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## Article

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# Pricing and Profit Distribution in Supply Chain Through Option Contracts

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## ABSTRACT

Supply chain can be simplified into two parts: upstream suppliers and downstream distributors. The authors use option contract to coordinate their relationship. But the instability of pure option contract where supplier and distributor deal only by contract makes it difficult for both sides to reach a consensus. They overcome the defect by combining operation model with wholesale price model, and the mix model can reach Pareto improvement because it will increase supplier and distributor's profit at the same time. The distribution of the increasing profit will be influenced by many internal factors. Among these internal factors, the risk aversion and bargaining power can affect the profit distribution between supplier and distributor to a large extent. This paper establishes the mathematical model and chooses risk aversion and bargaining power to analyze. They found that 1) the higher the risk aversion level of the distributor or supplier is, the more its profit will be, and 2) the one with more initiative in the negotiation will reap more profits from the other side in supply chain.

## KEYWORDS

Bargaining Power, Option Contract, Risk Aversion, Supply Chain

## INTRODUCTION

“While we have sold out of our initial supply, stores continue to receive iPhone 5 shipments regularly and customers can continue to order online and receive an estimated delivery date. We appreciate everyone's patience and are working hard to build enough iPhone 5 for everyone.” Apple said on September 24, 2012, after iPhone 5 was sold out. However, among Apple's two? Two display suppliers, Japan Display Inc. and South Korea's LG Display Co. both had trouble producing to meet the demand due to the sharp increase in Apple's order.

The same thing happened with Xiaomi, a large mobile phone manufacturer in China. Lei Jun, who is the CEO of Xiaomi, promised the customers that Mi9 would satisfy the customers' demand without delay. However, the fact was not in line with the announcement of Mr. Jun, and Mi9 sold out quickly. After Lei Jun announced that the goods would be replenished soon, Xiaomi's component suppliers were under huge production pressure for the replenishment of some components that were required 3 months ago.

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Along with the growth of E-commerce and express logistics, especially in China, where there are the largest Business-to-Business(B2B) platform and the largest logistics network in the world, the efficiency of trade is becoming higher, and the process from order to delivery becomes more time-saving. It also makes zero-stock transactions possible and necessary. Consider the following scenario: A firm acting as a distributor in the traditional supply chain used to buy products from upstream suppliers with wholesale prices at the beginning of the quarter. And during the sales quarter, they will store these products, try to handle these “dead stock,” and find a way to import more if these products are sold out. Therefore, in order to make more profits, the distributor hopes to change the order quantity at any time to cope with the constantly changing market demand.

As for suppliers in the upstream of the supply chain, in the “wholesale price model,” the suppliers make products according to the forecast of the market, then sell to distributors with the wholesale prices. The extra orders from retailers caused by the change in the market will create additional production pressure for the suppliers. Nowadays, one supplier may face many distributors, which makes the pressure caused by the changed orders even more serious in a B2B model. Being different from distributors who hope to have the chance to make real-time changes in the order, the suppliers, on the other hand, hope to confirm orders early and sell more products to the downstream of the supply chain. The conflict of interests between suppliers and distributors is difficult to solve by the “wholesale price model.”

By making new option contracts with suppliers, distributors will pay option prices to purchase the right—the right to buy a range of goods from suppliers with exercise prices sometime later and enforce parts of the right at different periods of the sales quarter. Suppliers sell the right and charge option prices. When an option is exercised, the supplier is obligated to sell the goods to the distributor with the contractual price (exercise price). Therefore, when the option contract is made, the future trading volume is fully considered by the two parties to prevent any of them from deciding the sales volume or production volume arbitrarily.

At the same time, option prices can reflect the share of the risks and benefits. It not only prevents one party from bearing the loss alone caused by market risks but also shares extra profits from market speculation. For suppliers, option prices can be used as a portion of the commercial profits before the transaction is done; even if distributors break option contracts or buy fewer products relatively, they can also get option prices which can reduce their risk of loss. For distributors, options can be waived if the market outlook is not good, and the maximum loss is just the price of buying the options. When the market prospect is good, distributors will buy more products to earn more profits; then there will be a part of profits which can be transferred to suppliers.

However, in the process of making options, it is impossible to fully predict the market demand. That is, the quantity of options and the final amount of sales are not the same. For retailers, when the quantity of options is less than the market demand, they still need to purchase at wholesale prices even after the execution of the options to maximize their profits. Therefore, the “option model” is not completely opposed to the “wholesale price model,” but they complement each other.

After applying the new “mixed model,” when commodity prices and the market demand change in a small range, the total profits of the supply chain are relatively steady after the goods are sold out. A competitive relationship still exists between distributors and suppliers when it comes to profit allocation. Both of them are expected to gain a greater share of the profits from the supply chain. Thus, even if suppliers and distributors reach a win-win cooperation through option contracts, there will still be a gap in their profits. Under the condition where the profits of the supply chain are steady, the enterprises’ internal factors, such as risk aversion and bargaining power, determine the profits of enterprises gotten from the whole chain and influence the profits of enterprises. In this paper, the Nash’s bargaining model (Nash,1953) is used to discuss the influence of different risk aversion on profit distribution. The Eliashberg’s model (Eliashberg,1986) is used to discuss the influence of bargaining power.

## LITERATURE REVIEW

The study about option contracts started early. Ritchken and Tapiero (1986) firstly suggested that the combination of option contracts with traditional ordering methods can resolve supply chain conflicts and strengthen supply chain cooperation. Cachon and Lariviere (2005) explored the option contracts that ensure credible demand forecast sharing across the supply chain with demand information asymmetry. Zhao et al. (2010) studied the supply chain coordination under option contracts and compared it to the basic wholesale price model considering the influence of risk aversion and bargaining power with random demand. To systematically explore the impact of the market demand, Brown and Lee (2003) analytically characterized the order quantity decision as a function of demand signal quality with a futures-options contract. They found that the number of futures decreases in demand signal quality, whereas the number of options increases in it.

Storing to meet market demand is an important issue. Zhao et al. (2018) researched the two-stage option contracts combined with the liquid spot market, so the retailer could purchase both from the spot market and the option contracts in order to prevent supply from falling short of demand. Detailed comparisons between the mixed market scenario (in which the spot market and the option contract market coexist) and the scenarios of the pure spot market or the pure option contract market are made, and thereby they analytically examined the effects of supply competition between the spot market and the option contract market. Köle (2012) considered the replenishment strategy of a buyer with two suppliers. Since its regular supplier is prone to disruptions, the buyer utilizes an options contract with a more expensive but perfectly reliable supply option to prevent undersupply when relying on one supplier. Mandal et al. (2017) studied the newsvendor's pricing and stocking decisions under reference point effects. The demand faced by the newsvendor is endogenous, and the customers may also decide to procure the product from an outside option. They characterized the firm's optimal pricing and stocking decisions. Their analysis revealed a threshold policy on the firm's ordering and pricing decisions while considering the impact of reference point effects. Option contracts can improve the performance of both companies across the supply chain but cannot completely eliminate the impact of overconfidence (Wang et al., 2021). Zhuo et al. (2018) find that supply chain coordination is not always achieved, contrasting with the result that properly designed option contracts can always coordinate a supply chain in the absence of risk considerations.

Pricing strategies, including how to determine retail prices, option prices, or option exercise prices in option contracts, are worth of discussion. Hu et al. (2018) considered a coordination problem under option contracts in a two-echelon supply chain, where the product retail prices, option prices, option exercise prices, and order quantity are optimized. The market demand is random and sensitive to product retail prices. Their analysis considered two types of contracts. One is a conventional option contract, where suppliers determine the option prices and exercise prices, and retailers determine the product retail prices and order quantity. The other type of contract is an option contract with a joint pricing mechanism, for which two supply chain players determine a relationship between the option exercise prices and product retail prices. For both types of contracts, they developed a newsvendor model to examine how joint pricing impacts the coordination and decisions of the supply chain. The study of Wang et al. (2006) investigated the impacts of customer returns and a bidirectional option contracting a newsvendor firm's refund prices and order decisions. The stochastic demand is refund price-sensitive. They showed that the optimal refund price increases with the option prices and decreases with both the wholesale prices and exercise prices, while the optimal order quantity increases with the exercise prices but decreases with both the wholesale prices and option prices. In a retailer-led supply chain, the retailer will set the option price as low as possible in order to shift more risks to the supplier (Liu et al., 2020).

In this article, it calculates the optimal order quantity and prices and finds a "mixed model" (in which the wholesale price model and the option contract model coexist) are both stable and profitable. Compared with the above scholars' research, this article innovates in: (1) proving the stability of the

model by calculating the conditions that the option or the wholesale prices must meet if the optimal trading volumes of suppliers and distributors are equal; (2) comparing the “mixed model” with the “pure option model” and the “pure wholesale price model” to determine which model can increase the profits of both sides; and (3) setting parameters to calculate the impact of risk preference and bargaining power on profit share.

## MODEL DESCRIPTION

Suppose that there is only one distributor and one supplier in the supply chain. The supplier sells products to the distributor, who then sells products to the customers. The market demand for the products is  $x$ , following a cumulative distribution function  $F(x)$ , with a probability density function  $f(x)$ ,  $x > 0$  and  $F(x)$  is strictly increasing in  $x$ . To make the calculation less complicated, we assume that  $F(x)$  is derivable and  $f(x)$  is integrable.

Under the “wholesale price model,” the quantity of products produced by the supplier is  $Q_{ws}$ , some or all of them are sold with the wholesale price  $w$  to the distributor. Then the distributor sells at the retail price  $p$  to consumers. Assume that  $p = w^*(1 + r)$ ,  $r$  is the rate of profits.

If the option contract model is used in the supply chain, the option price is assumed to be  $o$ , the exercise price is assumed to be  $e$ . It means the distributor buys  $N$  shares of the option at the price  $o$  per share. Within the duration of the option, the distributor has the right to buy  $N$  shares of products with the exercise price  $e$ . When the quantity of options meets the market demand, that is  $X \leq N$ , parameter  $Q$  is used to represent the amount of trading carried out by the two parties in the option. When market demand is larger, that is  $X > N$ , parameter  $q$  is used to represent additional transactions with the wholesale price.

In addition, the cost of each product manufactured by the supplier is expressed as  $c$ . If some of the goods are not sold out, the residual value obtained after recycling is expressed as  $v$ , and  $c > v$ . This paper mainly considers the profit change brought by options, assuming that: (1)  $r$ ,  $c$ ,  $v$  will not change after the option contract is signed; (2) suppliers can expand production at any time to meet the needs of distributors; and (3) products that cannot be sold out shall be returned to suppliers, and suppliers shall deal with the residual value  $v$ .

## Calculation of Profits

### *Profits Under the Pure Wholesale Price Model*

The wholesale price models are still widely used in supply chains in China. With suppliers trying to save on the costs, such as freight, staff, and storage costs, and publicity costs, other than production activities, they delegate the sales process to distributors. Distributors make money from the difference between the prices they buy and sell. In this paper, we calculate the profits of suppliers and distributors under the pure wholesale price model and then compare that with the pure option model and the mixed model.

### *Profit Expectation of a Distributor*

Distributors purchase goods according to market demand. That is, a distributor can increase or decrease the order quantity from the upstream according to the fluctuation of demand and reduce the quantity of goods that are unmarketable automatically. This will maximize the profit expectation. The profits of the distributor consist of two parts—the price at which goods are sold and the cost of buying from the supplier. The expectation of the distributor’s profits under the wholesale price model is as follows:

$$E_x \Pi_{wd}(Q_{wd}) = E_x \left[ p^* \min \{ Q_{wd}, x \} - w^* Q_{wd} \right] = p - w^* Q_{wd} - p^* \int_0^{Q_{wd}} F(x) dx \quad (1)$$

$Q_{wd}$  is the number of commodities held by the distributor. When the number of commodities held by the distributor is 0, the expected profit is 0. When the number of products that can be sold increases, the expected profits will also increase. If the number of goods held by the distributor exceeds the market demand, the redundant part of the goods cannot turn into normal retail profits.

As  $Q_{wd}$  increases, the profit expectation of the distributor goes up and then goes down. Take the derivative of the dependent variable  $E_x \Pi_{wd}(Q_{wd})$  with respect to  $Q_{wd}$ . Set the derivative to 0, and get the optimal quantity  $\bar{Q}_{wd}$  that makes  $E_x \Pi_{wd}(Q_{wd})$  maximal.

Set  $\frac{dE_x \Pi_{wd}(Q_{wd})}{dQ_{wd}} = 0$ , and get  $\bar{Q}_{wd} = F^{-1}\left(\frac{p-w}{p}\right) = F^{-1}\left(\frac{r}{1+r}\right)$ . Then substitute  $\bar{Q}_{wd}$  into equation (1), use  $\pi_{wd}$  to represent the maximum of  $E_x \Pi_{wd}(Q_{wd})$ .

$$\pi_{wd} = p - w * \bar{Q}_{wd} - p * \int_0^{\bar{Q}_{wd}} F(x) dx \quad (2)$$

### Profit Expectation of a Supplier

If a supplier produces and sells goods according to market demand, the loss caused by the unsellable part of products will be reduced. But inevitably, the quantity of goods sold by the supplier does not always equal to the total quantity of its output. At the same time, the supplier will also undertake the disposal of the remaining goods and get the residual value. Its profits consist of three parts—the selling price to the distributor, the marginal cost of producing goods, and the residual value of the remaining products. The equation is as follows:

$$\begin{aligned} E_x \Pi_{ws}(Q_{ws}) &= E_x \left[ w * \min\{Q_{ws}, x\} - c * Q_{ws} + v * \max\{Q_{ws} - x, 0\} \right] \\ &= w - c * Q_{ws} - (w - v) * \int_0^{Q_{ws}} F(x) dx \end{aligned} \quad (3)$$

$Q_{ws}$  is the number of products that the supplier produces. When the quantity of products produced by the supplier is 0, the expected profit is 0. The number of products that can be sold to the distributor is bound to increase as the output of the supplier increases, which will increase the supplier's profits. When the number of goods held by the supplier exceeds the demand, many goods cannot be changed into the normal selling profits, which can only be converted into residual value after processing so as to reduce the expected profits.

Thus, as  $Q_{ws}$  increases, the profit expectation of the distributor goes up and then goes down. Take the derivative of the dependent variable  $E_x \Pi_{ws}(Q_{ws})$  with respect to  $Q_{ws}$ . Set the derivative to 0, and get the optimal quantity  $\bar{Q}_{ws}$  that makes  $E_x \Pi_{ws}(Q_{ws})$  maximal.

Set  $\frac{dE_x \Pi_{ws}(Q_{ws})}{dQ_{ws}} = 0$ , and get  $\bar{Q}_{ws} = F^{-1}\left(\frac{w-c}{w-v}\right)$ . Then substitute  $\bar{Q}_{ws}$  into equation (3), use  $\pi_{ws}$  to represent the maximum of  $E_x \Pi_{ws}(Q_{ws})$ .

$$\pi_{ws} = w - c * \bar{Q}_{ws} - (w - v) * \int_0^{\bar{Q}_{ws}} F(x) dx \quad (4)$$

If the supply chain is stable in the long run, both parties must maximize their profits at the same time, which means  $\bar{Q}_{ud} = \bar{Q}_{us}$ , and  $\frac{w - c}{w - v} = \frac{r}{1 + r}$ . In other words, under the wholesale price model, the long-term stability of the supply chain can be achieved, which needs to be satisfied:

$$r = \frac{w - c}{c - v} \quad (5)$$

It is concluded from equation (5) that under a long-term stable “pure wholesale price model,” the profit margin of commodities is determined by the wholesale price, the marginal cost, and the residual value. As the wholesale price increases, the profit margin gets higher.

### Profit After the Application of an Option

**Profit Under the Pure Option Model.** A pure option model, in which distributors and suppliers no longer trade with the wholesale prices, is appropriate for the situation when  $w \geq e$  and  $x \leq N$ . It means the exercise price is lower than the wholesale price, and the quantity of options can meet the demand of consumers. For a distributor, the most cost-effective way is to buy from the upstream with options. At this point, both sides abandon the old “wholesale price model” and make a mutually acceptable  $(o, e)$ . The distributor buys the options with the price  $o$ , then exercises the options within the stipulated time to buy units of goods from the supplier. The profits include the difference between the buying and selling, and the option price. In the pure option model, the number of products purchased by the distributor from the supplier is set as  $Q_{od}$ . The profit expectation of the distributor is:

$$\begin{aligned} E_x \Pi_{od}(Q_{od}) &= E_x \left[ (w + wr - e) * \min\{Q_{od}, x\} - o * N \right] \\ &= (w + wr - e) * Q_{od} * F(N) - (w + wr - e) * \int_0^{Q_{od}} F(x) dx - oN \end{aligned} \quad (6)$$

Let  $\frac{dE_x \Pi_{od}(Q_{od})}{dQ_{od}} = 0$ , and get  $\bar{Q}_{od} = N$ , which means that when the distributor buys  $N$  units of

products from the manufacturer, the distributor gets the maximal expected profits under this model.

When the number of products purchased by the distributor from the upstream is less than  $\bar{Q}_{od}$  (i.e.,  $N$ ), the more it purchased, the greater the expectation of profit is. At this point, the distributor will increase the number of purchases until buying  $N$  units of options from the buyer.

The supplier will sell options to the distributor at unit price  $o$ . It is assumed that the distributor purchases goods in real-time according to the market demand. It is also assumed that the actual output that the supplier can sell to the distributor is  $Q_{os}$ . Then the supplier sells options in the number of  $\min\{Q_{os}, x\}$  with the exercise price  $e$ . There should be some products that cannot be sold. This part of the products is recycled by the supplier. Thus, the supplier’s profit equation includes these four parts: the option price, the sales income, the marginal cost of production, and the salvage value of the surplus products. The supplier’s profit expectation can be expressed as follows:

$$\begin{aligned} E_x \Pi_{os}(Q_{os}) &= E_x \left[ o * N + e * \min\{Q_{os}, x\} + v * \max\{Q_{os} - x, 0\} - c * Q_{os} \right] \\ &= o * N + e * Q_{os} * F(N) - c * Q_{os} - (e - v) * \int_0^{Q_{os}} F(x) dx \end{aligned} \quad (7)$$

$$\text{Let } \frac{dE_x \Pi_{os}(Q_{os})}{dQ_{os}} = 0, \text{ then one can get } \bar{Q}_{os} = F^{-1} \left( \frac{e * F(N) - c}{e - v} \right).$$

When the contract quantity limitation is less than  $\bar{Q}_{os}$ , the more the supplier produces, the more the expectation of profit is. At this point, the supplier will increase production and try to sell more options to increase  $N$ . When the contract quantity limitation is more than  $\bar{Q}_{os}$ , the more the supplier produces, the less the expectation of profit. At this point, the supplier will decrease production and try to sell less options to decrease  $N$ . So the optimal output of the supplier under the pure option model is  $\bar{Q}_{os}$ .

**Profit Under the Mixed Model.** The mixed model uses the option to purchase and, at the same time, uses the wholesale price to fill the vacancy, which is applicable to when  $w \geq e$  and  $x > N$ . Although the option's exercise price is cheaper than the wholesale price, and the option quantity  $N$  is not enough to meet market demand, the distributor will require the supplier to sell  $q$  units of goods with the wholesale price in order to seek greater profits. The profit function of the distributor is:

$$\begin{aligned} E_x \Pi_{md} &= E_x [w * (1 + r) * \min\{N + q, x\} - o * N - e * N - w * q] \\ &= w * (1 + r) * \left( N + q - N * F(N) - \int_N^{N+q} F(x) dx \right) - o * N - e * N - w * q \end{aligned} \quad (8)$$

$$\text{Let } \frac{dE_x \Pi_{md}}{dq} = 0, \text{ and get } q_{md}^* = F^{-1} \left( \frac{r}{1 + r} \right) - N.$$

That is, after the option is fully exercised, the distributor needs to buy additional products at the wholesale price as the quantity of goods is insufficient to meet the market demand. After calculation, the expected profit of the distributor is optimal when the number of products is purchased with the wholesale price, which is  $q_{md}^*$ . The total quantity of goods held by the distributor is  $F^{-1} \left( \frac{r}{r + 1} \right)$ , which is the same as the optimal quantity under the wholesale price model. Compared with the wholesale price model, the mixed model will not change the optimal purchase amount of the distributor.

In the mixed model, the supplier should not only use the option contract to sell the goods but also sell goods at the wholesale price. Its profit is made up of four parts—the option price and its exercise price, the wholesale price, the marginal cost of production, and the salvage value of the surplus products.

$$\begin{aligned} E_x \Pi_{ms} &= E_x [(o + e) * N + w * q - c * (N + q) + v * \max\{N + q - x, 0\}] \\ &= (o + e) * N + w * q - c * (N + q) + v * \left( \int_N^{N+q} F(x) dx - q * F(N) \right) \end{aligned} \quad (9)$$

$$\frac{dE_x \Pi_{ms}}{dq} = v * (F(N + q) - F(N)) + w - c$$

after calculation,  $\frac{dE_x \Pi_{ms}}{dq} > 0$  always stands up. That is, under the mixed model, the expected profit of the supplier increases monotonously, and there is no maximum theoretically.



Add equations (8) and (9), then get the expected profit of the entire supply chain, which is represented by  $E_x \Pi_{mc}$ .

$$E_x \Pi_{mc} = -c * (N + q) - v * \left[ q * F(N) + \int_N^{N+q} F(x) dx \right] + w * (1 + r) * \left[ (N + q) - N * F(N) - \int_N^{N+q} F(x) dx \right] \quad (10)$$

Take the derivative of equation (10) to get the optimal value of  $q$ , indicated as  $q_{mc}^*$ .

$$q_{mc}^* = F^{-1} \left( \frac{w * (1 + r) - v * F(N) - c}{w * (1 + r) - v} \right) - N \quad (11)$$

For the supply chain, when  $w \geq e$  and  $x > N$ , after executing the option, both parties in the supply chain can use the wholesale price  $w$  to trade  $q_{mc}^*$  of goods to achieve the best profit.

## ANALYSIS

### Parameter Relationship of the Option Model

When the “pure wholesale price model” can reach a balance, there is  $r = \frac{w - c}{c - v}$ . If the “pure option model” achieves a balance, the optimal amount of production for the distributor is  $N$ , where  $N$  is the maximum value of trading between the two sides. The output of the supplier is not less than  $N$ , so it can achieve the optimal profit at the same time.

When  $\bar{Q}_{om} \geq N$ , get  $F(N) \geq \frac{c}{v} > 1$ , which is out of the question. Therefore, it is difficult to achieve the optimal profit only through option contracts. Because in the mixed model, the retailer’s purchase quantity has the optimal value, while the supplier’s expected profit increases monotonously. Therefore, the optimal value of the trading volume of the supply chain will be more than the optimal purchase quantity of the retailer:

$$q_{mc}^* + N = F^{-1} \left( \frac{w * (1 + r) - v * F(N) - c}{w * (1 + r) - v} \right) \geq F^{-1} \left( \frac{r}{1 + r} \right)$$

we get:

$$F(N) \leq \frac{w - c}{v} + \frac{r}{r + 1} \quad (12)$$

Therefore, in order to use the option contract reasonably, the option quantity must satisfy equation (12). At this time, the mixed model can not only optimize the profits of the supply chain but also expand the optimal trading volume and increase the output of suppliers compared with the traditional “wholesale price model.”

## Profit Relationship in the Supply Chain

In order to express the profit share of the supplier and the distributor from the mixed model directly, the expected profit of the distributor and the supplier are expressed with  $\pi_{md}$  and  $\pi_{ms}$  when applying optimal trading volume  $q_{ms}^*$ .

$$\pi_{md} = E_x \Pi_{md}(q_{mc}^*) = w^*(1+r)^* \left[ (N + q_{mc}^*) - N^* F(N) - \int_N^{N+q_{mc}^*} F(x) dx \right] - wq_{mc}^* - (o + e)N \quad (13)$$

$$\pi_{ms} = E_x \Pi_{ms}(q_{mc}^*) = (o + e)^* N + w^* q_{mc}^* - c^* \left[ N + q_{mc}^* \right] + v^* \left[ -q_{mc}^* F(N) + \int_N^{N+q_{mc}^*} F(x) dx \right] \quad (14)$$

In order to represent the profit relationship between the distributor and the supplier directly, when the quantity of options, the wholesale price, the profit rate, and so on are confirmed, the element that most directly impacts on the profit distribution is  $o + e$ , let  $\sigma = o + e$ . Set parameters  $A$  and  $B$  so that:

$$A = w^*(1+r)^* \left[ (N + q_{mc}^*) - N^* F(N) - \int_N^{N+q_{mc}^*} F(x) dx \right] - w^* q_{mc}^*$$

$$B = w^* q_{mc}^* - c^*(N + q_{mc}^*) + v^* \left[ -q_{mc}^* F(N) + \int_N^{N+q_{mc}^*} F(x) dx \right]$$

Substituting the expressions of  $A$  and  $B$  into equations (13) and (14), you can view  $\pi_{md}$  and  $\pi_{ms}$  as functions of  $\sigma$ . Get the following relationships:

$$\pi_{md}(\sigma) = A - \sigma^* N \quad (15)$$

$$\pi_{ms}(\sigma) = \sigma^* N + B \quad (16)$$

Combined with the conclusions above, it can be seen that the value of  $\sigma$  affects the share distributed by both parties in the supply chain directly.

### Range of $\sigma$

The stability of the mixed model not only means it can make the supply chain achieve the maximal profit expectations but also be able to improve the profits of the distributor and the supplier at the same time compared with the model of the wholesale prices; otherwise, there is easily one party which refuses to sign contracts.

So  $\sigma$  must meet the following conditions: (1)  $\pi_{md}(\sigma) \geq \pi_{wd}$  and (2)  $\pi_{ms}(\sigma) \geq \pi_{us}$ . By solving the inequality. We get:

$$\frac{A - \pi_{wd}}{N} \geq \sigma \geq \frac{\pi_{ws} - B}{N} \quad (17)$$

For the convenience of expression, let  $\sigma_{min} = \frac{\pi_{ws} - B}{N}$ ,  $\sigma_{max} = \frac{A - \pi_{wd}}{N}$ , that is, the range of  $\sigma$  is  $[\sigma_{min}, \sigma_{max}]$ .

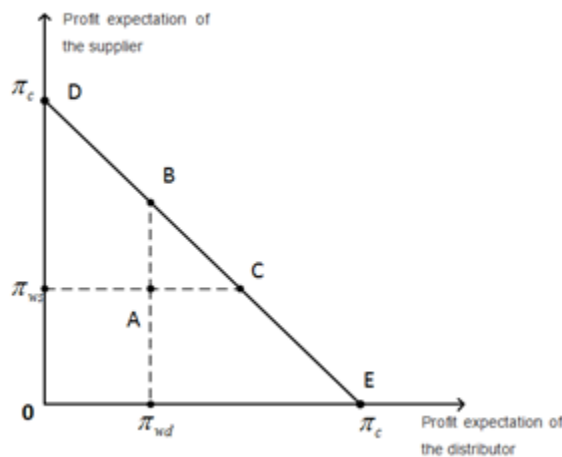
In the coordinate system shown in Figure 1, the  $x$ -axis represents the profit expectation of the distributor under the “mixed model,” and the  $y$ -axis represents the profit expectation of the supplier under the “mixed model.” The line DE represents the profit relationship which satisfies equations (15) and (16). The combination of maximal expected profits of the distributor and the supplier falls on DE and moves with the change in  $\sigma$ . The  $x$ -coordinate of point B is the maximal expected profits of the distributor under the wholesale price model. The  $y$ -coordinate of point C is the maximal expected profits that the supplier can get under the wholesale price model. From this figure, we can see: when the  $\sigma$  falls within  $[\sigma_{min}, \sigma_{max}]$ , the profit portfolio falls within BC; at this time, the supplier and the distributor gain more profit expectation than that under the wholesale price model. It can be seen that the mixed model achieves the Pareto improvement(Pareto,1897) of both profits.

### Selection of Option Contracts

#### Factors Affecting $\sigma$

Through the demonstration above, it can be seen that distributors and suppliers have sufficient reasons to choose the mixed model compared with the wholesale price model and the pure option model. Even under the same model, different  $\sigma$  will also affect the ratio of profit distribution. As a distributor, a lower value of  $\sigma$  means more revenue. As a supplier, a higher value of  $\sigma$  means more profit. Within the supply chain, there are different factors influencing  $\sigma$ . For example, risk aversion will affect the price that the option buyer is willing to pay for the option so as to influence the profit by changing  $\sigma$ . At the same time, the more bargaining power one has, the more additional revenue it gets from the other party. The next part of this paper focuses on how risk aversion and bargaining power influence  $\sigma$ .

Figure 1. The maximal expected value curve of profit



### Influence From Risk Aversion

Nash's Bargaining Model. The Nash's bargaining model, or Nash model, was developed by the economist John Forbes Nash in 1950. What the model means is—let  $X_i$  be the share of a cake that is given to the number  $i$  participant of the market.  $U_i(X_i)$  is the utility that participant  $i$  gets from it, simplified as  $U_i$ . Before the contract is signed, the original utility of the participant is  $d_i$ .  $U_i - d_i$  is the utility variation of *No.i* market member before and after the implementation of the contract.  $(U_1 - d_1) * (U_2 - d_2)$  is used to analyze the value of a program from a utility perspective. The solution that makes  $(U_1 - d_1) * (U_2 - d_2)$  maximal is called the “Nash solution,” which is the set of values that in theory maximize the effectiveness of the system composed of  $i$  market participants. In the article published by Zhao (2010), the Nash product was used to judge the optimal value. This equation mode is used for reference here.

The profit margin generated by the mixed model is:

$$\Delta\pi_d = A - \sigma N - \pi_{wd}, \Delta\pi_s = \sigma N + B - \pi_{ws}, \Delta\pi_c = A + B - \pi_{wd} - \pi_{ws} \quad (18)$$

where  $\Delta\pi_c$  is the profit margin of the whole supply chain before and after the application of the mixed model.

Before and after the application of the option model, the utility difference of the supplier is  $U_s(\Delta\pi_s)$ , and the utility difference of the distributor is  $U_d(\Delta\pi_d)$ . For the convenience of calculation, the following hypotheses are proposed for the utility equation.

$U_s(\Delta\pi_s)$  and  $U_d(\Delta\pi_d)$  are the second-order derivable functions.

Assuming that with the increase of  $\Delta\pi$ , the utility obtained by the market members will also increase, that is, the  $U_s(\Delta\pi_s)$  and  $U_d(\Delta\pi_d)$  are monotonic growth functions.

Assume that the growth amount of benefit is also 0 when the growth amount of income is 0. That means when  $\Delta\pi_i = 0$ ,  $U_i$  will be 0 ( $i = m, r$ ). From equation (18), we get that when  $\sigma$  is  $\sigma_{max}$ ,  $\Delta\pi_d = 0$ ,  $U_d = 0$ . When  $\sigma$  is  $\sigma_{min}$ ,  $\Delta\pi_s = 0$ ,  $U_s = 0$ .

Arrow-Pratt Risk Aversion. According to the theory “Arrow-Pratt Risk Aversion” that John W. Pratt and Kenneth Arrow put forward in 1964 and 1965, the degree of risk preference is determined by the following equation:

$$R_d(\Delta\pi_d) = -\frac{U_d''(\Delta\pi_d)}{U_d'(\Delta\pi_d)}, R_s(\Delta\pi_s) = -\frac{U_s''(\Delta\pi_s)}{U_s'(\Delta\pi_s)} \quad (19)$$

$R_d(\Delta\pi_d)$  is used to measure the risk preference of a distributor at the expected revenue increase of  $\Delta\pi_d$ . If  $R_d(\Delta\pi_d) > 0$ , the distributor is risk averse. As the absolute value of  $R_d(\Delta\pi_d)$  increases, the risk aversion is getting higher. If  $R_d(\Delta\pi_d) = 0$ , the distributor is risk neutral. If  $R_d(\Delta\pi_d) < 0$ , the distributor is risk oriented. As the absolute value of  $R_d(\Delta\pi_d)$  increases, the distributor is more risk appetite. It is the same for  $R_s(\Delta\pi_s)$ .

### Profit Distribution Based on Risk Aversion or Risk Neutrality

**Case 1:** Suppliers and distributors are both risk averse, which means  $R_d = -\frac{U_d''(\Delta\pi_d)}{U_d'(\Delta\pi_d)} > 0$ ,

$$U_d'(\Delta\pi_d) > 0, U_d''(\Delta\pi_d) < 0. \text{ And } R_s = -\frac{U_s''(\Delta\pi_s)}{U_s'(\Delta\pi_s)} > 0, U_s'(\Delta\pi_s) > 0, U_s''(\Delta\pi_s) < 0.$$

Let  $Q = U_d(\Delta\pi_d)U_s(\Delta\pi_s)$ , which is called “Nash product.” If there is a value of  $\sigma$ , which can maximize  $Q$ , the sum at this time can maximize the utility of both parties according to the Nash model. And then we take the derivative of  $Q$  with respect to  $\sigma$ , and get the value of  $\sigma$  that satisfies the condition that  $\frac{dQ}{d\sigma} = 0$ :

$$\frac{dQ}{d\sigma} = \frac{dU_d(\Delta\pi_d)}{d\Delta\pi_d} * \frac{d\Delta\pi_d}{d\sigma} * U_s(\Delta\pi_s) + \frac{dU_s(\Delta\pi_s)}{d\Delta\pi_s} * \frac{d\Delta\pi_s}{d\sigma} * U_d(\Delta\pi_d)$$

Let us put equation (18) into the derivative equation:

$$\frac{dQ}{d\sigma} = N * [-U_d'(\Delta\pi_d) * U_s + U_s'(\Delta\pi_s) * U_d]$$

It is hard to see what the magnitude is by taking one derivative and then taking the second derivative:

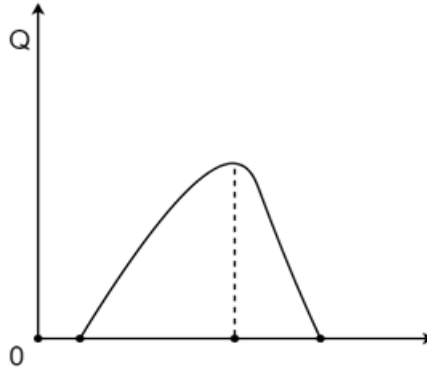
$$\frac{d}{d\sigma} \left( \frac{dQ}{d\sigma} \right) = N^2 [U_s * U_d'' + U_s'' * U_d - 2 * U_s' * U_d'] \tag{20}$$

$U_d'' < 0, U_s'' < 0$ . Therefore, the equation in parentheses of equation (20) is always negative and always makes (20) < 0. If the second derivative of  $Q$  is negative, then it is a convex function of  $\sigma$ .

Because when  $\sigma$  takes the maximum or minimum value, there will be  $U_s = 0$  or  $U_d = 0$ , which means  $Q = 0$ . So  $\sigma_{min}$  and  $\sigma_{max}$  are both the zeros of equation  $Q$ . Like Figure 2 shows, according to the mean value theorem, there must be  $\sigma^*$  in  $[\sigma_{min}, \sigma_{max}]$ , which makes  $\frac{dQ}{d\sigma^*} = 0$ .

Now to discuss the supplier’s share of revenue relative to the distributor. Suppose  $\Delta\pi$  is the profit increase of the supply chain after applying the mixed model. So, according to equation (18), it can be concluded that only when  $\sigma = \frac{\sigma_{min} + \sigma_{max}}{2}$ , the profit increase of the distributor and supplier is equal, both of them are  $\frac{1}{2} \Delta\pi$ . By comparing  $\sigma^*$  with  $\frac{\sigma_{min} + \sigma_{max}}{2}$ , we can compare the increase profits of both parties. Let  $\bar{\sigma} = \frac{\sigma_{min} + \sigma_{max}}{2}$ . As can be seen from Figure 2,  $Q$  is increasing with

Figure 2. Graph of  $Q$  as a function of  $\sigma$



$Q$  is increasing with  $\sigma$  monotonically when  $\sigma \leq \sigma^*$ . And  $Q$  is decreasing with  $\sigma$  monotonically when  $\sigma > \sigma^*$ . If  $Q'(\bar{\sigma}) > 0$ , then  $\sigma^* > \bar{\sigma}$ , which means  $\Delta\pi_s > \frac{1}{2}\Delta\pi > \Delta\pi_d$ , then the profits of the supplier are greater than that of the distributor. If  $Q'(\bar{\sigma}) < 0$ , that is  $\sigma^* < \bar{\sigma}$ , which means  $\Delta\pi_s < \frac{1}{2}\Delta\pi < \Delta\pi_d$ , then the profits of the supplier are less than that of the distributor:

$$Q'(\bar{\sigma}) = N * \left[ -U_d' \left( \frac{\Delta\pi}{2} \right) * U_s \left( \frac{\Delta\pi}{2} \right) + U_s' \left( \frac{\Delta\pi}{2} \right) * U_d \left( \frac{\Delta\pi}{2} \right) \right] \quad (21)$$

From equation (19), we can get  $U_d'(\Delta\pi_d) * R_d(\Delta\pi_d) = -U_d''(\Delta\pi_d)$ , integrating both sides of this equation, and getting  $\int_0^{\frac{\Delta\pi}{2}} R_d dU_d = -U_d' \left( \frac{\Delta\pi}{2} \right)$ . In a similar way,  $\int_0^{\frac{\Delta\pi}{2}} R_s dU_s = -U_s' \left( \frac{\Delta\pi}{2} \right)$ . Substitute into equation (21), then get:

$$Q'(\bar{\sigma}) = N * \left[ \int_0^{\frac{\Delta\pi}{2}} R_d dU_d * \int_0^{\frac{\Delta\pi}{2}} 1 dU_s - \int_0^{\frac{\Delta\pi}{2}} R_s dU_s * \int_0^{\frac{\Delta\pi}{2}} 1 dU_d \right] \\ = N * \int_D (R_d - R_s) dU_s dU_d \quad (22)$$

The interval for the double integral in equation (22) is:

$$\left\{ (\Delta\pi_r, \Delta\pi_m) : 0 \leq \Delta\pi_r \leq \frac{\Delta\pi}{2}, 0 \leq \Delta\pi_m \leq \frac{\Delta\pi}{2} \right\}$$

We can get the following conclusion from equation (22).

When  $R_s(\Delta\pi_s) > R_d(\Delta\pi_d)$ , the risk aversion level of the supplier is higher,  $Q'(\bar{\sigma}) < 0$ ,  $\sigma^* < \bar{\sigma}$ ,  $\Delta\pi_d > \frac{1}{2}\Delta\pi > \Delta\pi_s$ , we can infer that the distributor's profit increases and occupies a large amount in the total value of the system.

When  $R_s(\Delta\pi_s) < R_d(\Delta\pi_d)$ , the risk aversion level of distributor is higher,  $Q'(\bar{\sigma}) > 0$ ,  $\sigma^* > \bar{\sigma}$ ,  $\Delta\pi_d < \frac{1}{2}\Delta\pi < \Delta\pi_s$ , we can infer that the supplier's profit increases and occupies a large amount in the total value of the system.

When  $R_s(\Delta\pi_s) = R_d(\Delta\pi_d)$ , the risk aversion levels of the two are equal,  $Q'(\bar{\sigma}) = 0$ ,  $\Delta\pi_s = \frac{1}{2}\Delta\pi$ ,  $\Delta\pi_d = \frac{1}{2}\Delta\pi$ , we can infer that the profit increase of these two parties through the new model is equal.

The results are tested by numerical simulations, and the utility functions of the suppliers and distributors are shown in Table 1.

After assuming that  $\Delta\pi_d = 1000 - 50\delta$  and  $\Delta\pi_s = 50\delta - 400$ , we get the following figures shown in Figure 3.  $U_d = \sqrt{\Delta\pi_d}$ .

Comparing (a) with (b), or (c) with (d) in Figure 3, we can get that  $\sigma^*$  will be lower and increase the distributor's profit if the distributor's risk aversion is lower. It is the same as the supplier's risk aversion and profit.

So, if we use a new option contract when both parties are risk-averse, for those with lower risk aversion, the increase in profits is more than that of those with low risk aversion.

**Case 2:** Let one party be risk-averse and the other risk-neutral. Assume that the distributor's risk

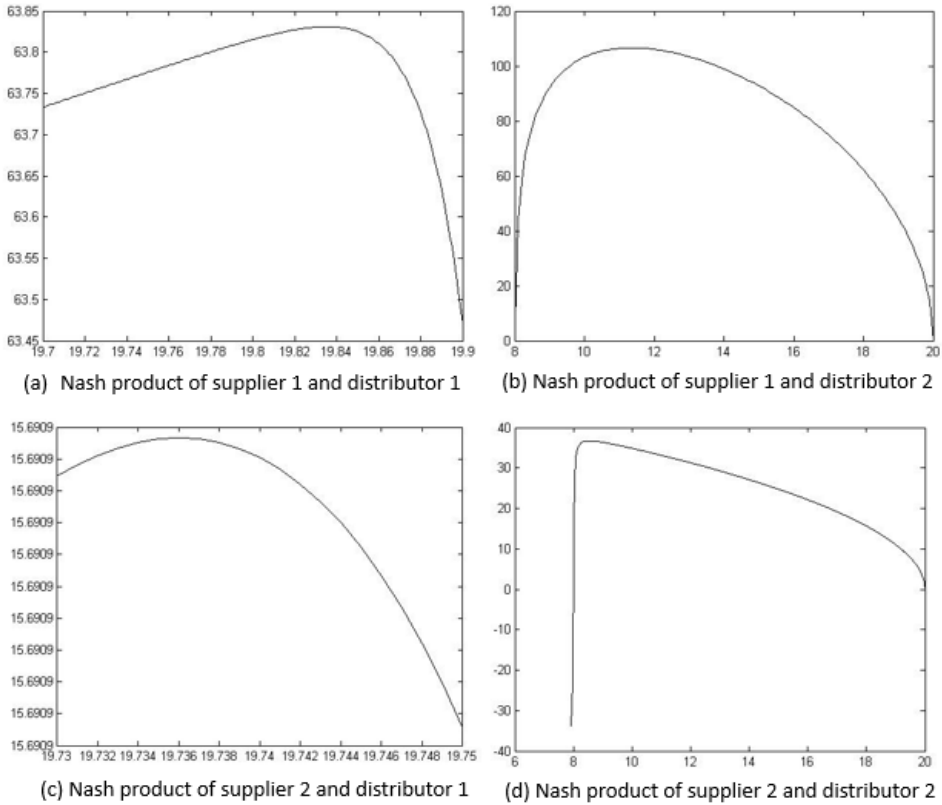
appetite still satisfies  $R_d = -\frac{U_d''(\Delta\pi_d)}{U_d'(\Delta\pi_d)} > 0$ ,  $U_d'(\Delta\pi_d) > 0$ ,  $U_d''(\Delta\pi_d) < 0$ . Being different

from the case 1,  $R_s = -\frac{U_s''(\Delta\pi_s)}{U_s'(\Delta\pi_s)} = 0$  and there will be  $U_s = a * \Delta\pi_s$ .

Table 1.

|               | Utility Function                | Risk Aversion                              |
|---------------|---------------------------------|--|
| Supplier 1    | $U_s = \ln(\Delta\pi_s + 1)$    | $R_s = 1 / (\Delta\pi_s + 1)$              |
| Supplier 2    | $U_s = \arctan(\Delta\pi_s)$    | $R_s = 2\Delta\pi_s / (1 + \Delta\pi_s^2)$ |
| Distributor 1 | $U_d = 10 - 10e^{-\Delta\pi_s}$ | $R_d = 1$                                  |
| Distributor 2 |                                 | $R_d = 1 / (2\Delta\pi_d)$                 |

Figure 3. Nash solutions



Substitute  $U_s = a * \Delta\pi_s$  into equation (20), and we will get:

$$\frac{d}{d\sigma} \left( \frac{dQ}{d\sigma} \right) = N^2 [a * \Delta\pi_s * U_d'' - 2 * U_d' * a]$$

Because  $U_d'(\Delta\pi_d) > 0$ ,  $U_d''(\Delta\pi_d) < 0$ , so we still have equation (20)  $< 0$ , the monotonicity and convexity of the function remain the same.

Substitute  $U_s = a * \Delta\pi_s$  into equation (22), and we will get  $Q'(\bar{\sigma}) = N * R_D * dU_m * dU_r > 0$ .

So  $\Delta\pi_s > \frac{1}{2} \Delta\pi > \Delta\pi_d$ , the increasing profits of the supplier are larger.

So, in a model which consists of these two market participants, if one party is risk-averse and the other one is risk-neutral, the latter can increase more profits by the mixed model.

Based on case 1 and case 2, it can be obtained that, after making the option contract and applying the mixed model in the supply chain, although Pareto improvement can be achieved in the profit, the increase in profits of both parties is not exactly the same. In terms of the market participant's risk appetite, the more risk aversion, the less increasing profits will be after the new "mixed model" is adopted by the market. However, the Nash model cannot introduce the influence of bargaining



power on profit distribution. In the following part, other models are cited to calculate the influence of bargaining power.

## Influence From Bargaining Power

### *Eliashberg Model*

According to the Eliashberg model put forward by J. Eliashberg in 1986, we can use the weighted sum of the utility of suppliers and distributors to identify the total utility of this system. Then set up a new utility equation and discuss the cases to realize the maximal utility and the profit distribution within the different cases of the supply chain:

$$U_c(\Delta\pi_d, \Delta\pi_s) = r_1 * U_d(\Delta\pi_d) + r_2 * U_s(\Delta\pi_s) \quad (23)$$

Equation (23) introduces the weighting factors of negotiation in the Eliashberg model— $r_1$  and  $r_2$ .  $r_1$  represents the negotiating weight of the distributor, the more the amount is, the more initiative the distributor can obtain in the negotiation.  $r_2$  represents the weight of the supplier's negotiation, the more the amount is, the more active the supplier can get in the negotiation. And  $r_1 + r_2 = 1$ .

### *Case Analysis*

**Case 3:** In a supply chain that applies the option model, the distributor's utility equation is

$$U_d(\Delta\pi_d) = \ln(1 + \alpha * \Delta\pi_d). \text{ The utility equation of the supplier is } U_s(\Delta\pi_s) = \ln(1 + \Delta\pi_s).$$

The total utility of the supply chain is:

$$U_c(\Delta\pi_d, \Delta\pi_s) = r_1 * \ln(1 + \alpha * \Delta\pi_d) + r_2 * \ln(1 + \beta * \Delta\pi_s) \quad (24)$$

Since  $\Delta\pi_d$  and  $\Delta\pi_s$  can be expressed by  $\sigma$ , equation (24) can be regarded as a function of independent variable  $\sigma$ . Let us take the derivative of  $U_c$  to solve the value of  $\sigma$  that optimizes  $U_c$ :

$$\frac{dU_c}{d\sigma} = N * \left( \frac{-r_1}{1 + \alpha * \Delta\pi_d} + \frac{r_2}{1 + \beta * \Delta\pi_s} \right) \quad (25)$$

Let equation (25) = 0, then we get:

$$\Delta\pi_d = \frac{r_1 - r_2 + r_1 * \beta * \Delta\pi}{\beta * r_1 + \alpha * r_2}; \Delta\pi_s = \frac{r_2 - r_1 + r_2 * \alpha * \Delta\pi}{\beta * r_1 + \alpha * r_2}$$

After calculation, the optimal value of  $\sigma$  at this point is:

$$\sigma^* = \frac{(A - \pi_{wd}) * \alpha * r_2 - (B - \pi_{us}) * \beta * r_1 - r_1 + r_2}{(\beta * r_1 + \alpha * r_2) * N}$$

When the profit increases of suppliers and distributors are equal, that is,  $\Delta\pi_d = \Delta\pi_s$ , it can imply that the condition is satisfied by the bargaining coefficient:

$$\frac{r_2}{r_1} = \frac{\alpha * \Delta\pi + 2}{\beta * \Delta\pi + 2} \quad (26)$$

Because there is no restriction for  $\sigma$  in the previous equation,  $\sigma^*$  is not necessary in the interval  $[\sigma_{min}, \sigma_{max}]$ . Then comparing  $\sigma^*$  with  $\sigma_{min}$  and  $\sigma_{max}$ , respectively, we get the following conclusions.

If  $\sigma^* < \sigma_{min}$ , that is,  $\frac{r_2}{r_1} < \frac{1}{\alpha * (A + B - \pi_{wd} - \pi_{ws}) + 1}$ , the supplier has low bargaining power, or the distributor has high bargaining power. In this case,  $\sigma$  takes the minimum value.  $\Delta\pi_r = \Delta\pi$ ,  $\Delta\pi_m = 0$ . After the mixed model is adopted, the increasing profits of the supplier are 0, and the total profits of the system are all owned by the retailer.

If  $\sigma^* > \sigma_{max}$ , that is,  $\frac{r_2}{r_1} > 1 + (A + B - \pi_{wd} - \pi_{ws}) * \beta$ , the supplier has high bargaining power, or the distributor has low bargaining power. In this case,  $\sigma$  takes the maximum value.  $\Delta\pi_r = 0$ ,  $\Delta\pi_m = \Delta\pi$ . After the mixed model is adopted, the increasing profits of the distributor are 0, and the total profits of the system are all owned by the supplier.

If  $\frac{\alpha * \Delta\pi + 2}{\beta * \Delta\pi + 2} < \frac{r_2}{r_1} \leq 1 + \Delta\pi * \beta$ , that is, in the negotiation process, the supplier's bargaining power is slightly higher than that of the distributor, but there is no absolute advantage.  $\Delta\pi_s > \frac{\Delta\pi}{2}$ , the supplier gets more profits. And the higher  $\frac{r_2}{r_1}$  is, the greater profits the supplier makes from the model.

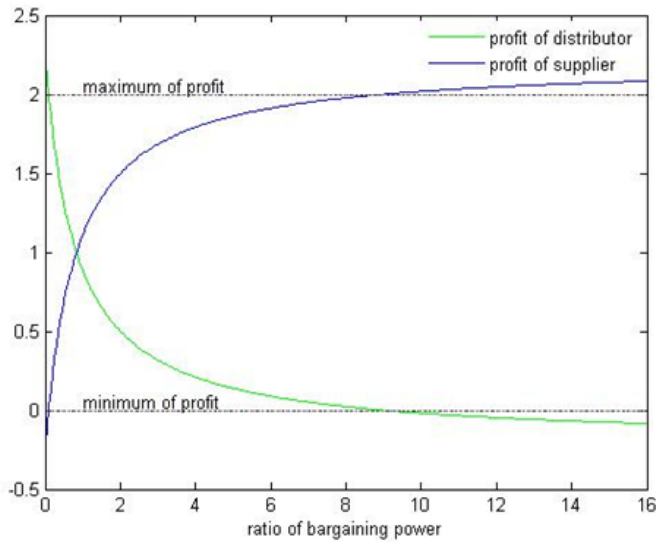
If  $\frac{\alpha * \Delta\pi + 2}{\beta * \Delta\pi + 2} \geq \frac{r_2}{r_1} > \frac{1}{1 + \Delta\pi * \alpha}$ , that is, in the negotiation process, the distributor's bargaining power is slightly higher than that of the supplier, but there is no absolute advantage.  $\Delta\pi_d > \frac{\Delta\pi}{2}$ , the distributor gets more profits. And the higher  $\frac{r_2}{r_1}$  is, the greater profits the distributor makes from the model.

If  $\frac{\alpha * \Delta\pi + 2}{\beta * \Delta\pi + 2} = \frac{r_2}{r_1}$ , both parties in the model have the same bargaining power.  $\Delta\pi_d = \Delta\pi_s = \frac{1}{2} \Delta\pi$ . The increase in profits of the new model is distributed equally between the two parties.

The results are tested by numerical simulations, setting concrete values as follows:  $\alpha = 5$ ;  $\beta = 4$ ; and  $\Delta\pi = 2$ . So, we can get Figure 4.

The x-axis is the ratio of bargaining power, which can be expressed as equation  $\frac{r_2}{r_1}$ , and the change in this figure is consistent with the analysis.

Figure 4. Influence of bargaining power on profit distribution



### Model Expansion

Combined with the case analysis, we can extend it to a general situation. In equation (23), the Arrow-Pratt risk measurement is substituted to determine the positive and negative sign of the equation after a classification according to the market investors' aversion or preference to risk.

We assume that both distributors and suppliers are risk-averse, which means:

$$R_d = -\frac{U_d''(\Delta\pi_d)}{U_d'(\Delta\pi_d)} > 0, U_d'(\Delta\pi_d) > 0, U_d''(\Delta\pi_d) < 0$$

$$R_s = -\frac{U_s''(\Delta\pi_s)}{U_s'(\Delta\pi_s)} > 0, U_s'(\Delta\pi_s) > 0, U_s''(\Delta\pi_s) < 0$$

Take the derivative of equation (23), and get:

$$\frac{dU_c}{d\sigma} = r_1 * \frac{dU_d(\Delta\pi_d)}{d\Delta\pi_d} * \frac{d\Delta\pi_d}{d\sigma} + r_2 * \frac{dU_s(\Delta\pi_s)}{d\Delta\pi_s} * \frac{d\Delta\pi_s}{d\sigma} = -N * r_1 * U_d' + N * r_2 * U_s' \quad (27)$$

Assume that there is  $\sigma^*$  that makes equation (27) equal to 0, which means  $r_1$  and  $r_2$  guarantee  $\sigma^*_{maxmin}$ . There will not be a situation in which the bargaining power of one party is too strong to take away all the increases in profits.

Setting  $f(\sigma) = \frac{U_d'(\Delta\pi_d)}{U_s'(\Delta\pi_s)}$ , combined with the condition of equation (27) = 0, we get

$f(\sigma^*) = \frac{r_2}{r_1}$ . The function shows the relationship between  $\frac{r_2}{r_1}$  and  $\sigma^*$ . Take the derivative to find out the specific relationship between the two variables:

$$f'(\sigma) = -\frac{N(U_d^n * U_s' + U_s^n * U_d')}{(U_s')^2} \quad (28)$$

When both parties are risk-averse, there will always be (28) > 0.

If  $r_2$  increases, it reveals that  $\frac{r_2}{r_1}$  is increasing, then  $\sigma^*$  will increase, then  $\Delta\pi_s \sigma^* = \sigma^* * N + B - \pi_{ws}$  will increase. That is, the greater the bargaining power of suppliers is, the more its profit share will be.

If  $r_1$  increases, it reveals that  $\frac{r_2}{r_1}$  is decreasing, then  $\sigma^*$  will decrease, then  $\Delta\pi_d \sigma^* = A - \sigma^* N - \pi_{wd}$  will increase. That is, the greater the bargaining power of distributors is, the more its profit share will be.

Under the condition that both sides are risk-averse, do the equivalent substitution and derivation operation based on the Eliashberg model, we learn that the party which has greater bargaining power can lead the optimal value of  $\sigma$  to the point which is good for its own profits. The bargaining power of suppliers is high, which will promote the reduction of  $\sigma^*$  and increase its own profits. The high bargaining power of the distributor will promote the increase of  $\sigma^*$  together with its profits. In theory, the increased bargaining power of a market participant would allow them to reap all the increases in supply chain profits.

## CONCLUSION

This paper's method of reasoning mainly was mathematical calculation. This paper cited the Arrow-Pratt risk aversion, the Nash model, and the Eliashberg model to simplify the supply chain which consists of one distributor and one supplier, and to calculate profits of the system. It was concluded that this mixed model can not only achieve the Pareto improvement by increasing the profits of the supplier and distributor but also can increase the turnover and expand the production, compared to the traditional wholesale price model or the pure option model. This advantage will attract market participants to sign option contracts.

Moreover, the mixed model has strong stability. It was calculated that if the number of options meets a certain range, it can not only attract both parties to sign option contracts but also can reach the optimal output after using option contracts. Although using options contracts to create the mixed model can increase the total profits of supply chain, how to divide the profits between distributors and suppliers would be affected by the internal factors for both sides. These "internal elements" dominate profit growth by influencing option prices and exercise prices of the contracts reached by both parties.

This paper mainly discussed two factors which are risk preference and bargaining power. The conclusions are as follows: (1) under option contracts, the higher the risk aversion level of the distributor or supplier is, the more its profits will be, and (2) when the two sides have the same degree of risk aversion, their bargaining power will affect the profit distribution. The more initiative in the negotiation, the more profits will be reaped from the supply chain, one of them may even get all the profit increases in the supply chain using option contracts.

These conclusions are valuable to both sides of the supply chain. For the participants in the supply chain, the "wholesale price model" is affected by a large number of uncertainties, while the pure option contract model lacks stability. The mixed model established in this paper reflects risk and benefit-sharing and has the ability to respond with flexibility to market changes.

When the supply chain is stable, the option price and the exercise price of the contract will be affected by the internal factors of the enterprise and determine the profit gap between the two parties. For the supplier and distributor, they can change their utility functions to increase profits by reducing the level of risk aversion, so they can achieve higher returns under the model of the steady state. Of course, the utility function of enterprises can also be modified by changing capital allocation and improving management mode.

The bargaining power of the supplier and distributor also influences the distribution results. According to calculations, a party with strong bargaining power can guide the value of an option contract to change. The bargaining power is affected by brand influence, resource holding, and industry competitiveness as well. As a party with weak bargaining power, it can change its utility function to prevent profits from being grabbed by the other party.

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