t+1}^i \propto f(r_{t+1}|M_{t+1} = m^i)\frac{1}{B}\sum_{b=1}^{B} P(M_{t+1} = m^i|M_t = m^{(b)}). \tag{8}$$

then the approximate likelihood function is given below:

$$g(r_1, \cdots, r_T; \Theta) \quad \approx \quad \prod_{t=1}^{T}\left[\frac{1}{B}\sum_{b=1}^{B} f(r_t|M_t = \hat{M}_t^{(b)})\right]. \tag{9}$$

The recursive particle filter procedure is detailed in the Appendix.

3 Empirical Results

3.1 Data Description

We collect commodity, stock indices, and foreign exchange rates data cover the period from 7/07/2014 to 25/01/2022. Daily returns are calculated as the log difference, denoted by $r_t = \ln(p_t) - \ln(p_{t-1})$, where p_t represents the prices of commodities, stock indices, and exchange rates. The commodities include natural gas, crude oil, soybean oil, corn, wheat, aluminum, soybean, copper, gold, silver and platinum. The stock indices pertain to the markets of eight countries: Malaysia, Australia, Russia, South Africa, Norway, Mexico, Canada, and Brazil. The exchange rates are for the currencies of these eight countries relative to the U.S. dollar. Data is obtained from Bloomberg terminal.

In selecting the currencies, stocks, and commodities for our study, we focus on countries that are major players in the export markets of these commodities. This choice is grounded in the substantial literature that highlights the critical role of export-oriented economies in the global commodity markets and their impact on related financial assets (Kocaarslan et al. (2017)). These countries are known for their substantial exports in these commodities, influencing global prices and market dynamics.

For example, Canada, Norway and Russia are among the world's largest oil and natural gas exporters, while Brazil is a significant exporter of various commodities, including soybeans, corn and crude oil.[4] Similarly, Australia's economy is closely tied to the export of metals such as aluminum and gold, as highlighted in the research by Golev and Corder (2016), which explores the importance of metal flows in the Australian economy. The influence of Mexico in the silver market is well-recognized, a point noted by Garner (2020). The choice of the South African Rand is motivated by South Africa's pivotal role in the platinum and gold markets.[5] The dominance of these countries in the commodity market renders their currencies and stock markets particularly sensitive to fluctuations in commodity prices. This interrelationship is evident in studies such as that by Kilian (2009), which highlights the impact of crude oil prices on macroeconomic variables, and in the work of Umar et al. (2021), which examines the relationship between equity markets and oil prices in some of these countries.

The summary statistics presented in the Table 1 provide a detailed snapshot of the behavior of stock indices (ST) and foreign exchange rates (FX) across eight countries, alongside the performance of eleven diverse commodities. For stock indices, we observe a spectrum of mean returns, with Russia displaying the highest at 0.039, while Malaysia shows a negative mean return of -0.011. This range indicates varying degrees of market performance across these nations. Volatility, as measured by standard deviation, is notably high in Brazil's stock market (1.641), suggesting significant market fluctuations, in contrast to Malaysia, which exhibits the lowest volatility (0.688). The skewness and kurtosis values across the board suggest deviations from a normal distribution, particularly in Canada and Brazil, where kurtosis values of 11.29 and 13.02, respectively, indicate the presence of outliers or extreme market movements. The Augmented Dickey and FullerDickey-Fuller (ADF) test results uniformly confirm the stationarity of the series across these markets, while the significant ARCH effects underscore the presence of volatility clustering.

[4]see, https://oec.world/en/profile/country/bra.

[5]see, https://commodity.com/data/south-africa/.

In examining the foreign exchange markets, the mean returns are more contained, with Brazil leading and Canada exhibiting the lowest. The volatility within these FX markets shows a marked contrast, with Russia's FX market displaying the highest standard deviation, denoting substantial volatility, while Canada's market is the least volatile. The skewness and kurtosis across these markets suggest non-normality, particularly in Russia and Norway, where the kurtosis indicates heavy-tailed distributions. The pronounced volatility clustering in these FX markets is evident from the ARCH test results, especially in Russia and Norway. In the commodities sector, the variation in mean returns, with some commodities like natural gas and platinum showing negative values, reflects diverse market trends. The standard deviation is markedly high for crude oil, suggesting significant price fluctuations, whereas gold exhibits the least volatility. The commodities demonstrate non-normal distributions, as indicated by the skewness and kurtosis values, with particularly high kurtosis in crude oil and natural gas, pointing towards potential outlier events.

3.2 Empirical Findings

We separate each time series data into two subsets (i.e., in-sample data used for estimation, and out-of-sample data for forecasting assessments). We estimate the in sample data from 07/07/2014 to 31/12/2018, and then perform out-of-sample evaluation of volatility forecasting using data from 01/01/2019 to 25/01/2022, based on the DCC-GARCH, and bivariate multifractal models.

The in-sample DCC-GARCH estimates are shown in Table 2 to 4. Specifically, we estimate the bivariate DCC-GARCH model of Engle (2002), which is an extension of the conventional GARCH model of Bollerslev (1986). With the univariate GARCH (1, 1) formulated as: $r = \mu + \epsilon_t$, and $\epsilon_t | I_{t-1} \sim N(0, \sigma_t)$, the volatility process follows: $\sigma_t = \omega + \alpha \cdot \epsilon_{t-1}^2 + \beta \cdot \sigma_{t-1}^2$. The dynamic conditional correlation (DCC) has a non-linear GARCH-type specification: $Q_t = (1 - a - b)\bar{Q} + a\epsilon_t \cdot \epsilon'_{t-1} + bQ_{t-1}$, where a and b are the so-called news and decay coefficients, respectively. $\bar{Q} = E[\epsilon_t \cdot \epsilon'_{t-1}]$ is the unconditional variance and covariance matrix of the standardized residuals (the unconditional covariance) and ρ_{12} represents the unconditional correlation coefficient in the matrix $\bar{Q}$. We find most of the unconditional returns are positive for stock indices and foreign exchange rates, while most of commodities returns are negative. Though most of GARCH persistent parameter β estimates are within a reasonable range, i.e., around 0.9, the GARCH reaction parameter estimates α for the Canada exchange rates, copper and platinum return volatilities are much lower, which are indicative of much less spikes in volatility. For the correlation coefficient ρ_{12}, we find that most of estimates for the stock markets and commodity markets correlation are positive, in contrast, most of estimates for the foreign exchange markets and commodity markets correlation are negative.

The bivariate multifractal models parameters estimates are presented in Tables 5 to 7. Note that the vector of bivariate multifractal model parameters consists of $\{m_{1,i}, \sigma_i, \rho, \lambda\}$, i stands for returns of stock index, foreign exchange rates and commodity prices, respectivly. We use the two-stage estimation procedure illustrated in the last section, that is, an ML estimator for the first group of parameters $\{m_{1,i}, \sigma_i\}$ and a simulation based ML estimator for the second group $\{\rho$ and $\lambda\}$. The latter are obtained through the particle filter approach keeping the first set of parameters at their ML-estimated values, and we then maximize the simulated likelihood using the Nelder-Mead algorithm.[6] The first four of these parameters (Table 5) could be identified by an estimator for a univariate multifractal model, while the remaining ones require the complete bivariate data set. In terms of 'fractality' of volatility as measured by the parameter m_1, we find that most of the estimates are around 1.2, which indicates assets returns exhibit reasonable persistence for stock markets, foreign exchange, and commodity markets. The unconditional volatility estimate of σ reveals that commodity returns are much volatile than those of the stock indices and foreign exchange rates.

In terms of the correlation of innovations ρ, we find that most of estimates for bivariate models of stock markets and commodity markets correlation are positive (Table 6), while for correlation estimates of the foreign exchange markets and commodity markets correlation are mostly negative (Table 7). We also observe that the correlations for pairs of oil and stock indices and pairs of oil and foreign exchange rates appearing to be more pronounced. Since the correlation across markets often pertain to the arrival of new volatility components, empirical estimates of λ show significant degree of co-movement for all pairs of assets return volatilites.

Table 8 to Table 16 present the out-of-sample forecasting performances at different horizons with a range of 1 day to 100 days. We report the relative mean square error (RMSE), i.e., the mean square error (MSE) divided by the respective statistics of the naive volatility predictor (derived using historical volatility). Therefore, any value smaller than 1 indicates an improvement relative to historical volatility. A glance at Table 8 reveals that the volatility forecasting performance of the univariate multifractal models is quite encouraging for stock indices, foreign exchange rates and commodities. Specifically, we observe most of the reported RMSEs are below 1 for 1, 5, 10 and 20 days horizons, except with some cases at the longer horizons of 50 and 100 days.

We then assess the forecasting performances of the bivariate models when using stock indices and commodities prices pairs, and foreign exchange rates and commodities prices pairs. Table 9 and 10 present volatility forecasting results based on DCC-GARCH models for the eight countries' stock indices and commodities data. We find that DCC-GARCH models perform well at short time horizons, while most of the RMSEs for longer horizons are above 1, for example, RMSEs of the stock markets and commodities of Canada and Malaysia at 10 to 100 days horizons. For the same pairs of data, Table 11 and 12 report the forecasting performances based on bivariate multifractal models. In general, bivariate multifractal models outperform the DCC-GARCH models statistically significant manner at long horizons under the RMSE criteria. The fact that these

[6]The two-stage approach provides a reduction in computation time, and it also makes the choice of larger number of cascade level k feasible. Note that, use $k = 8$ which is consistent with the existing literature.

gains are statistically significant relative to the DCC-GARCH models are also strongly confirmed by the dominant number of significant Diebold and Mariano (1995) test statistics. Turning to foreign exchange markets and commodities markets pair data, the forecasting results based on CC-GARCH models are reported in Table 13 and 14; the forecasting performances based on the bivariate multifractal models are provided in Table 15 and Table 16. Though our results are not entirely homogeneous, in general, bivariate multifractal models outperform the DCC models in most cases, and the multifractal models produce better forecasting performances by their ability to capture the same in a genuine, i.e., non-spurious-manner, by allowing for regime-switching cascades.

In addition, we are interested in studying if it is likely to exist a bidirectional relationship between the volatility process of the stock markets and commodity markets, the foreign exchange market and commodity returns, as discussed in the introduction. We find either stock markets or foreign exchange markets have major impacts on commodity markets, for instance, for those crude oil exporting countries: Malaysia, Norway, Mexico, Canada and Russia, including their stock market indices or exchange rates do not improve the oil price volatility forecasting. However, some commodity markets do significantly impact foreign exchange rates. As shown in Table 15, Canada exports not only crude oil, but also wheat, Aluminum and gold, we find including oil significantly improves the Canadian dollar exchange rates volatility forecasting at all horizons; For Malaysia, including natural gas also significantly improves the exchange rates volatility forecasting; For Mexican, silver price has a major impact on Mexican exchange rates volatility forecasting at short horizons.

3.3 Implications for risk management

The notion of correlation is central to modern risk management. In fact, correlation is used as a measure of dependence between financial assets in different markets. As a key input in volatility forecasting, dynamic portfolio management, and value-at-risk computation, a change in the correlation represents a risk. It is well-documented that while the correlations between financial assets may be low (or moderate) during low volatility regimes or in normal market circumstances due some arbitrage opportunities or heterogeneity in market participant behaviors, they are very strong during volatile periods or times of crisis. As result, an underestimation of correlation will lead to inaccurate estimates of portfolio volatility, and thus, to inaccurate value-at-risk forecasts, mispricing of short-lived options and inefficient capital allocation. This could on one side misguide financial institutions to hold less than the optimal amount of capital against potential losses from adverse market moves and on the other side also affect the financial decisions of market participants. Furthermore, a hidden correlation risk between commodity, FX and stock markets can negatively influence the flow of capital to a one of these markets that is experiencing a temporary liquidity crisis.

The correlation parameters in the bivariate MSM model are well estimated and allow us to refine our understanding of volatility linkages between commodity, stock and FX markets. Although the estimated correlation parameters are small, they are statistically significant different from zero at 5% confidence level and contribute to the improvement of volatility forecast accuracy of the bivariate MSM model over the DCC-GARCH and univariate multifractal model. While the DCC-GARCH model captures the time dependency between these markets, the parsimonious specification of the bivariate MSM consider several volatility and correlation states and allows us to model different market-dependent dynamics[7] and the occurrence of contagion phenomena. Our empirical results suggest that analysts should use the bivariate MSM model that seems to be more appropriate for modeling correlation structure between commodity, FX and stock markets and producing more accurate volatility forecasts.

4 Conclusion

This paper uses the bivariate MSM to reexamine volatility linkages between commodity, FX and stock markets. The parsimonious specification of the MSM framework permits to study the correlation between volatility components in several regimes. Using daily data on stock indices and FX rates from developed and emerging countries and a range of commodities such crude oil, natural gas, aluminum, copper, gold, silver, platinum, wheat, corn, soybean and soybean oil we estimate the two parameters, $\hat{\lambda}$ and $\hat{\rho}$ that characterize co-movement between volatility components and covariation between prices in stock and commodity (or FX and commodity) markets, respectively. The estimated correlation parameters across stock and commodity or exchange-rate and commodity pairs for all eight developed and emerging countries are statistically significant at 5 % confidence level, however remain low. We identify positive (negative) covariation between prices in stock and commodity markets (in FX and commodity markets). We have showed that the bivariate MSM model outperforms the DCC-GARCH and the univariate multifractal model at 1, 5, and 10 days forecasting horizons. Our results show that after the 2007-2008 global financial crisis there is an increase in interdependencies between commodity, FX and stock markets. As result, the bivariate MSM model seems to be capable to capture the dependency structures between these markets, and thus, produces more accurate volatility forecasts.

Our results have important implications for academics, investors and policy authorities. First, investors in commodity exporting countries can improve their portfolio allocation, pricing of derivative securities and risk management by accommodating the role of commodity market volatility into their models of volatility that are primarily multivariate in nature and capture long-memory and regime-changes, i.e., via the usage of bivariate MSM models. In this regard, from the perspective of an academician, our findings provide further evidence of the connectedness of stock and currency markets with the commodity sector, and hence highlights the influence of the process of financialization. Finally, since volatility provides high-frequency forecasts of uncertainty, which in turn

[7] The dynamics in calm and highly volatile markets may be different.

is known to contain leading information for economic activity, policymakers can nowcast low-frequency macroeconomic variables, and design appropriate policy responses in advance.

Future research can study the relationship between commodities, FX rates, stock indices and digital assets such as cryptocurrencies, NFTs, and DeFi. Given the important impact of digital assets on economic growth in the recent years, it would be interesting to reexamine co-movement in volatility and covariation in prices between the traditional financial markets (commodity, FX and stock markets) and digital financial markets and their implications for volatility forecasting, risk management, portfolio allocation and economic growth.

References

Adekoya, O. B. and J. A. Oloyide (2021). How COVID-19 drives connectedness among commodity and financial markets: Evidence from TVP-VAR and causality-in-quantiles techniques. *Resources Policy 70*, 101898.

Ali, S., E. Bouri, R. I. Czudaj, and S. J. H. Shahzad (2020). Revisiting the valuable role of commodities for international stock markets. *Resources Policy 66*, 101603.

Ashfaq, S., Y. Tang, and R. Maqbool (2019). Volatility spillover impact of world oil prices on leading asian energy exporting and importing economies' stock returns. *Energy 188*, 116002.

Bauwens, L., C. Hafner, and S. Laurent (2012). *Handbook of Volatility Models and Their Applications*, Chapter Volatility Models, pp. 1–45. John Wiley & Sons, Inc.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroscedasticity. *Journal of Econometrics 31*, 307 – 327.

Brennan, M. (1958). The supply of storage. *American Economic Review 48*, 50–72.

Calvet, L. and A. Fisher (2001). Forecasting multifractal volatility. *Journal of Econometrics 105*(1), 27 – 58.

Calvet, L. and A. Fisher (2004). Regime-switching and the estimation of multifractal processes. *Journal of Financial Econometrics 2*, 44–83.

Calvet, L. and A. Fisher (2006). Volatility comovement: a multifrequency approach. *Journal of Econometrics 131*, 179–215.

Chen, Y.-C., K. Rogoff, and B. Rossi (2010). Can exchange rates forecast commodity prices? *Quarterly Journal of Economics 125*, 1145–1194.

Cheng, I.-H. and W. Xiong (2014). Financialization of commodity markets. *The Annual Review of Financial Economics 6*, 419–441.

Creti, A., M. Joets, and V. Mignon (2013). On the link between stock and commodity markets' volatility. *Energy Economics 37*, 16–28.

Dai, X., Q. Wang, D. Zha, and D. Zhou (2020). Multi-scale dependence structure and risk contagion between oil, gold, and US exchange rate: a wavelet-based vine-copula approach. *Energy Economics 88*, 104774.

Deaton, A. and G. Laroque (1996). Competitive storage and commodity price dynamics. *Journal of Political Economy 104*, 896–923.

Delatte, A.-L. and C. Lopez (2013). Commodity and equity markets: Some stylized facts from a copula approach. *Journal of Banking & Finance 37*, 5346–5356.

Dickey, D. A. and W. A. Fuller (1979). Distribution of the estimators for autoregressive time series with a unit root. *Journal of the American Statistical Association 74*, 427–431.

Diebold, F. X. and R. S. Mariano (1995). Comparing predictive accuracy. *Journal of Business and Economic Statistics 13*, 253–263.

Ding, Q., J. Huang, and J. Chen (2021). Dynamic and frequency-domain risk spillovers among oil, gold, and foreign exchange markets: Evidence from implied volatility. *Energy Economics 102*, 105514.

Du, X., L. Y. Cindy, and D. J. Hayes (2011). Speculation and volatility spillover in the crude oil and agricultural commodity markets: A bayesian analysis. *Energy Economics 33*, 497–503.

Engle, R. (1982). Autoregressive conditional heteroscedasticity with estimates of the variance of united kingdom inflation. *Econometrica 50*, 987–1007.

Engle, R. (2002). Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. *Journal of Business and Economic Statistics 20*, 339–350.

Francq, C. and J. M. Zakoïan (2019). *Structure, Statistical Inference and Financial Applications* (2nd Edition ed.)., Chapter Multivariate GARCH Processes, pp. 273–316. Wiley.

Frankel, J. (2006). The effect of monetary policy on real commodity prices. NBER Working Paper 12713.

Fry-McKibbin, R. and K. McKinnon (2023). The evolution of commodity market financialization: Implications for portfolio diversification. *Journal of Commodity Markets 32*, 100360.

Garner, R. L. (2020). *In Mines of Silver and Gold in the Americas* (1st Edition ed.)., Chapter Long-term silver mining trends in Spanish America: A comparative analysis of Peru and Mexico, pp. 225–262. Routledge.

Golev, A. and G. Corder (2016). Modelling metal flows in the australian economy. *Journal of Cleaner Production 112*, 4296–4303.

Gruber, J. W. and R. J. Vigfusson (2018). Interest rates and the volatility and correlation of commodity prices. *Macroeconomic Dynamics 22*, 600–619.

Hicks, J. (1939). *Value and Capital*. Cambridge, UK: Oxford Univ. Press.

Hirshleifer, D. (1988). Residual risk, trading costs, and commodity futures risk premia. *The Review of Financial Studies 1*, 173–193.

Kaldor, N. (1939). Speculation and economic stability. *The Review of Economic Studies 7*, 1–27.

Keynes, J. (1923). Some aspects of commodity markets. *The Manchester Guardian Commercial, European Reconstruction Series 13*, 784–786.

Kilian, L. (2009). Not all oil price shocks are alike: Disentangling demand and supply shocks in the crude oil market. *American Economic Review 99*, 1053–1069.

Kocaarslan, B., R. Sari, A. Gormus, and U. Soytas (2017). Dynamic correlations between BRIC and US stock markets: The asymmetric impact of volatility expectations in oil, gold and financial markets. *Journal of Commodity Markets 7*, 41–56.

Kumar, A. S. and S. Anandarao (2019). Volatility spillover in crypto-currency markets: Some evidences from garch and wavelet analysis. *Physica A: Statistical Mechanics and its Applications 524*, 448–458.

Liu, R., R. Demirer, R. Gupta, and M. Wohar (2020). Volatility forecasting with bivariate multifractal models. *Journal of Forecasting 39*, 155–167.

Liu, R. and R. Gupta (2022). Investors'uncertainty and forecasting stock market volatility. *Journal of Behavioral Finance 23*, 327–337.

Lux, T. (2008). The Markov-switching multifractal model of asset returns: GMM estimation and linear forecasting of volatility. *Journal of Business and Economic Statistics 26*, 194–210.

Lux, T. and M. Segnon (2018). *The Oxford Handbook of Computational Economics and Finance*, Chapter Multifractal Models in Finance: Their Origin, Properties, and Applications, pp. 203–247. Oxford University Press.

Mandelbrot, B., A. Fisher, and L. Calvet (1997). A multifractal model of asset returns. Cowles Foundation Discussion Papers 1164, Cowles Foundation for Research in Economics, Yale University.

Mandelbrot, B. B. (1974). Intermittent turbulence in self similar cascades; divergence of high moments and dimension of the carrier. *Journal of Fluid Mechanics 62*, 331–358.

Mensi, W., M. Beljid, A. Boubaker, and S. Managi (2013). Correlations and volatility spillovers across commodity and stock markets: Linking energies, food, and gold. *Economic Modelling 32*, 15–22.

Nguyen, D. K., A. Sensoy, R. M. Sousa, and G. S. Uddin (2020). U.S. equity and commodity futures markets: Hedging or financialization? *Energy Economics 86*, 104660.

Rossi, B. (2012). The changing relationship between commodity prices and prices of other assets with global market integration. Paper presented at the a seminar organized by the IMF. http://www. imf. org/external/np/seminars/eng/2011/tur/pdf/rossi. pdf. Accessed Sept.

Sadorsky, P. (2014). Modeling volatility and correlations between emerging market stock prices and the prices of copper, oil and wheat. *Energy Economics 43*, 72–81.

Singleton, K. (2014). Investor flows and the 2008 boom/bust in oil prices. *Management Science 60*, 300–318.

Tang, K. and W. Xiong (2012). Index investment and financialization of commodities. *Financial Analysts Journal 68*, 54–74.

Telser, L. (1958). Futures trading and the storage of cotton and wheat. *Journal of Political Economy 66*, 233–255.

Uddin, G. S., J. A. Hernandez, S. J. H. Shahzad, and S. H. Kang (2020). Characteristics of spillovers between the us stock market and precious metals and oil. *Resources Policy 66*, 101601.

Umar, Z., N. Trabelsi, and A. Zaremba (2021). Oil shocks and equity markets: The case of GCC and BRICS economies. *Energy Economics 96*, 105155.

Wang, W. and Y.-W. Cheung (2023). Commodity price effects on currencies. *Journal of International Money and Finance 130*, 102745.

Wen, D. and Y. Wang (2021). Volatility linkages between stock and commodity markets revisited: Industry perspective and portfolio implications. *Resources Policy 74*, 102374.

Zaharieva, M. D., M. Trede, and B. Wilfling (2020). Bayesian semiparametric multivariate stochastic volatility with application. *Econometric Reviews 39*, 947–970.

Table 1: Summary statistics.

ST	Malaysia	Australia	Russia	Sth. Africa	Norway	Mexico	Canada	Brazil			
mean	-0.011	0.012	0.039	0.017	0.029	0.008	0.015	0.036			
Std.dev	0.688	0.998	1.139	1.106	1.109	0.964	0.978	1.641			
Min	-0.251	-1.182	-0.780	-0.780	-0.898	-0.510	-1.864	-1.048			
Max	12.746	16.826	11.345	12.413	11.259	7.688	50.116	17.523			
Skewness	-5.405	-10.203	-8.713	-10.227	-9.832	-6.638	-13.176	-15.994			
Kurtosis	6.626	6.766	7.435	7.261	5.842	4.744	11.294	13.023			
ADF	-29.071	-31.962	-30.809	-31.326	-31.949	-31.854	-14.838	-31.373			
ARCH	480.785	673.766	314.494	650.122	494.469	420.205	703.637	629.510			
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)			
FX	Malaysia	Australia	Russia	Sth. Africa	Norway	Mexico	Canada	Brazil			
mean	0.014	0.014	0.042	0.018	0.019	0.023	0.008	0.045			
Std.dev	0.386	0.609	1.173	0.994	0.726	0.814	0.476	1.060			
Min	-0.501	0.315	0.328	0.239	0.759	0.773	0.004	0.016			
Max	10.899	5.294	53.133	4.182	10.543	9.758	4.386	5.377			
Skewness	-3.470	-2.233	-17.346	-5.081	-4.066	-4.190	-1.969	-5.965			
Kurtosis	1.988	3.910	17.001	4.905	7.255	7.977	2.104	7.112			
ADF	-29.200	-30.032	-30.942	-30.555	-29.659	-30.805	-31.915	-32.897			
ARCH	389.028	182.056	742.848	90.593	502.482	167.106	120.448	69.537			
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)			
	Gas	Crude oil	Soyb. oil	Corn	Wheat	Alumi	Soybean	Copper	Gold	Silver	Platinum
mean	-0.002	-0.010	0.025	0.021	0.021	0.025	0.002	0.016	0.017	0.006	-0.019
Std.dev	3.140	3.288	1.372	1.534	1.723	1.179	1.287	1.303	0.861	1.616	1.475
Min	0.099	-2.958	-0.126	-0.968	0.284	0.060	-0.834	-0.145	-0.191	-0.764	-0.551
Max	7.043	76.601	5.143	16.913	3.693	5.423	10.761	4.599	6.946	13.521	10.576
Skewness	-18.055	-60.168	-9.278	-19.100	-5.974	-7.910	-10.495	-6.929	-5.863	-16.119	-13.638
Kurtosis	19.798	31.963	6.648	7.718	6.578	5.975	6.426	5.003	4.967	7.698	10.092
ADF	-34.412	-21.841	-29.902	-31.786	-31.761	-30.848	-19.451	-32.339	-31.466	-29.739	-28.941
ARCH	138.667	278.872	195.368	37.060	36.328	204.577	62.024	43.766	96.769	216.301	406.559
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Note: This table reports stock indices (ST) and foreign exchange rates (FX) of eight countries and eleven commodities returns. The specific statistics of reported are the mean, standard deviation, minimum (min.) and maximum (max.), skewness and kurtosis of returns, followed by ADF test statistics, i.e.,Augmented Dickey and Fuller (DF, 1981) unit root test. The optimal lag length in the ADF regression is chosen using the Schwarz Information criterion (SIC), namely we begin with a maximum of 20 lags and then use the SIC to select the optimal lag length. The last row reports the heteroskedasticity test statistics: we filter the return series using an autoregressive model with an order of 12 and then implement the Lagrange-Multiplier (LM) test examining the null-hypothesis of no ARCH, with the LM test statistic and the p-value (in parentheses) reported in the table.

Table 2: In-sample estimates DCC GARCH models.

	Malaysia	Australia	Russia	Sth. Africa	Norway	Mexico	Canada	Brazil			
ST											
μ	-0.006	0.016	0.041	0.016	0.05	0.012	0.017	0.069			
	(0.014)	(0.02)	(0.029)	(0.025)	(0.025)	(0.021)	(0.016)	(0.04)			
ω	0.005	0.01	0.033	0.019	0.043	0.032	0.013	0.08			
	(0.002)	(0.004)	(0.014)	(0.008)	(0.027)	(0.011)	(0.005)	(0.029)			
α	0.904	0.898	0.918	0.909	0.855	0.85	0.854	0.908			
	(0.02)	(0.02)	(0.023)	(0.018)	(0.055)	(0.029)	(0.028)	(0.023)			
β	0.086	0.091	0.053	0.072	0.103	0.107	0.124	0.056			
	(0.017)	(0.018)	(0.014)	(0.014)	(0.03)	(0.02)	(0.024)	(0.015)			
	Malaysia	Australia	Russia	Sth. Africa	Norway	Mexico	Canada	Brazil			
FX											
μ	0.007	0.025	0.022	0.041	0.022	0.033	0.022	0.024			
	(0.007)	(0.017)	(0.025)	(0.027)	(0.018)	(0.021)	(0.014)	(0.028)			
ω	0.001	0.002	0.04	0.014	0.003	0.062	0.219	0.123			
	(0.000)	(0.001)	(0.01)	(0.007)	(0.002)	(0.019)	(0.012)	(0.059)			
α	0.904	0.976	0.829	0.928	0.963	0.75	0.000	0.743			
	(0.013)	(0.009)	(0.023)	(0.017)	(0.013)	(0.048)	(0.001)	(0.085)			
β	0.095	0.019	0.148	0.062	0.03	0.16	0.142	0.159			
	(0.013)	(0.007)	(0.022)	(0.014)	(0.009)	(0.029)	(0.038)	(0.045)			
	Gas	Crude oil	Soyb. oil	Corn	Wheat	Alumi	Soybean	Copper	Gold	Silver	Platinum
μ	0.013	-0.028	-0.027	0.014	-0.001	-0.024	-0.002	-0.024	-0.002	-0.03	-0.065
	(0.066)	(0.059)	(0.033)	(0.036)	(0.049)	(0.032)	(0.032)	(0.036)	(0.022)	(0.037)	(0.033)
ω	0.083	0.104	0.002	0.037	0.109	0.198	0.024	1.197	0.002	0.006	0.928
	(0.042)	(0.040)	(0.003)	(0.014)	(0.062)	(0.051)	(0.008)	(0.042)	(0.002)	(0.006)	(0.245)
α	0.908	0.912	0.982	0.923	0.929	0.726	0.926	0.192	0.979	0.982	0.173
	(0.014)	(0.019)	(0.006)	(0.016)	(0.029)	(0.055)	(0.012)	(0.268)	(0.006)	(0.007)	(0.192)
β	0.089	0.071	0.016	0.057	0.035	0.126	0.061	0.052	0.018	0.015	0.12
	(0.014)	(0.014)	(0.006)	(0.012)	(0.013)	(0.028)	(0.011)	(0.026)	(0.005)	(0.005)	(0.04)

Note: This table reports the in-sample estimates of GARCH(1, 1) models for stock indices (ST), foreign exchange rates (FX) of eight countries and commodity prices. Since it is a two-stage estimation for bivariate DCC GARCH models, the parameters estimates of $\{\mu, \omega, \alpha, \beta\}$ for univariate GARCH models, which are estimated in the first stage.

Table 3: In-sample estimates DCC-GARCH models for stock indices and commodities.

	Australia				Brazil					Canada				
	ST	Gas	Alumi	Gold	ST	Oil	Corn	Soyb1	Soyb2	ST	Oil	Wheat	Alumi	Gold
ρ_{12}		-0.040	0.081	-0.076		0.248	0.138	0.132	0.156		0.417	0.081	0.171	0.044
		(0.031)	(0.030)	(0.036)		(0.070)	(0.031)	(0.030)	(0.057)		(0.070)	(0.030)	(0.072)	(0.043)
a		0.069	0.000	0.000		0.007	0.031	0.000	0.006		0.019	0.035	0.005	0.038
		(0.031)	(0.000)	(0.000)		(0.005)	(0.019)	(0.000)	(0.006)		(0.007)	(0.047)	(0.003)	(0.020)
b		0.000	0.805	0.812		0.986	0.738	0.833	0.982		0.973	0.000	0.993	0.831
		(0.248)	(0.123)	(1.150)		(0.012)	(0.077)	(0.394)	(0.025)		(0.009)	(1.761)	(0.004)	(0.055)

	Malaysia			Mexico					Norway			
	ST	Gas	Oil	ST	Oil	Copper	Gold	Silver	ST	Gas	Oil	Alumi
ρ_{12}		-0.001	0.058		0.203	0.187	-0.046	0.094		0.031	0.347	0.178
		(0.028)	(0.031)		(0.050)	(0.041)	(0.063)	(0.040)		(0.028)	(0.028)	(0.027)
a		0.044	0.022		0.008	0.013	0.016	0.011		0.000	0.032	0.000
		(0.029)	(0.033)		(0.006)	(0.010)	(0.009)	(0.010)		(0.000)	(0.030)	(0.000)
b		0.375	0.367		0.982	0.944	0.964	0.952		0.830	0.798	0.850
		(0.259)	(0.549)		(0.012)	(0.027)	(0.018)	(0.045)		(0.425)	(0.283)	(0.904)

	Russia							South Africa			
	ST	Gas	Oil	Wheat	Alumi	Gold	Platinum	ST	Alumi	Gold	Platinum
ρ_{12}		0.055	0.208	0.017	0.131	-0.014	0.095		0.210	-0.042	0.121
		(0.027)	(0.047)	(0.035)	(0.074)	(0.041)	(0.042)		(0.031)	(0.095)	(0.035)
a		0.000	0.026	0.000	0.023	0.024	0.004		0.097	0.006	0.003
		(0.000)	(0.037)	(0.000)	(0.024)	(0.012)	(0.022)		(0.037)	(0.003)	(0.117)
b		0.840	0.861	0.848	0.910	0.880	0.844		0.019	0.990	0.811
		(0.668)	(0.285)	(1.859)	(0.040)	(0.036)	(0.238)		(0.426)	(0.007)	(2.720)

Note: This table reports the in-sample estimates of DCC GARCH models for stock indicies (ST) of eight countries and commodity prices. Since DCC model of Engle (2002) has a non-linear GARCH type specification for the conditional correlation: $Q_t = (1-a-b)\bar{Q} + a\epsilon_t \cdot \epsilon'_{t-1} + bQ_{t-1}$. a and b are the so-called news and decay coefficients, respectively. $\bar{Q} = E[\epsilon_t \cdot \epsilon'_{t-1}]$ is the unconditional variance matrix of the standardized residuals (the unconditional correlation) and ρ_{12} represents the unconditional correlation coefficient in matrix $\bar{Q}$, which are estimated in the second stage.

Table 4: In-sample estimates DCC-GARCH models for exchange rates and commodities.

	Australia				Brazil					Canada				
	FX	Gas	Alumi	Gold	FX	Oil	Corn	Soyb1	Soyb2	FX	Oil	Wheat	Alumi	Gold
ρ_{12}		-0.027	-0.176	-0.397		-0.171	-0.166	-0.163	-0.227		-0.446	-0.124	-0.181	-0.349
		(0.027)	(0.146)	(0.049)		(0.067)	(0.034)	(0.035)	(0.055)		(0.055)	(0.031)	(0.032)	(0.047)
a		0.000	0.010	0.046		0.007	0.011	0.008	0.019		0.006	0.002	0.069	0.054
		(0.005)	(0.007)	(0.023)		(0.007)	(0.009)	(0.009)	(0.008)		(0.003)	(0.016)	(0.037)	(0.021)
b		0.847	0.985	0.903		0.987	0.945	0.937	0.960		0.990	0.876	0.000	0.866
		(0.466)	(0.020)	(0.058)		(0.016)	(0.039)	(0.040)	(0.019)		(0.004)	(0.054)	(0.733)	(0.049)
	Malaysia			Mexico					Norway					
	FX	Gas	Oil	FX	Oil	Copper	Gold	Silver	FX	Gas	Oil	Alumi		
ρ_{12}		-0.010	-0.133		-0.222	-0.107	-0.186	-0.245		-0.035	-0.335	-0.158		
		(0.042)	(0.034)		(0.075)	(0.060)	(0.051)	(0.045)		(0.032)	(0.036)	(0.041)		
a		0.008	0.000		0.020	0.013	0.055	0.029		0.060	0.038	0.024		
		(0.006)	(0.000)		(0.007)	(0.007)	(0.021)	(0.023)		(0.036)	(0.016)	(0.012)		
b		0.971	0.802		0.967	0.967	0.846	0.903		0.543	0.870	0.920		
		(0.012)	(1.530)		(0.011)	(0.016)	(0.062)	(0.115)		(0.232)	(0.060)	(0.039)		
	Russia							South Africa						
	FX	Gas	Oil	Wheat	Alumi	Gold	Platinum	FX	Alumi	Gold	Platinum			
ρ_{12}		-0.049	-0.410	-0.025	-0.091	-0.124	-0.262		-0.136	-0.342	-0.405			
		(0.029)	(0.130)	(0.032)	(0.041)	(0.253)	(0.036)		(0.048)	(0.044)	(0.027)			
a		0.000	0.012	0.049	0.067	0.005	0.010		0.010	0.056	0.043			
		(0.000)	(0.007)	(0.031)	(0.052)	(0.004)	(0.015)		(0.008)	(0.040)	(0.032)			
b		0.839	0.982	0.619	0.238	0.993	0.892		0.973	0.852	0.605			
		(0.298)	(0.014)	(0.128)	(0.118)	(0.009)	(0.064)		(0.021)	(0.137)	(0.414)			

Note: This table reports the in-sample estimates of DCC GARCH models for foreign exchange rates (FX) of eight countries and commodity prices. Since DCC model of Engle (2002) has a non-linear GARCH type specification for the conditional correlation: $Q_t = (1-a-b)\bar{Q} + a\epsilon_t \cdot \epsilon'_{t-1} + bQ_{t-1}$. a and b are the so-called news and decay coefficients, respectively. $\bar{Q} = E[\epsilon_t \cdot \epsilon'_{t-1}]$ is the unconditional variance matrix of the standardized residuals (the unconditional correlation) and ρ_{12} represents the unconditional correlation coefficient in matrix $\bar{Q}$, which are estimated in the second stage.

Table 5: In-sample estimates bivariate multifractal models.

	Malaysia	Australia	Russia	Sth. Africa	Norway	Mexico	Canada	Brazil			
ST											
m_1	1.254	1.218	1.293	1.296	1.284	1.274	1.236	1.244			
	(0.026)	(0.025)	(0.028)	(0.028)	(0.028)	(0.026)	(0.031)	(0.026)			
σ	0.803	1.47	0.712	0.56	0.866	1.042	1.082	0.962			
	(0.033)	(0.026)	(0.031)	(0.032)	(0.031)	(0.037)	(0.033)	(0.036)			
	Malaysia	Australia	Russia	Sth. Africa	Norway	Mexico	Canada	Brazil			
FX											
m_1	1.167	1.243	1.157	1.542	1.248	1.195	1.363	1.195			
	(0.022)	(0.035)	(0.033)	(0.042)	(0.036)	(0.036)	(0.041)	(0.043)			
σ	0.604	1.043	0.498	1.252	0.783	0.662	1.473	0.985			
	(0.015)	(0.024)	(0.028)	(0.025)	(0.032)	(0.036)	(0.027)	(0.033)			
	Gas	Crude oil	Soyb. oil	Corn	Wheat	Alumi	Soybean	Copper	Gold	Silver	Platinum
m_1	1.263	1.228	1.209	1.217	1.256	1.179	1.301	1.152	1.172	1.267	1.291
	(0.031)	(0.031)	(0.026)	(0.038)	(0.041)	(0.026)	(0.041)	(0.033)	(0.026)	(0.026)	(0.028)
σ	2.826	1.237	0.796	1.266	2.307	1.113	1.381	1.16	1.759	1.354	1.395
	(0.042)	(0.045)	(0.038)	(0.033)	(0.038)	(0.032)	(0.031)	(0.026)	(0.028)	(0.028)	(0.032)

Note: This table reports the in-sample estimates for univariate multifractal models for stock index(ST), foreign exchange (FX) and commodities returns. Since we adopt two-stage estimation for bivariate MSM models, the first step i sthe maximum likelihood estimates of $\{m_1, \sigma\}$ for each individual time series data, numbers in parentheses are standard errors.

Table 6: In-sample estimates of bivariate multifractal models for stock markets indices and commodities prices.

	Australia				Brazil					Canada				
	ST	Gas	Alumi	Gold	ST	Oil	Corn	Soyb1	Soyb2	ST	Oil	Wheat	Alumi	Gold
ρ		-0.061	0.084	-0.086		0.281	0.129	0.113	0.175		0.412	0.059	0.181	0.007
		(0.013)	(0.013)	(0.016)		(0.020)	(0.021)	(0.021)	(0.025)		(0.028)	(0.025)	(0.021)	(0.016)
λ		0.135	0.154	0.135		0.135	0.135	0.135	0.108		0.15	0.15	0.135	0.15
		(0.024)	(0.025)	(0.025)		(0.025)	(0.026)	(0.031)	(0.026)		(0.024)	(0.022)	(0.025)	(0.020)
	Malaysia			Mexico					Norway					
	ST	Gas	Oil	ST	Oil	Copper	Gold	Silver	ST	Gas	Oil	Alumi		
ρ		0.013	0.055		0.224	0.193	-0.042	0.081		0.041	0.347	0.178		
		(0.006)	(0.015)		(0.018)	(0.018)	(0.016)	(0.011)		(0.016)	(0.020)	(0.022)		
λ		0.135	0.224		0.120	0.153	0.145	0.177		0.160	0.187	0.166		
		(0.011)	(0.015)		(0.012)	(0.012)	(0.019)	(0.021)		(0.022)	(0.022)	(0.021)		
	Russia							South Africa						
	ST	Gas	Oil	Wheat	Alumi	Gold	Platinum	ST	Alumi	Gold	Platinum			
ρ		0.057	0.221	0.027	0.144	-0.017	0.106		0.232	-0.062	0.112			
		(0.015)	(0.027)	(0.028)	(0.031)	(0.036)	(0.021)		(0.022)	(0.028)	(0.027)			
λ		0.185	0.169	0.177	0.185	0.135	0.176		0.135	0.122	0.150			
		(0.022)	(0.024)	(0.024)	(0.026)	(0.026)	(0.031)		(0.025)	(0.025)	(0.026)			

Note: This table reports the in-sample estimates for the bivariate mutifractal models for stock indices (*ST*) of eight countries and commodity prices. Since we adopt two-stage estimation for bivariate MSM models, it is the second step of particle filter apprach (with number of particles being 10000), here we report the extra bivariate parameters estimates of $\{\rho, \lambda\}$, and numbers in parentheses are standard errors.

Table 7: In-sample estimates bivariate multifractal models for exchange rates and commodities prices.

	Australia				Brazil					Canada				
	FX	Gas	Alumi	Gold	FX	Oil	Corn	Soyb1	Soyb2	FX	Oil	Wheat	Alumi	Gold
ρ		-0.025	-0.177	-0.321		-0.217	-0.166	-0.157	-0.198		-0.488	-0.119	-0.177	-0.276
		(0.010)	(0.015)	(0.018)		(0.022)	(0.016)	(0.021)	(0.027)		(0.032)	(0.028)	(0.030)	(0.033)
λ		0.187	0.165	0.135		0.137	0.135	0.180	0.133		0.146	0.137	0.195	0.140
		(0.020)	(0.020)	(0.022)		(0.023)	(0.021)	(0.031)	(0.028)		(0.020)	(0.032)	(0.035)	(0.021)

	Malaysia			Mexico					Norway			
	FX	Gas	Oil	FX	Oil	Copper	Gold	Silver	FX	Gas	Oil	Alumi
ρ		-0.009	-0.096		-0.274	-0.101	-0.121	-0.214		-0.049	-0.314	-0.158
		(0.005)	(0.015)		(0.022)	(0.020)	(0.025)	(0.021)		(0.027)	(0.027)	(0.028)
λ		0.167	0.164		0.135	0.15	0.135	0.15		0.135	0.165	0.15
		(0.031)	(0.035)		(0.038)	(0.033)	(0.036)	(0.031)		(0.037)	(0.036)	(0.035)

	Russia							South Africa			
	FX	Gas	Oil	Wheat	Alumi	Gold	Platinum	FX	Alumi	Gold	Platinum
ρ		-0.047	-0.41	-0.014	-0.089	-0.036	-0.183		-0.142	-0.265	-0.406
		(0.012)	(0.015)	(0.020)	(0.017)	(0.018)	(0.011)		(0.015)	(0.017)	(0.021)
λ		0.135	0.135	0.115	0.135	0.15	0.158		0.135	0.215	0.157
		(0.027)	(0.025)	(0.025)	(0.020)	(0.026)	(0.031)		(0.033)	(0.036)	(0.036)

Note: This table reports the in-sample estimates of bivariate mutifractal models for foreign exchange rates (FX) of eight countries and commodity prices. Since we adopt two-stage estimation for bivariate MSM models, it is the second step of particle filter apprach (with number of particles being 10000), here we report the extra bivariate parameters estimates of $\{\rho, \lambda\}$, and numbers in parentheses are standard errors.

Table 8: The performance metrics for out-of-sample volatility forecasts based on the univariate multifractal model.

	Horizons	Gas	Oil	Corn	Wheat	Almum	Soyb. oil	Soybeam	Copper	Gold	Silver	Platinum
Commodities												
	1	0.936	0.978	0.979	0.992	0.945	0.920	0.958	0.964	0.948	0.913	0.955
	5	0.937	0.990	0.980	0.992	0.943	0.948	0.959	0.986	0.965	0.956	0.971
	10	0.951	0.994	0.984	0.999	0.951	0.958	0.976	0.994	0.983	0.967	0.978
	20	0.965	0.997	0.989	1.001	0.961	0.971	0.984	0.992	0.998	0.985	0.987
	50	0.991	1.001	0.997	1.000	0.977	0.986	0.99	0.996	1.001	0.989	0.994
	100	1.003	1.000	1.001	1.003	0.989	0.996	0.993	0.994	0.997	0.992	0.998
		Malaysia	Australia	Russia	Sth Africa	Norway	Mexico	Canada	Brazil			
ST												
	1	0.911	0.898	0.915	0.919	0.896	0.847	0.943	0.916			
	5	0.955	0.959	0.953	0.958	0.954	0.914	0.982	0.972			
	10	0.973	0.979	0.977	0.975	0.974	0.942	0.992	0.990			
	20	0.989	0.997	0.996	0.995	0.997	0.968	1.001	1.002			
	50	0.996	1.002	1.010	1.003	1.008	0.985	1.003	1.006			
	100	1.002	1.002	1.004	1.002	1.000	0.993	1.002	1.003			
		Malaysia	Australia	S.Africa	Norway	Mexico	Russia	Canada	Brazil			
FX												
	1	0.458	0.948	0.940	0.948	0.837	0.645	0.905	0.922			
	5	0.479	0.975	0.954	0.980	0.901	0.690	0.924	0.940			
	10	0.510	0.985	0.966	0.996	0.946	0.732	0.951	0.952			
	20	0.549	0.997	0.983	1.001	0.990	0.759	0.969	0.965			
	50	0.618	1.004	0.990	1.004	1.013	0.847	0.989	0.998			
	100	0.682	1.005	0.987	1.003	1.015	0.929	0.996	1.011			

Note: This table reports the relative MSE for the out-of-sample volatility forecasts based on the univariate multifractal models.

Table 9: The performance metrics for out-of-sample volatility forecasts from the DCC GARCH model.

	Australia						Brazil							
	ST	Gas	ST	Alumi	ST	Gold	ST	Oil	ST	Corn	ST	Soyb1	ST	Soyb2
1	0.711	0.944	0.714	0.959	0.713	0.965	0.793	0.925	0.794	0.987	0.794	0.882	0.793	0.956
5	0.845	0.954	0.847	0.968	0.842	0.976	0.933	0.973	0.939	0.985	0.936	0.898	0.936	0.969
10	0.989	0.978	0.989	0.980	0.988	0.991	0.990	0.983	0.998	0.991	0.998	0.905	0.998	0.993
20	1.079	0.986	1.062	0.992	1.057	1.010	1.013	0.985	1.017	0.988	1.017	0.919	1.013	1.004
50	1.042	1.019	1.038	1.010	1.062	1.013	1.002	1.018	1.008	1.002	1.010	0.944	1.010	1.012
100	1.026	1.025	1.029	1.010	1.026	0.998	0.999	1.006	1.011	1.007	1.011	0.969	1.011	1.015

	Canada								Malaysia					
	ST	Oil	ST	Wheat	ST	Alumi	ST	Gold	ST	Gas	ST	Oil		
1	0.727	0.927	0.724	0.986	0.725	0.963	0.731	0.965	0.862	0.945	0.861	0.926		
5	0.909	0.974	0.910	0.991	0.912	0.968	0.917	0.979	0.937	0.952	0.936	0.971		
10	1.045	0.984	1.042	0.997	1.047	0.982	1.042	0.992	1.033	0.975	1.034	0.986		
20	1.087	0.990	1.083	0.999	1.087	0.997	1.073	1.009	1.059	0.984	1.057	0.991		
50	1.029	1.018	1.023	1.005	1.032	1.004	1.022	1.006	1.017	1.020	1.012	1.011		
100	1.003	1.013	1.011	1.006	1.014	1.006	1.019	1.015	1.016	1.027	1.011	1.010		

	Mexico								Norway					
	ST	Oil	ST	Copper	ST	Gold	ST	Silver	ST	Gas	ST	Oil	ST	Alumi
1	0.779	0.925	0.778	0.982	0.771	0.965	0.778	0.947	0.831	0.944	0.831	0.929	0.832	0.965
5	0.856	0.970	0.851	0.996	0.858	0.979	0.858	0.965	0.892	0.954	0.897	0.973	0.894	0.967
10	0.896	0.983	0.896	0.998	0.896	0.995	0.899	0.969	0.943	0.978	0.945	0.984	0.944	0.984
20	0.970	0.985	0.971	1.005	0.971	1.008	0.973	0.976	1.002	0.986	1.009	0.985	1.012	0.995
50	0.987	1.018	0.982	1.002	0.982	1.011	0.987	0.983	1.004	1.020	1.009	1.018	1.015	1.003
100	0.998	1.006	0.998	1.093	1.001	0.998	0.993	0.986	1.003	1.029	1.008	1.009	1.012	1.005

Note: This table reports the relative MSE for the out-of-sample volatility forecasts based on the DCC model,*st* denotes stock index for each countries.

Table 10: The performance metrics for out-of-sample volatility forecasts from the DCC-GARCH model.

	Russia											
	ST	Gas	ST	Oil	ST	Wheat	ST	Alumi	ST	Gold	ST	Platinum
1	0.858	0.942	0.858	0.925	0.859	0.983	0.858	0.963	0.855	0.967	0.853	0.925
5	0.912	0.956	0.912	0.972	0.911	0.991	0.914	0.968	0.907	0.978	0.902	0.955
10	0.968	0.977	0.968	0.984	0.966	0.997	0.965	0.984	0.974	0.995	0.972	0.986
20	0.997	0.986	1.002	0.988	0.997	1.009	0.995	0.997	0.997	1.008	0.997	0.991
50	1.006	1.019	1.009	1.018	1.007	1.007	1.006	1.001	1.006	1.013	1.013	0.989
100	1.002	1.023	1.008	1.002	1.008	1.010	1.008	1.006	1.006	1.019	1.012	0.987

	South Africa					
	ST	Alumi	ST	Gold	ST	Platinum
1	0.791	0.962	0.789	0.965	0.795	0.928
5	0.878	0.966	0.878	0.977	0.877	0.957
10	0.973	0.984	0.975	0.994	0.970	0.984
20	1.028	0.997	1.021	1.010	1.021	0.998
50	1.015	1.001	1.019	1.015	1.018	0.990
100	1.006	1.006	1.018	0.991	1.016	0.991

Note: This table reports the relative MSE for the out-of-sample volatility forecasts based on the DCC model, *st* denotes stock index for each countries.

Table 11: The performance metrics for out-of-sample volatility forecasts from the bivariate multifractal model.

	Australia						Brazil							
	ST	Gas	ST	Alumi	ST	Gold	ST	Oil	ST	Corn	ST	Soyb1	ST	Soyb2
1	0.895	0.932*	0.897	0.947*	0.896	0.939*	0.914	0.977	0.917	0.976*	0.918	0.918	0.917	0.958
5	0.927	0.931*	0.929	0.946*	0.929	0.951*	0.952	0.985	0.955	0.974*	0.957	0.934	0.955	0.968
10	0.949*	0.945*	0.954*	0.954*	0.953*	0.974*	0.975*	0.984	0.977*	0.974*	0.979*	0.935	0.977*	0.988*
20	0.988*	0.954*	0.991*	0.961*	0.991*	1.004*	0.983*	0.988	1.003*	0.973*	1.003	0.944	1.003*	1.011
50	1.005*	0.989*	1.006*	0.985*	1.005*	1.024	1.016	1.004*	0.984*†	0.993*	1.005*	0.946*	1.016	1.012
100	1.014*	1.009*	1.015*	0.961*	1.015*	0.980*	1.018	1.002*	0.991*†	1.000*	1.008*	0.957	1.018	0.996*
	Canada								Malaysia					
	ST	Oil	ST	Wheat	ST	Alumi	ST	Gold	ST	Gas	ST	Oil		
1	0.942	0.976	0.941	0.988	0.943	0.940*	0.943	0.945*	0.857*	0.936*	0.859	0.978		
5	0.968	0.985	0.963	0.994	0.968	0.941*	0.967	0.958*	0.936	0.933*	0.935	0.985		
10	0.980*	0.984	0.975*	1.004	0.980*	0.946*	0.978	0.980*	1.030	0.944*	1.027*	0.985		
20	0.999*	0.988*	0.998*	1.011	0.999*	0.962*	0.999	1.004*	1.050*	0.953*	1.050*	0.988*		
50	1.008*	1.004*	1.009*	1.019	1.009*	0.985*	1.009*	0.993*	1.007*	0.989*	1.007*	1.004*		
100	0.994*†	1.002*	1.003*	1.040	1.009*	0.967*	1.010*	0.998*	1.002*	1.008*	1.002*	1.002*		
	Mexico								Norway					
	ST	Oil	ST	Copper	ST	Gold	ST	Silver	ST	Gas	ST	Oil	ST	Alumi
1	0.830†	0.977	0.837	0.964*	0.834	0.946*	0.832	0.913*	0.890†	0.930*	0.901	0.977	0.889	0.944*
5	0.888	0.985	0.886	0.990*	0.883	0.959*	0.880	0.943*	0.932	0.930*	0.938	0.985	0.929	0.941*
10	0.906	0.984	0.904	1.006	0.901	0.980*	0.901	0.954*	0.945	0.944*	0.953	0.984	0.943	0.950*
20	0.958*	0.988	0.961*	1.005	0.957*	1.006	0.955*	0.996	0.997	0.954*	0.995*	0.988	0.997*	0.963*
50	0.975*	1.004*	0.968*	1.027	0.967*	1.020	0.968*	0.991	1.032	0.988*	1.032	1.004*	1.032	0.984*
100	1.037	1.002	1.035	1.000*	1.036	0.980*	1.037	0.960*	1.024	1.009*	1.025	1.002*	1.025	0.962*

Note: This table reports the relative MSE for the out-of-sample volatility forecasts based on the bivariate multifractal model, *st* denotes stock index for each countries. $*$ indicates an improvement of the bivariate MF model against the DCC GARCH model at the 5% level; † indicates the best forecast of each country's ST when combining different commodities at the 5% level; All comparisons are based on the test statistics of Diebold and Mariano (1995).

Table 12: The performance metrics for out-of-sample volatility forecasts from the bivariate multifractal model.

	Russia											
	ST	Gas	ST	Oil	ST	Wheat	ST	Alumi	ST	Gold	ST	Platinum
1	0.908	0.932*	0.895	0.978	0.906	0.992	0.906	0.957*	0.903	0.945*	0.901	0.955
5	0.935	0.930*	0.931	0.985	0.933	0.994	0.933	0.944*	0.933	0.957*	0.935	0.966
10	0.962*	0.943*	0.957*	0.984	0.959*	1.004	0.959*	0.955*	0.959*	0.979*	0.961*	0.970*
20	0.993	0.953*	0.993*	0.988	0.990*	1.008	0.991	0.968*	0.993	1.004*	0.992	0.979*
50	1.034	0.991*	1.000*	1.004*	1.033	1.011	1.033	0.988*	1.035	1.007*	1.033	0.983*
100	1.030	1.012*	1.001*	0.997*	1.029	1.029	1.029	0.960*	1.031	0.981*	1.029	0.981*
	South Africa											
	ST	Alumi	ST	Gold	ST	Platinum						
1	0.905	0.941*	0.904	0.946*	0.906	0.952						
5	0.929	0.943*	0.928	0.959*	0.931	0.962						
10	0.946*	0.952*	0.946*	0.981*	0.948*	0.968*						
20	0.986*	0.964*	0.986*	1.005*	0.987*	0.977*						
50	1.010*	0.986*	0.998*	1.006*	1.010*	0.982*						
100	0.998*	0.967*	1.011*	0.981*	1.007*	0.981*						

Note: This table reports the relative MSE for the out-of-sample volatility forecasts based on the bivariate multifractal model, *st* denotes stock index for each countries. $*$ indicates an improvement of the bivariate MF model against the DCC GARCH model at the 5% level; † indicates the best forecast of each country's ST when combining different commodities at the 5% level; All comparisons are based on the test statistics of Diebold and Mariano (1995).

Table 13: The performance metrics for out-of-sample volatility forecasts based on DCC GARCH models.

	Australia						Brazil							
	FX	Gas	FX	Alumi	FX	Gold	FX	Oil	FX	Corn	FX	Soyb1	FX	Soyb2
1	0.957	0.944	0.955	0.963	0.953	0.968	0.934	0.925	0.926	0.987	0.936	0.882	0.933	0.953
5	0.981	0.954	0.979	0.968	0.971	0.977	0.956	0.973	0.951	0.985	0.951	0.898	0.953	0.969
10	0.994	0.978	0.991	0.982	0.992	0.994	0.972	0.983	0.970	0.991	0.974	0.905	0.975	0.993
20	1.012	0.986	1.010	0.997	1.011	1.009	0.989	0.985	0.983	0.988	0.986	0.919	0.982	1.007
50	1.032	1.019	1.031	1.004	1.025	1.003	1.003	1.018	1.010	1.002	1.006	0.944	1.006	1.011
100	1.034	1.025	1.035	1.009	1.035	0.985	1.002	1.006	1.011	1.007	1.011	0.969	1.012	1.011
	Canada								Malaysia					
	FX	Oil	FX	Wheat	FX	Alumi	FX	Gold	FX	Gas	FX	Oil		
1	0.978	0.927	0.978	0.986	0.978	0.963	0.978	0.967	0.424	0.944	0.495	0.925		
5	0.992	0.974	1.004	0.991	1.007	0.968	1.008	0.976	0.483	0.954	0.609	0.970		
10	1.002	0.984	1.010	0.997	1.012	0.982	1.011	0.996	0.511	0.978	0.792	0.984		
20	1.003	0.990	1.008	0.999	1.003	0.997	1.002	1.009	0.548	0.986	0.904	0.988		
50	1.011	1.018	1.002	0.998	1.003	1.004	0.998	1.006	0.617	1.025	0.939	1.018		
100	1.012	1.013	1.001	1.002	1.005	1.006	1.006	0.989	0.739	1.029	1.102	1.002		
	Mexico								Norway					
	FX	Oil	FX	Copper	FX	Gold	FX	Silver	FX	Gas	FX	Oil	FX	Alumi
1	0.769	0.925	0.758	0.982	0.755	0.968	0.753	0.947	0.934	0.944	0.941	0.929	0.937	0.962
5	0.816	0.970	0.804	0.996	0.805	0.979	0.814	0.965	0.983	0.954	0.985	0.973	0.981	0.966
10	0.912	0.983	0.905	0.998	0.911	0.992	0.917	0.969	1.018	0.978	1.012	0.984	1.017	0.984
20	0.980	0.985	0.987	1.005	0.982	1.008	0.984	0.976	1.024	0.986	1.025	0.985	1.024	0.997
50	0.998	1.018	1.012	1.002	1.009	1.003	1.007	0.983	1.024	1.020	1.028	1.018	1.026	1.001
100	0.998	1.006	1.012	1.093	1.011	0.990	1.013	0.986	1.017	1.029	1.020	1.009	1.019	1.006

Note: This table reports the relative MSE for the out-of-sample volatility forecasts based on the DCC model,FX denotes foreign exchange rate each country's currency to the US dollar.

Table 14: The performance metrics for out-of-sample volatility forecasts from the DCC-GARCH model.

	Russia											
	FX	Gas	FX	Oil	FX	Wheat	FX	Alumi	FX	Gold	FX	Platinum
1	0.653	0.942	0.653	0.925	0.658	0.983	0.657	0.963	0.653	0.967	0.653	0.925
5	0.705	0.956	0.710	0.972	0.711	0.991	0.707	0.968	0.705	0.977	0.705	0.955
10	0.755	0.977	0.751	0.984	0.758	0.997	0.756	0.984	0.755	0.994	0.755	0.986
20	0.752	0.986	0.759	0.988	0.762	1.009	0.759	0.997	0.752	1.008	0.752	0.991
50	0.829	1.019	0.825	1.018	0.833	1.007	0.822	1.001	0.829	1.013	0.829	0.989
100	0.885	1.023	0.879	1.002	0.876	1.007	0.879	1.006	0.885	1.019	0.885	0.981

	South Africa					
	FX	Alumi	FX	Gold	FX	Platinum
1	0.943	0.962	0.943	0.967	0.951	0.925
5	0.959	0.966	0.955	0.979	0.957	0.952
10	0.975	0.984	0.971	0.994	0.973	0.986
20	1.013	0.997	1.013	1.008	1.010	0.998
50	1.023	1.001	1.024	1.005	1.021	0.989
100	1.028	1.006	1.028	0.989	1.022	0.986

Note: This table reports the relative MSE for the out-of-sample volatility forecasts based on the DCC model,FX denotes foreign exchange rate each country's currency to the US dollar.

Table 15: The performance metrics for out-of-sample volatility forecasts from the bivariate multifractal model.

	Australia						Brazil							
	FX	Gas	FX	Alumi	FX	Gold	FX	Oil	FX	Corn	FX	Soyb1	FX	Soyb2
1	0.952	0.934*	0.951	0.940*	0.949	0.950	0.923*	0.978	0.917*	0.975*	0.921*	0.918	0.923*	0.959
5	0.972*	0.932*	0.972*	0.940*	0.970	0.957	0.944*	0.985	0.940*	0.974*	0.942*	0.934	0.942*	0.968
10	0.981*	0.946*	0.982*	0.950*	0.978	0.977*	0.961*	0.984	0.959*	0.973*	0.956*	0.936	0.957*	0.989
20	0.998*	0.955*	0.999*	0.964*	0.994	1.003*	0.976*	0.989	0.972*	0.972*	0.967*	0.945	0.967*	0.997*
50	1.021*	0.990*	1.020*	0.986*	1.015	1.017*	0.985*	1.004*	0.988*	0.993*	0.986*	0.946	0.991*	1.003*
100	1.022*	1.010*	1.025*	0.966*	1.021*	0.972*	0.992*	1.003	1.001*	1.002*	0.997*	0.956*	1.002*	1.002*
	Canada								Malaysia					
	FX	Oil	FX	Wheat	FX	Alumi	FX	Gold	FX	Gas	FX	Oil		
1	0.903*†	0.977	0.947*	0.990	0.942*	0.945*	0.935*	0.947*	0.448†	0.932*	0.655	0.978		
5	0.924*†	0.984	0.949*	0.995	0.951*	0.944*	0.950*	0.957*	0.471*†	0.930*	0.776	0.985		
10	0.945*†	0.983	1.021*	1.006	1.025	0.954*	1.015	0.977*	0.496*†	0.945*	0.862	0.985		
20	0.964*†	0.988	1.065	1.011	1.071	0.966*	1.073	1.005*	0.543*†	0.952*	0.935	0.989		
50	1.007*†	1.003*	1.141	1.015	1.141	0.988*	1.146	1.021	0.600*†	0.988*	1.040	1.004*		
100	1.007*†	1.002*	1.203	1.038	1.206	0.970*	1.217	0.978*	0.694*†	1.008*	1.244	1.003		
	Mexico								Norway					
	FX	Oil	FX	Copper	FX	Gold	FX	Silver	FX	Gas	FX	Oil	FX	Alumi
1	0.839	0.977	0.826	0.963*	0.828	0.944*	0.816*†	0.910*	0.945	0.931*	0.950	0.977	0.948	0.942*
5	0.890	0.984	0.868	0.989*	0.875	0.956*	0.860*†	0.942*	0.970*	0.932*	0.977*	0.985	0.973*	0.943*
10	0.929	0.984	0.915	1.008	0.915	0.978*	0.908*†	0.953*	0.993*	0.944*	0.995*	0.984	0.995*	0.954*
20	0.984	0.988	0.986	1.007	0.983	1.005	0.980*†	0.995	1.003*	0.952*	1.001*	0.989	1.003*	0.968*
50	1.054	1.004*	1.051	1.029	1.050	1.022	1.051	0.992	1.015*	0.991*	1.015*	1.004*	1.015*	0.990*
100	1.101	1.002	1.100	1.002*	1.098	0.979*	1.100	0.960*	1.017	1.005*	1.018	1.003*	1.017	0.970*

Note: This table reports the relative MSE for the out-of-sample volatility forecasts based on the bivariate multifractal model, FX denotes foreign exchange rate each country's currency to the US dollar. $*$ indicates an improvement of the bivariate MF model against the DCC GARCH model at the 5% level; † indicates the best forecast of each country's FX when combining different commodities at the 5% level; All comparisons are based on the test statistics of Diebold and Mariano (1995).

Table 16: The performance metrics for out-of-sample volatility forecasts from the bivariate multifractal model.

	Russia											
	FX	Gas	FX	Oil	FX	Wheat	FX	Alumi	FX	Gold	FX	Platinum
1	0.635*†	0.931*	0.638*	0.978	0.645*	0.989	0.645*	0.942	0.640*	0.947*	0.645*	0.951
5	0.675*	0.932*	0.673*	0.984	0.685*	0.994	0.682*	0.944	0.678*	0.956*	0.668*	0.962
10	0.729*	0.945*	0.731*	0.984	0.739*	1.006	0.732*	0.952	0.730*	0.979*	0.718*	0.968*
20	0.732*	0.956*	0.732*	0.989	0.740*	1.008	0.734*	0.964	0.733*	1.007	0.720*	0.978*
50	0.794*	0.992*	0.780*	1.003*	0.805*	1.014	0.796*	0.985	0.796*	1.022	0.773*	0.981*
100	0.821*	1.009*	0.818*	1.003	0.835*	1.036	0.826*	0.964	0.827*	0.981*	0.804*	0.982

	South Africa					
	FX	Alumi	FX	Gold	FX	Platinum
1	0.940	0.939*	0.938*†	0.944*	0.947	0.950
5	0.954*	0.942*	0.945*†	0.953*	0.958	0.961
10	0.967*	0.951*	0.964*	0.975*	0.970	0.967*
20	1.001	0.966*	0.995*	0.984*	0.996*	0.976*
50	1.022	0.984*	1.019*	0.985*	1.010*	0.978*
100	1.020	0.963*	1.022*	0.977*	1.010*	0.979*

Note: This table reports the relative MSE for the out-of-sample volatility forecasts based on the bivariate multifractal model, FX denotes foreign exchange rate each country's currency to the US dollar. $*$ indicates an improvement of the bivariate MF model against the DCC GARCH model at the 5% level; † indicates the best forecast of each country's FX when combining different commodities at the 5% level; All comparisons are based on the test statistics of Diebold and Mariano (1995).

A Particle filter

In the second stage of estimation, we adopt simulation-based maximum likelihood (SML) approach proposed by Calvet and Fisher (2006). Specifically, by keeping the first set of parameters at their ML-estimated values, we maximize the simulated likelihood using particle filter. The two-stage approach provides a reduction in computation time against a complete SML approach, and it also makes the choice of larger number of cascade level k feasible. Note that, use $k = 8$ which is consistent with the existing literature.

Recall the conditional probability $\pi_t^i = P(M_t = m^i | r_1, \cdots, r_t)$, and due to $\sum_{i=1}^{4^n} \pi_t^i = 1$; by Bayesian updating, we get

$$\pi_{t+1} = \frac{f(r_{t+1}|M_{t+1} = m^i) \otimes (\pi_t A)}{\sum f(r_{t+1}|M_{t+1} = m^i) \otimes (\pi_t A)}. \tag{A1}$$

In order to reduce the computational burden, instead of explicitly evaluating the exact $4^k \times 4^k$ elements within the transition matrix, the particle filter uses an approximation to the prediction probability density $P(M_t = m_t^i | r_{t-1})$, by using the discrete support of a finite number B of particles. Denoting by $m^{(b)}$ the volatility state of any particle $b = 1, \ldots, B$, the one-step-ahead conditional probability is approximated by:

$$\pi_t^i \propto f(r_t | M_t = m^i) \frac{1}{B} \sum_{b=1}^{B} P(M_t = m^i | M_{t-1} = m^{(b)}). \tag{A2}$$

As can be seen, Eq (A2) provides a discrete approximation of the conditional densities by filtering the particles. This approximation not only simplifies the maximum likelihood estimation by avoiding evaluation of the infeasible dimensions of the transition matrix, but also provides a practical solution for multi-step forecasting. For instance, to obtain one-step ahead π_{t+1}^i, we use particle filter with sampling/importance resampling (SIR), cf. Rubin (1987), Pitt and Shephard (1999), i.e., simulating each $m^{(b)}$ one-step forward and re-weighting using an importance sampler as follows:

1. Simulate the Markov chain one-step-ahead to obtain $\hat{M}_{t+1}^{(1)}$ given $M_t^{(1)}$. Repeat B times to generate draws $\hat{M}_{t+1}^{(1)}, \hat{M}_{t+1}^{(2)}, \ldots, \hat{M}_{t+1}^{(B)}$.

2. This preliminary step only uses information available at date t, and must therefore be adjusted to account for the new return. Drawing a random number q from 1 to B with probabilities of:

$$P(q = b) = \frac{f(r_{t+1} \mid M_{t+1} = m^{(b)})}{\sum_{i=1}^{B} f(r_{t+1} \mid M_{t+1} = m^{(i)})}. \tag{A3}$$

3. We then select $M_{t+1}^{(1)} = \hat{M}_{t+1}^{(q)}$, and repeat B times to obtain B draws to get the new $M_{t+1}^{(1)}, \ldots M_{t+1}^{(B)}$, which have been adjusted to account for the new realizations.

This recursive procedure provides a discrete approximation to Bayesian updating, which is computationally convenient in large state spaces.