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Semi-Nonparametric Estimation of Energy Demand in Tunisia

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ABSTRACT

In this paper, we have studied the possibilities of substitution between different forms of energy at the sectoral level in Tunisia. Indeed, we estimated a globally flexible functional form of Fourier type using the iterative Zellner's seemingly unrelated regression (ITSUR). Contrary to several previous studies in the literature, we calculated the price (proper and crossed) and income elasticities, the elasticities of substitution in the sense of Allen, as well as the elasticities of substitution in the sense of Morishima for the period (1980-2017). The results show that price and income elasticities differ from one energy product to another and from one sector to another, depending on the destination (final or intermediate) and the nature of use. Additionally, the findings suggest that energy demand is more sensitive to changes in income rather than changes in price. It is noteworthy that the absolute value of price elasticity is lower than that of income elasticity. Implementing an appropriate pricing policy can help limit the income effect and encourage economic agents to make efforts to save energy. Our results also indicate that any pricing policy aiming to reduce energy consumption must consider the possibilities of substituting between products. Otherwise, this policy may yield results contrary to the intended objectives.

Keywords: Fourier Flexible Functional Form, Iterative Zellner's Seemingly Unrelated Regression, Energy Demand, Income and Price Elasticities, Elasticities of Substitution

JEL Classifications: C3, Q41, Q42

1. INTRODUCTION

The energy modeling literature has been using regular and classical functions such as Cobb-Douglas type functions or Constant Elasticity of Substitution (CES) functions for several years and decades (Arrow et al., 1961 and Uzawa 1962). These classical functions are characterized by their conformity with the neoclassical conditions relative to those of the maximization program in the theory of the constrained consumer.

However, many researchers have represented the production process by a constant elasticity of substitution function (Arrow et al., 1961). This function makes it possible to generalize classical functions of the Leontief, Cobb-Douglas, and linear type. It is characterized by an elasticity of substitution between the inputs respectively null (perfect complement), unitary, and infinite (perfect substitute).

Because of its manageability, this function is widely used in macroeconomic models and in the econometric analysis of producers (Van der Werf, 2008). It requires a limited number of parameters to be calibrated or estimated econometrically. The limit of the CES function lies in the fact that it imposes a constant elasticity of substitution along the isoquant (all other things being equal). The CES function lacks generality when one seeks to analyze a system with more than two factors of production. It requires a common elasticity of substitution between all the factors. This limitation appeared specifically constraining at the time of the oil shocks of the 1970s when it became essential to design various substitution properties between the factors of production: Energy, capital, and labor (Artus and Peyroux, 1981).

The limits related to CES shapes have helped the appearance of new so-called flexible functions. These forms impose fewer constraints on the production structure. Under these categories, we mainly find the Generalized Leontief (LG) function proposed by Diewert (1971), the Normalized Quadratic (QN) function developed by Diewert and Wales (1987) and the Transcendental Logarithmic (Translog) function developed by Christensen et al. (1973). These functions are local approximations by a second-order limited Taylor expansion of any production function, but they lead to distinct functional forms. "Both theoretically and empirically, the choice between these flexible forms is in practice tricky (Caves and Christensen, 1980; Despotakis, 1986)".

Locally flexible models make it possible to have elasticities at a given point (Barnett et al., 1992). They have played an important role in the enrichment of micro-econometrics due to the fusion between neoclassical microeconomic theory with econometric applications.

However, despite the properties and specificities of these models, and as proven by Caves and Christensen (1980), Guilkey and Lovell (1980), Barnette and Lee (1985) and Barnett et al. (1985, 1987), these models are unable to give correct approximations of reality by moving away from the point of approximation.

Barnett and Jonas (1983); Barnett et al. (1985), Barnett et al. (1985), and Barnett et al. (1985, 1987) brought about a partial solution to overcome this problem. They proposed a model known by the Laurent miniflex model (The Miniflex Laurent Model) which is based on the serial development of Laurent. Barnett and Serletis (2000) presented a brief description of these models. However, the Miniflex Laurent model is locally flexible and satisfies the regularity conditions (monotonicity and quasi-concavity) required for the producer along a region and not in a global way. To overcome this problem, semi-nonparametric functional forms, which are globally flexible and in which asymptotic inferences are free from any specification error, have been developed.

Semi-nonparametric functions can cause a global asymptotic approximation for complex economic relations. Indeed, through global approximation, the flexible functional form is able, in the limit, to approximate the underlying unknown generating function at all points, and thus produce arbitrarily precise elasticities at all data points. In other words, the semi-nonparametric forms make it possible to have global approximations of the true function. They converge towards the true function at any point of its domain of definition when the number of parameters, which varies with the order of approximation, increases indefinitely. In addition, these forms are characterized by their good approximations to the gradient and the Hessian of the true function, which are necessary to accurately determine the factors' demand functions and the elasticities of substitution.

The main semi-nonparametric forms are: The Fourier flexible functional form (FFF), introduced by Gallant (1981), and the AIM form (The asymptotically Ideal Model) introduced by Barnett and Jonas (1983), used and explained by Barnett and Yue (1988). These two semi-nonparametric forms have received a lot of attention from economists and modelers such as, for example, the work of Galant and Tauchen (1989), Fisher and Fleissig (1997), Havenner and Saha (1999), Fleissig et al. (2000), Fisher et al. (2001), Fleissig and Swofford (1996, 1997), Drake et al. (2003), Serletis and Shamoradi (2005; 2008), Chen and Ludvigson (2009), Sarwar et al. (2011), Ewis and Fisher (1985), Fisher (1992), Fleissig (1997), Jones et al. (2008), Fleissig and Jones (2015), Fleissig (2016), Anderson et al. (2019) and Fleissig (2021).

This paper emphasizes a semi-nonparametric functional form of the Fourier type in order to study the possibilities of substitution between the energy forms at the sectoral level. The singularity of this form lies in its power to verify the global flexibility condition in its structure and in the asymptotic inferences being generally free from any specification error (Serletis and Shahmoradi, 2005; 2008, Barnett and Jonas (1983); Barnett et al. (1985), Barnett et al. 1985; 1987, Gallant and Nychka, 1987). There is a lack of econometric analysis on inter-fuel substitution possibilities in Tunisia. Therefore, this paper aims to fill this gap in the empirical literature by examining the possibilities of substitution between the different forms of energy in Tunisia. Based on the estimation results, we will calculate the price and income elasticities as well as the elasticities of substitution between the different energy forms. The achieved results will be used to develop some suggestions and strategies that can help control and rationalize energy demand in Tunisia.

2. METHODOLOGY

To analyze the inter-energy links at the sectoral level, we will estimate a factor demand system resulting from a flexible Fourier function.

2.1. Introducing Flexible Fourier Form (FFF)

Our objective is to estimate a system of demand equations derived from an indirect utility function. The main advantage of using the indirect utility approach is that prices are considered as exogenous variables in the estimation process, and the system of demand shares can be easily inferred by applying Roy's identity.

Classical flexible functional forms are essentially based on a limited expansion of the second order in the Taylor series. However, much work has shown the inability of classical functional forms to provide a correct approximation of reality when significantly deviating from the point of approximation. These functional forms can provide an approximation of reality in a neighborhood of the approximation point, but concerns and research arise regarding the relevance of these results in econometric analysis when using a precise flexible form, especially on samples with relatively high empirical variance.

The work of Wales (1977), Guilkey and Lovell (1980), Guilkey et al. (1983) showed the inability of classical functional forms to give correct approximations of reality when one moves away from the point of approximation. Increasing the number of parameters to be estimated in order to increase the order of approximation in the Taylor series of the usual shapes can be a solution to overcome this problem, but it can generate a risk of approximation. For this reason, Gallant (1981; 1982) developed a new flexible form allowing to limit the errors made by the classical forms. This function is based on a Fourier series expansion which is known by the flexible Fourier form (FFF). This form is the most used in semi-nonparametric approaches (Mitchell and Onvural, 1996, Huangard and Wang, 2004).

The flexible Fourier form is a global approximation of the true function. It converges to the true function at any point of its domain of definition when the number of parameters, which varies with the order of approximation, increases indefinitely. In practice, the number of parameters of the Fourier form is finite, but the approximation error is bounded.

The Fourier form uses a distance called the Sobelev norm to measure the error of approximation. The Sobelev norm measures the distance of an approximate function g(x) using the real function g*(x).

The p-type Sobolev norm is then defined by the following relation:

$$\| g^* - g \|_{m,p,w} = \left(\sum_{|U^*|<m} \int \left| D^U g^* - D^U g \right|^P dw(x)\right)^{\frac{1}{P}} \quad \| 1 < \mathrm{P} < \infty$$

Where: m: Represents the maximum order of the partial differentiation of the function of interest g.
D^U: Designates the partial differentiation.
W(x): Is the continuous probability distribution function admitting a bounded density function.
U: Is the approximation region (it is the domain over which we seek to approximate g).

If g presents a bad approximation of g* or one of its derivatives of order m, the Sobolev norm will attribute a large value to the approximation error.

When p tends to infinity, the Sobolev norm will take the following form:

$$\| g^* - g \|_{m,p,w} = \sum_{|U^*|<m} Sup \left| D^U g^* - D^U g \right|$$

According to Devezeaux de Lavergne et al. (1990), the Sobolev norm is well suited to measure the distance of the true function from its approximation in the sense that the errors made on the partial derivatives of order less than or equal to m are involved in the calculation of the approximation error.

The function g(x) is said to be flexible in the Sobolev sense when it satisfies the following condition:

$$\lim_{k\to\infty} k^{m-r-\varepsilon} \| g(x) - g^*(x) \|_{r,p,w} = 0; \ \forall \varepsilon > 0$$

Where: r < m: Is the maximum order of partial derivation of g;
k: Is the dimension of the parameter vector θ ($g^*(x)=g_k(x/\theta)$).

This condition implies that a flexible form in the Sobolev sense makes it possible to obtain a good approximation of the true function at any point of the approximation region.

- The notion of multi-index:

The notion of multi-index makes it possible to significantly reduce the complexity of the notation necessary to designate the partial differentiation of high order and the multivariate expansion in Fourier series. On the other hand, a multi-index is an N-dimensional vector with integer components. The length of a multi-index is defined by:

$$|K|^* = \sum_{i=1}^{N} |K_i|$$

Suppose λ is a multi-index with non-negative components. The partial differentiation of the function f(x) is given by**:**

$$D^\lambda f = \frac{\partial^{|\lambda|^*}}{\partial x_1^{\lambda_1} \partial x_2^{\lambda_2} \ldots\ldots\ldots\ldots\ldots\ldots \partial x_N^{\lambda_N}} f(x)$$

- The Fourier form:

A Fourier series expansion of multivariate real-valued functions of order k is defined by the following writing:

$$\sum_{|K|^* \le K} a_k e^{ik'x}$$

Where: $e^{ik'x} = \cos inus(k'x) + i \sin us(k'x)$

a_k: Are coefficients with complex values and they take the following form:

$$a_k = u_k + iv_k$$

With: u_k and v_k are real numbers.

An exception made for multi-indexes 0=⟦(0,……,0)⟧', multi-indexes of order k with $|K|^* \le K$ will have multi-indexes of opposite sign, (−k).

Therefore: a_0 = real value

$$a_k = \overline{a}_{-k}$$

and: $v_0 = 0$; $u_k = u_{-k}$; $v_k = -v_{-k}$

This allows us to say that: $\sum_{|k|^*} a_k e^{ik'x}$ is real.

With these restrictions, we deduce that:

$a_0 e^{i0'x} = u_0$: is real

$a_k e^{ik'x} + a_{-k} e^{-ik'x} = 2\, u_k \cos(k'x) - 2\, v_k \cos(k'x)$: is also real.

The sum $\sum_{|K|^* \le K} a_k e^{ik'x}$ can be rewritten as a double sum as follows:

$$\sum_{\alpha=1}^{A} \sum_{j=-J}^{J} a_{j\alpha}\, e^{ijk'_{\alpha} x}$$

The main idea is to build a set of multi-indexes $\{k^{\alpha}\}$ and to choose the values of A and J in such a way:

$\{k: |k|^* \le K\} \subset \{jk_{\alpha}: \alpha = 1, \ldots, A;\ j = 0, \pm 1 \ldots, \pm J\}$

By imposing the restrictions of the single-sum form on the double-sum form, we obtain the following results:

$a_{0\alpha} \in R$, with:
$V_{0\alpha} = 0$
$u_{j\alpha} = u_{-j\alpha}$
$v_{j\alpha} = -v_{-j\alpha}$
$a_{j\alpha} = \bar{a}_{-j\alpha}$

and $a_{j\alpha} = u_{j\alpha} + iv_{j\alpha}$; α=1,2,………,A; j=–J,……… 0,………, J

$a_{-j\alpha} = u_{j\alpha} - iv_{j\alpha}$; α=1,2,………, A; j = –J,……… 0,………, J

These restrictions will allow us to rewrite the double-sum form as follows:

$$\sum_{\alpha=1}^{A} \sum_{j=-J}^{J} a_{j\alpha} e^{ijk'_{\alpha} x} =$$

$$\sum_{\alpha=1}^{A} \left\{ \begin{array}{l} u_{0\alpha} + \sum_{j=1}^{J} [\left(u_{j\alpha} + iv_{j\alpha}\right)[\cos\left(jk'_{\alpha} x\right) + i \sin\left(jk'_{\alpha} x\right)] \\ + \left(u_{j\alpha} - iv_{j\alpha}\right)\left[\cos\left(jk'_{-\alpha} x\right) + i \sin\left(jk'_{-\alpha} x\right)\right]] \end{array} \right\}$$

$$= \sum_{\alpha=1}^{A} \left\{ u_{0\alpha} + 2 \sum_{j=1}^{J} [u_{j\alpha} \cos\left(jk'_0 x\right) - v_{j\alpha}(jk'_0 x)] \right\}$$

The previous expression is a Fourier series expansion of a multivariate function with arbitrary real values. This development is characterized by its flexibility in the sense of Sobolev.

Gallant (1981) pointed out that introducing a linear term (b′x) in a Fourier approximation of a periodic function can considerably reduce the number of sine and cosine terms without deteriorating the quality of the approximation performed. Moreover, he showed that to impose concavity restrictions, it is necessary to add a quadratic term of the form x′cx.

In light of the above, the flexible Fourier form takes the following form:

$$g_k(x) = a_0 + b'x + \frac{1}{2} x'cx + \sum_{\alpha=1}^{A} \sum_{j=-J}^{J} a_{j\alpha} e^{ijk'_{\alpha} x}$$

With: $a_{j\alpha} = \bar{a}_{-j\alpha}$

$$c = -\sum_{\alpha=1}^{A} a_{0\alpha} k_{\alpha} k'_{\alpha}$$

a_0; $a_{0\alpha}$ and b are reals

The derivative functions of $g_k(x)$ are given by the following equations:

$$\left(\partial / \partial x\right) g_k(x) = b + cx + i \sum_{\alpha=1}^{A} \left(\sum_{j=-J}^{J} j a_{j\alpha} e^{ijk'_{\alpha} x} \right) k_{\alpha}$$

$$\left(\partial^2 / \partial x \partial x'\right) g_k(x) = -\sum_{\alpha=1}^{A} \left(a_{0\alpha} + \sum_{j=-J}^{J} j^2 a_{j\alpha} e^{ijk'_{\alpha} x}\right) k_{\alpha} k'_{\alpha}$$

The standard flexible form is defined by a second-order approximation at a given point of the true production function or cost. This definition is also satisfied for the Fourier form, this is guaranteed when A is large enough.

In this paper, we will follow the procedure already explained by Gallant (1981), Serletis and Shahmoradi (2005-2008) to develop an indirect utility function using Fourier series:

$$h(v) = u_0 + b'v + \frac{1}{2} v'cv + \sum_{\alpha=1}^{A} \left\{ u_{0\alpha} + 2 \sum_{j=1}^{J} [(u_{j\alpha} \cos\left(jk'_{\alpha} v\right) - w_{j\alpha} \sin\left(jk'_{\alpha} v\right)] \right\}$$

with: $c = -\sum_{\alpha}^{A} = 1 u_{0\alpha}^{A}\ k_{\alpha}\, k'_{\alpha}$

Where: $V = \frac{p}{m}$: Represents the price normalized by the total energy expenditure.
$m = P'X$; With $X = (X_1, X_2, X_3)$: Is a vector containing the demand for energy forms,
$P = (p_1, p_2, p_3)$: Is the vector of energy prices and m is the total energy expenditure.
k_{α}: Is a multi-index of n vectors composed by integers.
u_0; {b}, {u} et {w}: Are unknown parameters to be estimated.

The parameters A and J represent respectively the numbers of the terms and the degree of approximation. They determine the degree of Fourier polynomials.

Referring to Gallant (1981), the length of a multi-index is defined as follows:

$$|k_{\alpha}|^* = \sum_{i=1}^{n} |k_{i\alpha}|$$

This multi-index reduces the complexity of the notation required to express the high-order partial differentiation.

The Flexible Fourier Functional Form (FFF) has the ability to perform approximations in the sense of the Sobolev norm which imparts nonparametric properties on the functional form. This is the reason why the (FFF) is considered as a semi-nonparametric functional form (Serletis, 2007, Serletis and Shahmoradi, 2005; 2008 and Serletis and Rangel-Ruiz, 2005).

Applying Roy's identity gives us the demand share equations of energy form i relative to the indirect utility function:

$$S_i = \frac{v_i(\partial h(v)/\partial v_i)}{v'(\partial h(v)/\partial v_i)} = \varphi_i(v,\ \theta)$$

With: $\theta = \{u_0, b_i, u_{j\alpha}, W_{j\alpha}\}$ and I =1,2,3 forms of energy.

The Fourier demand system for a given sector(s) is written as follows:

$$S_i = \frac{v_i b_i - \sum_{\alpha=1}^{A}(u_{0\alpha} v' k_\alpha + 2\sum_{j=1}^{J} j\,[u_{j\alpha} \sin(jk'_\alpha v) + w_{j\alpha} \cos(jk'_\alpha v)])k_{i\alpha} v_i}{b'v - \sum_{\alpha=1}^{A}(u_{0\alpha} v' k_\alpha + 2\sum_{j=1}^{J} j\,[u_{j\alpha} \sin(jk'_\alpha v) + w_{j\alpha} \cos(jk'_\alpha v)])k'_\alpha v} = \varphi_i(v,\ \theta)$$

For: i = 1,2,3 forms of energy.

Eastwood and Gallant (1991) and Huber (1981) showed that Fourier functions give consistent and asymptotically normal estimators when the number of parameters to be estimated is equal to the effective number of observations raised to the power of ($\frac{2}{3}$).

In our case, for n = 3 forms of energy and T = 34, the effective number of observations is equal to: [T*(n-1)] = [(34*2) = 68]. As we are going to estimate (n-1) budget shares then we must also estimate approximately: [$(68)^{\frac{2}{3}} \approx 17$] parameters.

By imposing the restriction $\sum_{i=1}^{n-1} b_i = b_n$, the Fourier demand system has: (n-1)b, $Au_{0\alpha}$, $AJu_{j\alpha}$, et AJw_α parameters to estimate, for a total of parameters equivalent to (n-1) + A(1+2J).

By setting [(n-1) + A (1 + 2J)] equal to 17, we choose a value for A equals to 5 and for J equals to 1. This also determines the elementary multi-indexes used in this part, as shown below in Table 1:

2.2. Calculation of Elasticities

Based on the results of the estimation of the flexible form of Fourier, we then calculate the price and income elasticities and the elasticities of substitution in the sense of Allen-Uzawa and Morishima.

Table 1: Elementary multi-indexes

α	1	2	3	4	5
V_1	1	0	0	1	1
V_2	0	1	0	1	0
V_3	0	0	1	0	1
$\lvert k_\alpha \rvert$*	1	1	1	2	2

The calculation of the elasticities of substitution will allow us to study the substitutability or the complementarity between the different energy forms.

2.2.1. Price and income elasticities of demand

The primary and particular interest in estimating economic relations based on the demand-share approach is to know the magnitudes by which the arguments of the underlying functions affect the quantities demanded. This is classically and completely expressed in terms of elasticities: Price, income and substitution.

These elasticities can be calculated directly from the demand share equations of the form of energy i for a sector (s) and this by writing in the left side:

$$X_{is} = \frac{S_i m_s}{P_i} \qquad \text{With: } i = 1,2,3$$

In particular, the income elasticity of demand for energy form i for a sector(s) can be written as:

$$n_{ij} = \frac{p_j}{S_i}\frac{\partial S_i}{\partial p_j} - \delta_{ij},\ i, j = 1,2,3$$

The cross-price elasticity measures the sensitivity of the demand for energy i following the variation in the price of energy j. It is given as follows:

With:= $\delta_{ij} = \begin{cases} 1 \textit{ if } i = j \\ 0 \textit{ if not} \end{cases}$

2.2.2. The elasticity of substitution

From an energy policy point of view, measuring the elasticity of substitution between goods is of great importance (Uri, 1982). There are two methods used for the calculation of these elasticities and which are: the elasticity of substitution in the sense of Allen and Morishima.

Regarding the partial elasticity in the sense of Allen between two variables, this is calculated as follows:

$$\sigma_{ij}^{A} = n_{im} + \frac{n_{ij}}{s_j}$$

This elasticity is interpreted as follows: If $\sigma_{ij}^{A} > 0$: goods i and j are substitutable; If $\sigma_{ij}^{A} < 0$: goods i and j are complementary and If $\sigma_{ij}^{A} = 0$: goods i and j are independent.

Allen's elasticity of substitution is the traditional and most widely used measure in the literature. It is used to measure substitution behaviors as well as structural instability in several contexts. Despite the simplicity of this elasticity, it suffers from a number of limitations. Indeed, in the case where we have more than two goods, the Allen elasticity can become less informative, and also, we risk having ambiguous and complex relations between the goods.

The Morishima elasticity (ESM) in this case becomes the correct measure of the elasticity of substitution. It is given by the following formula:

$$\sigma_{ij}^{M} = S_i(\sigma_{ij}^{A} - \sigma_{ii}^{A})$$

With: S_i: is the share of energy form i in the total energy expenditure in a given sector (s).

Morishima's concept of elasticity finds its foundation in the notion of Hicksian substitutability. It is, the change in the utilization ratio of goods when relative prices change.

Morishima elasticities are equal to the difference between the direct price elasticity and the cross-price elasticity. They then take into account the relative adjustment of the quantity demanded in the face of changes in the price of goods (Ball and Chambers, 1982).

All other things being equal, a variation in the price ratio Pi/Pj, generates a variation in the ratio of use of goods (Xi/Xj). Assuming the level of production is constant, the two goods are considered complementary (respectively substitutable) 'Morishima' if an increase in the ratio Pi/Pj causes a decrease (respectively an increase) in the ratio (Xi/Xj). In addition, the ESM has the particularity of being asymmetrical; this means that $\sigma_{ij}^{M} \neq \sigma_{ji}^{M}$ (Blackbory and Russel, 1989).

3. DATABASES AND COMPUTATIONAL CONSIDERATIONS

Our empirical study focuses on the semi-nonparametric modeling of substitution possibilities between energy forms at the sectoral level using the Fourier flexible functional form. We present in the following, a description of our database and the computational considerations taken into account during the estimations.

3.1. Databases of Econometric Analysis

Our study is based on annual data covering the period from 1980 to 2017. The data that we will use in this section concern:

- Energy demand by sector (industry, transport and residential) for the following forms: Electricity, natural gas, heavy fuel oil, domestic fuel oil, gas oil, LPG, premium unleaded petrol, and diesel. These data are provided by the National Agency for Energy Management (ANME) and the Directorate General for Energy (DGE) for the period from 1980 to 2017.
- Data on energy prices, also covering the period from 1980 to 2017, are provided by the National Agency for Energy Management (ANME).
- Data published by the Central Bank concerning indicators of economic activity and price indices (base year 1990).

3.2. Computational Considerations

Our study is inspired by the works of Gallant and Gollub (1984), Serletis and Shahmoradi (2005–2008) and Sarwar et al. (2011). The sectors selected for the estimates are industrial, transport and residential sectors.

The forms of energy used by sector in our estimates are:

- For the industrial sector, the forms retained are heavy fuel oil, electricity, and natural gas.
- For the transport sector, the forms retained are premium unleaded petrol, diesel, and natural gas.
- For the residential sector, the forms used are LPG and electricity. Natural gas is the main competitor to these products in this sector.

The system of demand shares that we plan to estimate for a given sector(s) is written as follows:

$$S_{1t} = \varphi_1(v_t, \theta) + \varepsilon_{1t}$$

$$S_{2t} = \varphi_2(v_t, \theta) + \varepsilon_{2t}$$

With ε_{it}: is an error term verifying the assumptions of the classical model.

4. RESULTS OF FFF ESTIMATES

We present in this section the results of the estimates of the budget share equations by using the Fourier method, as well as the different price, income, and substitution elasticities.

4.1. Results of Estimation

To estimate our model for each sector, we used a sample composed of 34 annual observations covering the period from 1980 to 2017. The method used to estimate the budget share equations is the iterative procedure of Zellner (1962): [Zellner's Iterative Seemingly Unrelated Regression (ITSUR)]. To perform our regressions, we utilized the "PROC Model" command in the SAS software. A summary of the results is presented in Table 2.

The analysis of the results from the estimation of the Fourier flexible form at the sector level reveals that the majority of the estimated parameters are statistically significant[1]. This suggests that the demand adjustments are satisfactory. However, Durbin Watson's statistics indicate the existence of an error autocorrelation problem. This is why we have introduced first-order autoregressive processes (AR[1]) into the regressions, following the example of Berndt and Savin (1975) and Fleissig and Serletis (2002). The explanatory powers of each equation are high.

The regularity conditions are checked as follows:

- Positivity and monotonicity are well verified in all sectors. Indeed, We have ensured that all the estimated values of the shares are positive ($\hat{S}_t \geq 0$).

1 According to Fleissig and Serletis (2002, p83) and Fleissig and Rangel-Ruiz (2005), as there are many parameters to be estimated in the model, it is therefore not important whether all the parameters are statistically significant or not, what is important is that the model fits the data well.

Table 2: Results of the estimation of Fourier parameters at sector level

Estimated Parameters	Residential	Transport	Industrial
b_1	0.461362 (21.28)***	0.749103 (65.50)***	0.135327 (5.76)***
b_2	0.545447 (25.64)***	0.006897 (0.56)	0.089816 (3.61)***
u_{01}	−0.00027 (−2.69)**	0.00174 (13.32)***	0.002817 (3.71)**
u_{02}	0.000371 (6.17)***	−0.00017 (−2.34)**	0.00162 (2.43)**
u_{03}	−0.00196 (−5.21)***	−0.00062 (−7.38)***	0.00198 (8.01)***
u_{04}	0.000127 (3.09)***	0.000031 (1.30)	−0.0011 (−0.44)
u_{05}	0.000584 (5.81)***	0.000398 (21.30)***	−0.00315 (−3.82)***
u_{11}	0.000422 (0.13)	−0.00277 (−1.01)	−0.00387 (−1.03)
u_{12}	−0.00229 (−0.93)	0.000248 (1.19)	−0.00175 (−0.37)
u_{13}	−0.00319 (−2.85)**	−0.00144 (−2.38)**	−0.00909 (−1.16)
u_{14}	−0.0003 (−0.46)	0.001123 (2.77)**	−0.00858 (−1.73)
u_{15}	0.000711 (1.19)	0.000314 (1.25)	0.001089 (0.56)
w_{11}	0.003162 (1.67)	−0.00193 (−0.79)	−0.01355 (−3.19)***
w_{12}	0.000445 (0.19)	0.000845 (3.85)***	0.001346 (0.45)
w_{13}	−0.00048 (−0.73)	0.001938 (2.38)**	−0.00779 (−0.80)
w_{14}	−0.01033 (−2.48)**	−0.00117 (−0.52)	0.001866 (1.46)
w_{15}	−0.00028 (−0.40)	−0.00598 (−4.65)***	−0.01021 (−2.04)*
R^2S1	0.9625	0.8876	0.9133
R^2S2	0.9756	0.8817	0.9746
DWS1	1.6330	1.9039	1.8883
DWS2	1.6744	1.9180	1.8898

Figures in parentheses are Student ratios; (*): Significance at the 10% threshold; (**): Significance at 5%; (***): Significance at the 1% level.

- The concavity condition requires that the Slutsky matrix be negative semidefinite. This condition is checked by performing a Cholesky factorization of this matrix and checking if the Cholesky values are not positive.

The estimated parameters of our model are not easily interpreted in economic theory. We will explore their economic content through the calculation of price and income elasticities and essentially through the elasticities of substitution.

4.2. Calculation of Elasticities

This section presents a comparative analysis of energy substitutions at the sectoral level. The comparison is based on the study of own-price, cross-price and income elasticities as well as on the examination of the substitutability-complementarity relationships between the forms of energy.

4.2.1. Price and income elasticities

Table 3 presents the results of the calculation of price and income elasticities evaluated at the mean point of the sample of observations for three sectors (industrial, transport and residential), as well as for each of three energy products.

All elasticities in the current study were obtained using the following numerical differentiations: $(\partial si/\partial m)$ and $(\partial si/\partial pj)$ for i, j = 1, 2, 3.

In the light of the results obtained from calculating the price and income elasticities at the mean point of the sample of observations, a few remarks need to be made:

- Generally, energy demand is more responsive to changes in income than to changes in price (the price elasticity is lower in absolute value compared to the income elasticity).
- The price and income elasticities vary depending on the energy product and sector, based on the destination (final or intermediate) and the nature of use.
- In economic theory, own price elasticities must be negative. This condition is necessary to ensure the concavity of the cost function. Our results (presented in table 3) are all negative, consistent with the theory of the producer. These findings align with economic theory. Overall, our results are in line with economic theory, demonstrating consistency across sectors and models.

In the Industrial sector:

- The demand for electricity and Heavy Fuel Oil is quite elastic with respect to income (the income elasticities are respectively of the order of 1.2284 and 1.5810. These are the superior goods). This implies that these two products, all other things being equal, are likely to increase at a faster rate than that of industrial value added.
- The analysis of direct-price elasticity reveals that the demand for natural gas is highly elastic to its relative price. In fact, a 10% increase in the price of natural gas results in a 18.81% reduction in consumption of the product. This elasticity is significantly higher than that of Heavy Fuel Oil and electricity, mainly due to the absence of captive use of gas in the industrial sector.
- The cross-elasticities between Heavy Fuel Oil and electricity are very low and have negative signs. This implies that an increase in the price of one of these two products leads to a drop in the demand for the other product ($n_{EL/Fu} = -0.1185$ and $n_{Fu/El} = -0.1467$).
- In addition, the demand for natural gas is slightly sensitive to variations in the price of electricity, while the latter is inelastic when it comes to an increase in the price of natural gas ($n_{EL/GN} = 0.0454$ and ($n_{GN/EL} = 0.3250$).

Transport sector:

- The demand for regular gasoline and diesel is very sensitive to a variation in income (with income elasticities respectively

equal to 1.5476 and 1.0288). This implies that an improvement in the level of economic activity leads to an increase in the consumption of diesel and regular gasoline.
- The demand for unleaded gasoline is very sensitive to changes in its price. Indeed, it has a direct-price elasticity of (−1.5834), which implies that a 1% increase in the price of gasoline would save 1.583% of its demand.
- The demand for diesel is elastic relative to the evolution of its price ($n_{D/D}$ = −0.9592). On the other hand, the demand for normal gasoline is not very sensitive to changes in its price.
- The analysis of cross-elasticities shows an inelastic demand for diesel and unleaded gasoline in response to changes in the price of normal gasoline. The elasticity values are very close to zero ($n_{D/EN}$ = 0.0818 et $n_{ESP/EN}$ = 0.0134). However, the demand for regular gasoline is slightly sensitive to variations in these two products.
- In addition, we notice that an increase in the price of unleaded gasoline (respectively diesel) leads to a decrease in the demand for diesel (respectively unleaded gasoline). The demand for these two products is mutually sensitive.

Residential sector:
- In this sector, electricity and natural gas are sensitive to income (income elasticities are equal to 1.1693 and 1.3179, respectively).
- The demand for LPG is quite sensitive to changes in its price ($n_{GPL/GPL}$ = −2.3476). This means that a 1% increase in the price of LPG would cause a 2.3476% drop in demand for this product).
- The demand for natural gas is less sensitive to changes in its price than LPG and electricity.
- We note that the demand for electricity is weakly sensitive to an increase in the price of natural gas and vice versa ($n_{EL/GN}$ = 0.1505 et $n_{GN/EL}$ = 0.1306).
- LPG demand is more sensitive to changes in the price of natural gas than electricity demand

4.2.2. Substitutability-complementarity relationships

The following Table 4 gives estimates of the Allen and Morishima elasticities of substitution:

Our analysis strategy will be as follows: we will analyze the diagonal terms of the Allen matrix to examine the proper elasticities of substitution of different forms of energy. Since the Allen partial elasticity yields ambiguous results for the off-diagonal terms of the matrices, we will use the elasticities of substitution in the sense of Morishima to determine the nature of the relationship existing between the energy forms at the sector level.

The Allen and Morishima elasticities of substitution that were calculated for the four sectors are shown respectively in the above table. Several points can be drawn:
- The results differed from one sector to another.
- In Allen matrices, the diagonal elements are all negative, which implies that the rise in the price of an energy product will tend to reduce the intensity of this same product. However, the majority of off-diagonal elements for the Allen and Morishima matrices are positive.
- We can generally observe fairly wide possibilities of substitution between the different energy products used by the different sectors, with a few notable exceptions. They mainly concern heavy fuel oil and electricity in the industrial sector.

Table 3: Average price and income elasticities at the sector level

	Sector	Income elasticity	Price elasticity		
		n_i	n_{i1}	n_{i2}	n_{i3}
Electricity	Industrial sector	1.2284	−0.8508	−0.1185	0.0454
Heavy fuel oil		1.5810	−0.1467	−0.9326	1.5095
Natural gas		0.4310	0.3250	0.4805	−1.8183
Premium Gasoline	Transport sector	0.4993	−1.5834	−0.6498	0.0134
Diesel		1.0288	−0.7447	−0.9592	0.0818
Natural gas		1.5476	−0.5224	0.3341	−0.1750
LPG	Residential sector	0.6757	−2.3476	0.1505	0.9160
Electricity		1.1693	0.3546	−0.9084	0.1507
Natural gas		1.3179	0.3924	0.1306	−0.6982

Table 4: Average Allen and Morishima elasticities of substitution

	Sectors	Allen's elasticity			Morishima's elasticity		
		σ^A_{i1}	σ^A_{i2}	σ^A_{i2}	σ^A_{i3}	σ^M_{i1}	σ^M_{i2}
Electricity	Industrial sector	−3.1579	−4.1250	0.8418		−0.3664	0.7173
Heavy fuel oil			−1.4595	3.1476	−0.0153		1.2938
Natural gas				−3.2440	0.9518	1.5994	
Premium Gasoline	Transport sector	−3.2377	−0.7289	0.3146		1.1492	1.6167
Diesel			−0.7842	1.6604	0.0493		1.2821
Natural gas				−2.1597	0.1716	0.2304	
LPG	Residential sector	−7.4679	2.2729	3.6375		0.5674	1.0975
Electricity			−2.2502	−2.0714	0.1617		−0.6699
Natural gas				−6.2314	0.8618	0.3592	

- In the industrial sector, natural gas and electricity are substitutable. The elasticity of substitution is of the order of: $\sigma^M_{GN/EL}$ = 0.951 and $\sigma^M_{EL/GN}$ = 0.717. There is also a possibility of substantial substitution between heavy fuel oil and natural gas ($\sigma^M_{GN/FL}$ = 1.599 and $\sigma^M_{FL/GN}$ = 1.293). It is worth noting that there is a complementarity between electricity and heavy fuel oil $\sigma^M_{FL/EL} = -0.015$; $\sigma^M_{EL/FL} = -0.366$. This can be partially explained by the of heavy fuel oil in the production of electricity, which is not accounted for in our current database. According to the International Energy Agency, the share of fuel oil consumption in the electricity production was approximately 73% in 2012.
- In the transport sector, we notice an asymmetric substitutability between unleaded gasoline and diesel $(\sigma^M_{ES/DI} > (\sigma^M_{\frac{DI}{ES}} = 0.049))$. This may reflect the cost of replacing unleaded petrol engines with diesel in the transport sector. In addition, we notice the existence of a substitutability between normal gasoline with unleaded gasoline and diesel.
- In the residential sector, LPG and natural gas are highly substitutable ($\sigma^M_{GPL/GN}$ = 1.097 and $\sigma^M_{GN/GPL}$ = 0.861) and electricity and LPG are slightly substitutable ($\sigma^M_{GPL/EL} = 0.567$ and $\sigma^M_{EL/GPL} = 0.161$).
- As we have already mentioned, gas and electricity are substitutable in the industrial sector. The sensitivity of electricity demand to the price of gas is very low. If we examine the gas-electricity cross-price elasticities more closely, we find that the situation is markedly different. The demand for gas in this sector is quite sensitive to the price of electricity, indicating competition between the two energies based on a price differential.
- In the literature, there have been several previous studies that aimed to examine the symmetry between energy substitutions (e.g. Gately and Huntington [2002]; Griffin and Schulman [2005]). In our case, we have noticed the existence of an asymmetrical relation between several forms of energy substitution. The degree of asymmetry varies across different sectors. In our study, the asymmetry generally occurs without a change of sign. However, we observed that in the residential sector, there is an asymmetry with an opposite sign ($\sigma^M_{EL/GN} = -0.669$ et $\sigma^M_{GN/EL} = 0.359$).

5. CONCLUSION AND POLICY IMPLICATIONS

Substitution between forms of energy is an important topic that has been questioned for many years, which is why governments around the world are looking to set policies to reduce carbon emissions or steer economies towards or away from certain fuels.

In this regard, it should be noted that the results of most energy and climate change policy models are very sensitive to elasticity parameters, in particular the elasticity of inter-energy substitution.

Thus, a detailed analysis of the possibilities of substitution between the different forms of energy in various branches of activity is of great importance for a better understanding of the problems and energy needs in Tunisia. From an environmental and economic point of view, a study of the possibilities of substitution between different types of energy is relevant.

Referring to Serletis and Shahmoradi (2005, 2008), in this paper we have estimated a semi-nonparametric and globally flexible functional form of the Fourier (FFF) type at the sectoral level. The purpose of this study is to examine the possibilities of substitutions between different forms of energy. This form is the sum of a quadratic form and a Fourier series expansion. It converges to the true function at any point within its defined domain, which sets it apart from classical functional forms like the Translog, which only provide an approximation in the vicinity of a point. The FFF has the advantage of having a greater number of parameters that theoretically ensures a wider range of applicability compared to the Translog form.

In our estimations, the data used covers the period from 1980 to 2017. The sectors used in our analysis are transport, industry and residential. Calculations of the different elasticities (price and income) indicate that energy demand is generally more sensitive to income than to price (the absolute value of price elasticity is lower than income elasticity). We also notice the existence of an asymmetric relation of substitution between several forms of energy. The degree of asymmetry differs from one sector to another. This leads us to conclude that for certain sectors, an adequate price policy would counteract the income effect and encourage economic agents to make efforts to save energy. Our results also indicate that any pricing policy aiming to reduce energy consumption must pay attention to the possibilities of substitution between products. Otherwise, this policy may lead to results contrary to the intended objectives.

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