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## Article

### Investigation of the influence of the heterogeneous permeability distribution on the oil phase displacement processes

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# INVESTIGATION OF THE INFLUENCE OF THE HETEROGENEOUS PERMEABILITY DISTRIBUTION ON THE OIL PHASE DISPLACEMENT PROCESSES

The object of research is the filtration processes of displacement of the oil phase under the influence of an injection well in a heterogeneous porous medium. It is possible to evaluate and take into account the effect of reservoir heterogeneity on the distribution of reservoir pressure (and, consequently, on the intensity of the filtration process) using numerical modeling of filtration processes based on the piezoelectric equation. To solve the non-stationary anisotropic problem of piezoconductivity, it is proposed to apply the combined finite-element-difference method of M. Lubkov, which makes it possible to take into account the inhomogeneous distribution of permeability inside the anisotropic oil-bearing formation and at its boundaries, and to adequately calculate the distribution of reservoir pressure. The use of the combined finite-element-difference method allows to combine the advantages of the finite-element method and the finite difference method: to model geometrically complex areas, to find the value at any point of the object under study. At the same time, the use of an implicit difference scheme when finding the nodal values of the grid provides high reliability and convergence of the results.

The simulation results show that the distribution of the pressure field between the production and injection wells significantly depends on their location, both in the isotropic landslide and in the anisotropic oil-bearing reservoir. It is shown that the distance between the wells of more than 1 km levels out the effectiveness of the impact of the injection well on the oil filtration process. The influence of the permeability of the oil phase in the shear direction dominates the influence of the permeability in the axial directions (affects the pressure decrease by 4–9.5%). In the case of a landslide-isotropic reservoir, the wells should be located in the shear (diagonal) direction, which will provide the lowest level of drop in the average reservoir pressure (by 4%).

Based on the information obtained, for the effective use of anisotropic low-permeability formations, it is necessary to place production and injection wells in areas with relatively low anisotropy of the formation permeability, especially to avoid places with the presence of landslide permeability of the formation. The location of the wells is important so that, on the one hand, there is no blockage of oil from the side of reduced permeability, and on the other hand, rapid depletion of the formation from the side of increased permeability does not occur. And also the mutual exchange between the production and injection wells did not stop. When placing a system of production and injection wells in anisotropic formations of an oil field, it is necessary to carry out a systematic analysis of the surrounding anisotropy of the formations in order to place them in such a way that would ensure effective dynamics of filtration processes around these wells. Using the method used, it is possible to predict the impact of an injection well on the distribution of reservoir pressure in the reservoir.

**Keywords:** oil-bearing reservoirs, anisotropic filtration processes, piezoconductivity equation, finite element-difference method, reservoir pressure distribution.

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## 1. Introduction

Currently, the problems of increasing and effectively maintaining a stable level of oil production remain urgent. For this, in practice, various modern technologies are used to increase the intensification of filtration of the oil phase near an operating production well [1–3]. On the other hand,

taking into account the formation anisotropy is an important factor in oil production. In this situation, methods of computer modeling of low-permeability anisotropic oil-bearing reservoirs are in demand, since they allow to get an idea of the filtration processes around production and injection wells in various practical cases. They also allow the assessment and consideration of uncertainties arising

from insufficient information about the structure and properties of the reservoir outside the wells. This information can be obtained in a relatively inexpensive way and used for effective analysis, control and management of the oil production process.

Today, for modeling filtration processes in a porous medium, numerical methods are most widely used [4–6] (finite difference method, finite element method, boundary element method, method of straight lines, etc.). The combined finite-element-difference method used in this work for solving the non-stationary anisotropic problem of piezoconductivity, taking into account the inhomogeneous distribution of permeability inside the anisotropic oil-bearing formation and at its boundaries, makes it possible to adequately calculate the distribution of reservoir pressure. The use of the combined finite element-difference method allows to combine the advantages of the finite-element method [7] and the method of finite differences [8]: to model geometrically complex areas, to find the value at any point of the object under study. At the same time, the use of an implicit difference scheme when finding the nodal values of the grid provides high reliability and convergence of the results. Verification of the combined finite-element-difference method was confirmed by its approbation when comparing the results of test examples, as well as the results of solving geophysical problems from different areas of geodynamics with the known results [9].

## **2. The object of research and its technological audit**

*The object of research* is the filtration processes of displacement of the oil phase under the action of an injection well in a heterogeneous reservoir medium. For the effective use of injection wells in order to increase oil production, it is advisable to evaluate and take into account the effect of the heterogeneity of the porous reservoir on the intensity of the filtration process. This can be done using numerical modeling of the reservoir pressure distribution based on the piezoelectric equation. To solve the non-stationary anisotropic problem of piezoconductivity, it is proposed to apply a combined finite-element-difference method [9]. This allows to combine the advantages of the finite element method and the finite difference method, take into account the heterogeneous distribution of permeability within an anisotropic oil-bearing formation and at its boundaries, and adequately calculate the distribution of reservoir pressure.

## **3. The aim and objectives of research**

*The aim of research* is to simulate the field of pressure distribution in an anisotropic porous medium between production and injection wells. To achieve this aim, it is necessary to complete the following tasks:

1. Formulate the mathematical formulation of the non-stationary anisotropic filtration problem.
2. Apply the combined finite-element-difference method [9] to decouple the formulated problem.
3. Construct a complete heterogeneous porous medium penetration study for reservoir tracing within the study area.

## **4. Research of existing solutions to the problem**

Modeling the field of pressure distribution in a porous medium is based on the solution of the non-stationary equa-

tion of piezoconductivity with the established initial and boundary conditions [4, 10]. For this, various methods can be used: analytical, approximate analytical and numerical.

Analytical and approximate analytical methods have a low degree of versatility [10], that is, they are focused on solving narrow classes of problems, in particular, it is inexpedient to use these methods to solve a non-stationary anisotropic problem of piezoelectricity. Today, the most widely used numerical methods (finite difference method, finite element method, boundary element method, method of straight lines, etc.). The boundary element method is the most effective in solving problems in unbounded areas [4], that is, when establishing adequate boundary conditions for a porous formation, it is impossible to solve the piezoconductivity equations using the boundary element method (there is no decoupling theory). The advantages of the finite difference method are the relatively easy construction of an algorithm for solving the problem and its software implementation [11]. Disadvantages – the problematic use on irregular grids, the rapid growth of requirements for computing technology with an increase in the dimension of the problem (an increase in the number of unknown variables). The finite element method is a leader in solving problems on the geometrically complex structure of the model [12, 13]. Nevertheless, the main disadvantages are the time required for calculations, as well as the requirements for the amount of information memory of the computer [14]. Finally, when using both the finite element method and the finite element method, the problem of correct formulation of boundary conditions arises (most often, homogeneous boundary conditions are chosen, and to reduce the error – rather distant ones) [4, 10]. A numerical algorithm for solving the equation of piezoconductivity [9] makes it possible to take into account the inhomogeneous distribution of permeability both within an anisotropic oil-bearing formation and at its boundaries.

Also, the use of the combined finite-element-difference method [9] allows to combine the advantages of the finite-element method [12] and the finite difference method [15]. Thus, it is possible to model geometrically complex areas and find the value at any point of the object under study [7, 12]. The use of an implicit difference scheme in finding the nodal values of the grid provides high reliability and convergence of the results [8]. Therefore, a promising direction is the use of the finite-element-difference technique for solving filtration problems in order to simplify and simultaneously increase the accuracy of calculations.

## **5. Methods of research**

Theoretical research methods include a systematic analysis of the information used, numerical modeling based on a combined finite-element-difference method, methods of visual presentation of the information received.

The study was carried out for productive anisotropic oil-bearing formations, in which the gas content is negligible compared to oil. Assuming that the average thickness of the oil-bearing porous reservoir is much less than the horizontal dimensions of the area under consideration, it is sufficient to use a two-dimensional unsteady anisotropic model of piezoconductivity [4, 10]. In this case, the general formulation of the non-stationary anisotropic problem of piezoconductivity, taking into account the condition of oil permeability at the boundary of the region, in the Cartesian

coordinate system  $(x, y)$ , which is associated with the boundaries of the region, has the form [9]:

$$\frac{\partial P}{\partial t} = \frac{1}{c} \left( k_{xx} \frac{\partial^2 P}{\partial x^2} + k_{yy} \frac{\partial^2 P}{\partial y^2} + 2k_{xy} \frac{\partial P}{\partial x} \frac{\partial P}{\partial y} \right) + \gamma, \tag{1}$$

$$P(t=0) = P_0, \tag{2}$$

$$k_b \text{grad} P = \alpha(P - P_b), \tag{3}$$

where (1) – non-stationary anisotropic equation of piezoelectric conductivity; (2) – initial condition; (3) – boundary condition for the infiltration of the oil phase at the boundaries of the considered area;  $P(x, y, t)$  – pressure as a function of coordinates and time;  $c = \eta(m\beta_1 + \beta_2)$  – coefficient of the piezoelectric support;  $k_{xx}, k_{yy}, k_{xy}$  – anisotropic coefficients of permeability of the oil phase;  $\eta$  – dynamic viscosity of oil;  $m$  is the porosity of the oil-bearing formation;  $\beta_1$  – oil crush factor;  $\beta_2$  – coefficient of compression of the skeleton of the rocks of the oil-bearing formation;  $\gamma$  – parameter of the intensity of oil production in the well;  $P_0$  – initial pressure in the formation;  $\alpha$  – coefficient of infiltration of the oil phase at the boundaries of the considered area;  $P_b$  – pressure at the boundaries of the considered area;  $k_b$  – coefficient of permeability of the oil phase at the boundaries of the considered area.

To solve the non-stationary anisotropic problem of piezoelectric conductivity (1)–(3), a variational finite element method is used, which leads to the solution of the variational equation of piezoconductivity:

$$\delta I(P) = 0, \tag{4}$$

where  $I(P)$  – functional of the anisotropic piezoconductivity problem (1)–(3), which is represented in the form:

$$I(P) = \frac{1}{2} \iint_S \left[ k_{xx} \left( \frac{\partial P}{\partial x} \right)^2 + k_{yy} \left( \frac{\partial P}{\partial y} \right)^2 + 2k_{xy} \frac{\partial P}{\partial x} \frac{\partial P}{\partial y} + 2 \int_{P_0}^P \frac{\partial P}{\partial t} dP - 2\gamma P \right] dx dy - \frac{1}{2} \int_L \alpha(P - 2P_b) P dl, \tag{5}$$

where  $S$  – cross-sectional area of the area that is being investigated;  $L$  – contour covering the area  $S$ ;  $dl$  – contour element.

When solving the variational equation (4), an eight-node isoparametric quadrangular finite element is used [9]. The Cartesian system  $(x, y)$  is used as a global coordinate system, where all finite elements are united into which the area  $S$  is divided. The normalized coordinate system  $(\xi, \eta)$  [9] is used as a local coordinate system, where approximation functions  $\varphi_i$  based on quadratic polynomials are determined within the finite element and numerical integration is carried out. In this system, the coordinates, pressure, initial reservoir pressure, pressure at the boundaries of the area, the coefficient of oil infiltration within the area, as well as the derivatives of pressure along the coordinates are approximated as follows:

$$x = \sum_{i=1}^8 x_i \varphi_i; y = \sum_{i=1}^8 y_i \varphi_i; P = \sum_{i=1}^8 P_i \varphi_i; P_0 = \sum_{i=1}^8 P_{0i} \varphi_i;$$

$$P_b = \sum_{i=1}^8 P_{bi} \varphi_i; \alpha = \sum_{i=1}^8 \alpha_i \varphi_i;$$

$$\frac{\partial P}{\partial x} = \sum_{i=1}^8 P_i \Psi_i; \frac{\partial P}{\partial y} = \sum_{i=1}^8 P_i \Phi_i; \Psi_i = \frac{1}{|J|} \left( \frac{\partial \varphi_i}{\partial \eta} \frac{\partial y}{\partial \xi} - \frac{\partial \varphi_i}{\partial \xi} \frac{\partial y}{\partial \eta} \right);$$

$$\Phi_i = \frac{1}{|J|} \left( \frac{\partial \varphi_i}{\partial \xi} \frac{\partial x}{\partial \eta} - \frac{\partial \varphi_i}{\partial \eta} \frac{\partial x}{\partial \xi} \right), \tag{6}$$

where  $J = \frac{\partial y}{\partial \xi} \frac{\partial x}{\partial \eta} - \frac{\partial y}{\partial \eta} \frac{\partial x}{\partial \xi}$  – transition Jacobian between systems  $(x, y)$  and  $(\xi, \eta)$ .

Based on the variational equation (4) and assuming that the nodal values of the derivatives of pressure with respect to time  $dP_i/dt$  are known by values but do not vary, let's compose a system of differential equations for the  $n$ -th node of the  $p$ -th finite element in the form:

$$\frac{\partial I_p}{\partial P_n} = \sum_{i=1}^8 \left\{ H_{ni}^p \frac{dP_i}{dt} + (A_{ni}^p + Q_{ni}^p) P_i - Q_{ni}^p P_0 \right\} - \gamma_n^p = 0,$$

$$H_{ij}^p = \int_{-1}^1 \int_{-1}^1 c^p \varphi_i \varphi_j |J| d\xi d\eta;$$

$$A_{ij}^p = \int_{-1}^1 \int_{-1}^1 (k_{xx}^p \Psi_i \Psi_j + k_{yy}^p \Phi_i \Phi_j + k_{xy}^p \Psi_i \Phi_j) |J| d\xi d\eta;$$

$$Q_{ij}^p = \int_L \alpha \varphi_i \varphi_j dl,$$

$$\gamma_i^p = \int_{-1}^1 \int_{-1}^1 \gamma^p \varphi_i |J| d\xi d\eta. \tag{7}$$

To solve the system of linear differential equations of the first order (7) with initial conditions from (6), the finite difference method is used, in which the time derivative is approximated on the basis of an implicit difference scheme:

$$\frac{dP}{dt} = \frac{P(t + \Delta t) - P(t)}{\Delta t}. \tag{8}$$

Substituting expression (8) into system (7), let's obtain the following system of linear algebraic equations:

$$\sum_{i=1}^8 \left\{ \left( \frac{1}{\Delta t} H_{ni}^p + A_{ni}^p + Q_{ni}^p \right) P_i(t + \Delta t) - \left( \frac{1}{\Delta t} H_{ni}^p P_i(t) - Q_{ni}^p P_0 \right) \right\} - \gamma_n^p = 0, \tag{9}$$

$(n = 1 - 8).$

Making additions to equations (9) for all finite elements, let's obtain a global system of linear algebraic equations, for allows starting unknown pressure values at time  $t + \Delta t$  through their values at time. The placement of global systems of linear algebraic equations is carried out on the basis of the numerous Gauss method without choosing the main element [9]. As a result of the solution, the pressure

is set at all nodal points of the finite element mesh. According to the found nodal values, the pressure will change at an arbitrary point in the oil-bearing formation of the investigated area at a given moment in time. The use of the quadratic approximation and the implicit difference scheme leads to the choice of the accuracy and stability of numerous solutions to the problem.

**6. Research results**

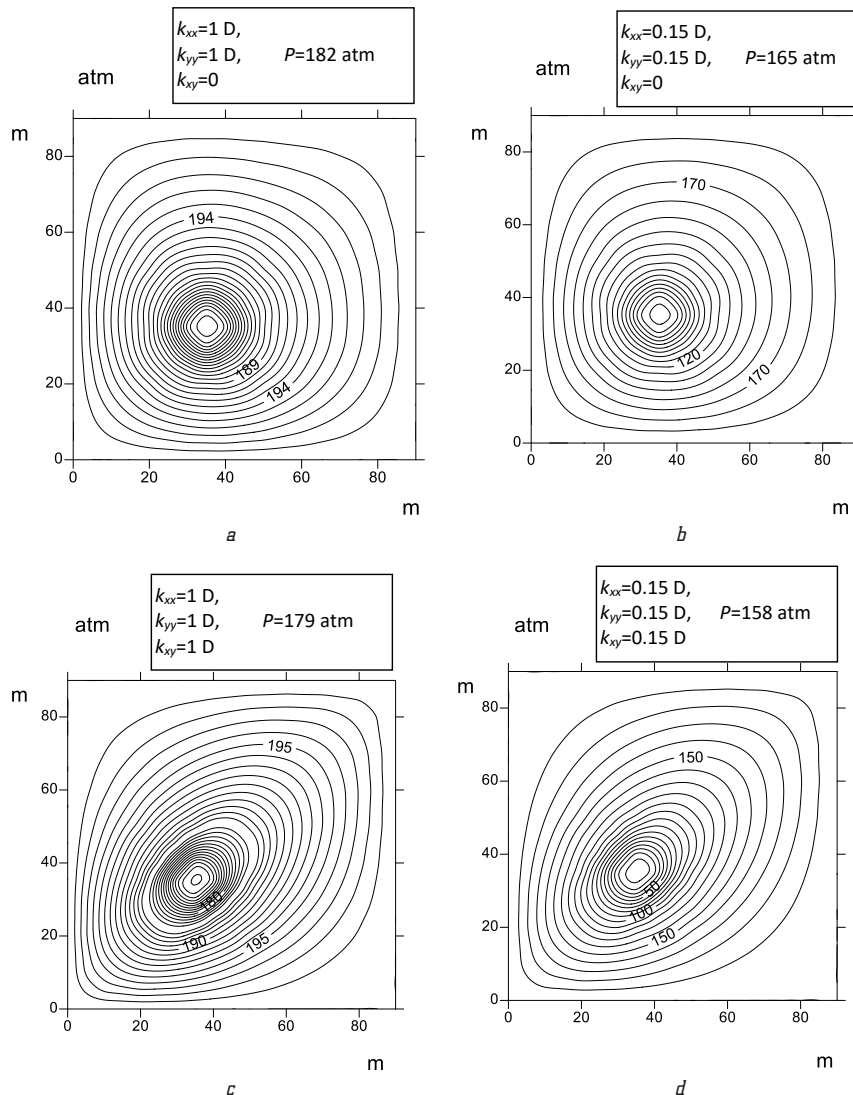
The initial data for the obtained results of the study are given in Table 1. The anisotropy of the porous medium was set by excellent permeability in the  $xx$  (horizontal),  $yy$  (vertical) and  $xy$  (diagonal) directions. The mesh for calculating the model includes 81 eight-node isoparametric quadrangular finite elements.

The research results in Fig. 1 show the change in the distribution of reservoir pressure near the production well, depending on the magnitude of the permeability and the type of isotropy of the oil-bearing reservoir ( $P$  is the value of the average reservoir pressure, atm).

**Table 1**

Initial data for modeling

Name, designation	Value	Units
Oil-bearing reservoir area $S$	90×90	m <sup>2</sup>
Porosity coefficient $m$	0.2	–
Dynamic viscosity of oil $\eta$	10 <sup>-3</sup>	Pa·s
Compression ratio of oil $\beta_1$	10 <sup>-9</sup>	Pa <sup>-1</sup>
Compression ratio of the rock skeleton $\beta_2$	10 <sup>-10</sup>	Pa <sup>-1</sup>
The coefficient of the piezoelectric support of the formation $c$	0.3·10 <sup>-12</sup>	S
Initial reservoir pressure $P_0$	20·10 <sup>6</sup>	Pa
Average production rate of production well $Q$	100	m <sup>3</sup> /day
Oil infiltration coefficient across the boundaries of the considered area $\alpha$	0.001	m
Time from well start $t$	86400	s



**Fig. 1.** Distribution of the set pressure ( $P$  – its average value in the modeled area) around the production well at various parameters of the permeability of the oil-bearing formation:  $a$  – isotropic formation with a permeability of 1 D;  $b$  – isotropic formation with permeability 0.15 D;  $c$  – the presence of landslide isotropy in the formation with a permeability of 1 D;  $d$  – the presence of landslide isotropy in the formation with a permeability of 0.15 D

The most intense filtration process towards the producing well occurs precisely in an absolutely isotropic case, if oil is uniformly flowing into the well from all sides (radially).

This can be seen in Fig. 1, *a, b*, where the average reservoir pressures are  $P=18.2$  MPa with a permeability of 1 D and  $P=16.5$  MPa with a permeability of 0.15 D. In the case of landslide isotropy of the reservoir permeability (Fig. 1, *c, d*), oil quickly approaches in the shear (diagonal) direction, while the axial directions are partially blocked, and in the shear direction of the formation, its rapid depletion occurs. This is evidenced by a decrease in the average reservoir pressure (by 2 % at a permeability of 1 D (Fig. 1, *c*) and by 4.25 % at a permeability of 0.15 D (Fig. 1, *d*)).

Further, the changes in the pressure distribution in the isotropic and isotropic-displacement formations were considered when the injection well was located at different distances  $R$  from the producer. For each of the layers, 20 constructed models of pressure distribution fields were obtained for different values of  $R$  (10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 120, 130, 140, 150, 200, 500, 800, 1000 and 1010 m). Based on the results of the data obtained, graphs of the dependence of the mean reservoir pressure on the distance between the production and injection wells were constructed (Fig. 2).

The results of modeling reservoir pressure fields at various distances between the production and injection wells  $R$  showed the following:

a) for the effective use of low-permeability (permeability 0.1 D) isotropic (Fig. 2, *a*) formations, the injection well should be located no further than 110 m from the producing one;

b) in low-permeability isotropic-shifting (Fig. 2, *b*) formations, the distance between the producing and injection formations should be equal to no more than 100 m.

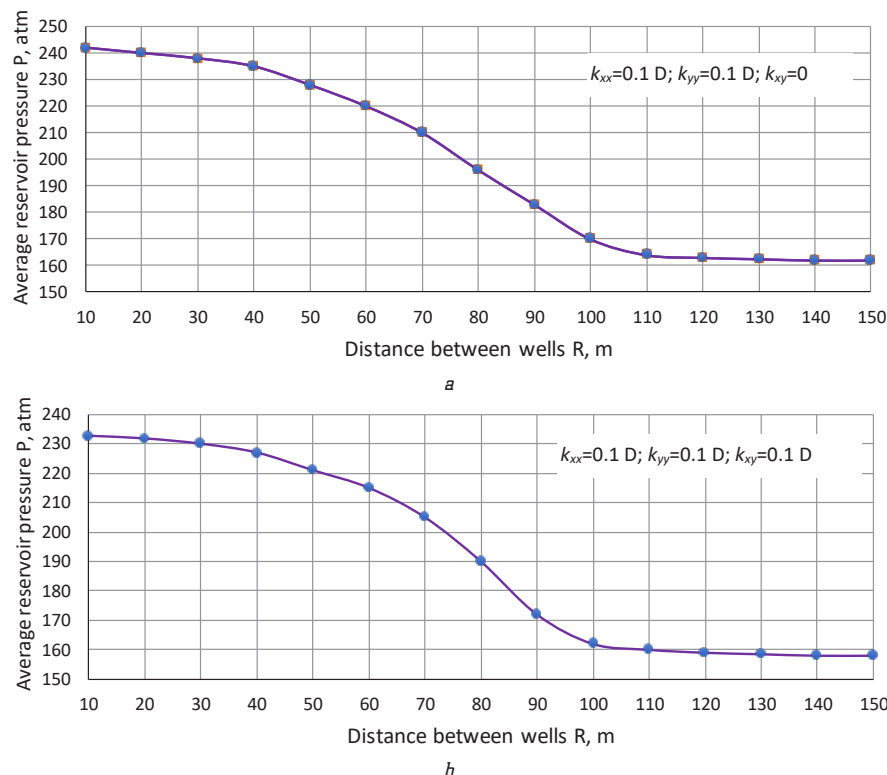
It is also found that placing an injection well at a distance of more than 1 km will negate its efficiency.

In Fig. 3, it is possible to identify the distribution of reservoir pressures between the production and injection wells in absolutely isotropic and isotropic-displacement formations at different distances  $R$  between them.

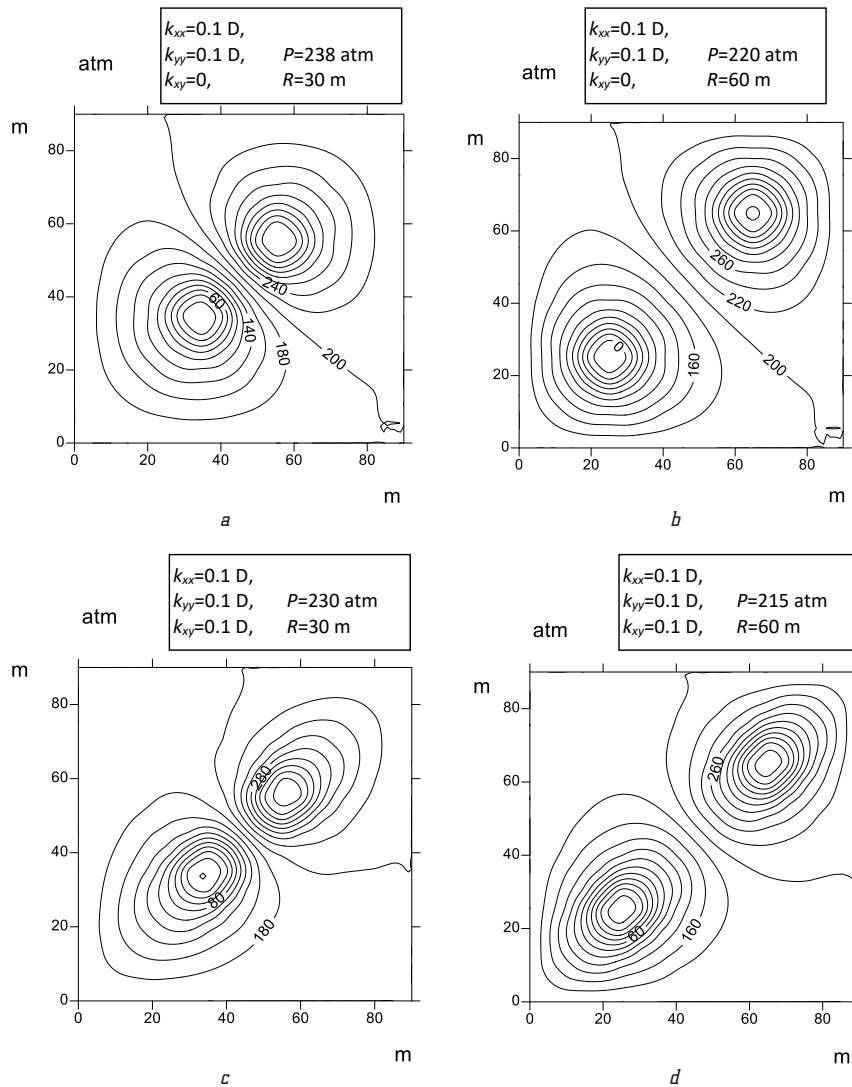
The most active filtration process with a permeability deterioration of 0.1 D occurs precisely in an absolutely isotropic case (Fig. 3, *a, b*), as evidenced by high average reservoir pressures  $P$  (for all modeled values of  $R$  – Fig. 2, *a*). But when the production and injection wells are located in an isotropic-shear formation exactly in the shear (diagonal) direction (Fig. 3, *c, d*), the intensity of the filtration process practically does not decrease (for all modeled values of  $R$  – Fig. 2, *b*). This arrangement provides the smallest decrease in the average pressure of the isotropic-landslide layer compared to the fully isotropic layer (by an average of 4 %), while the location of the wells parallel to the  $x$  axis provides the worst filtration conditions (pressure drop by an average of 9.5 %).

In Fig. 4, it is possible to identify the intensity of filtration processes between the production and injection wells in an anisotropic oil-bearing formation in the presence and absence of landslide permeability ( $R=30$  m).

It is possible to see that the intensity of the filtration process between the production and injection wells depends on their mutual arrangement in the anisotropic oil-bearing formation, when the best mutual exchange between them is observed. For example, the mutual exchange, characterized by high average reservoir pressures, in cases (Fig. 4, *b, d*) is better than in cases (Fig. 4, *a, c*).



**Fig. 2.** Reduction of the average reservoir pressure with an increase in the distance between the production and injection wells: *a* – in the case of an absolutely isotropic reservoir; *b* – in the case of an isotropic landslide formation (in the simulation, the production and injection wells were located along the diagonal of the coordinate grid (in the  $xy$  direction))



**Fig. 3.** Distribution of the set pressure in absolutely isotropic and isotropic-shift oil-bearing reservoirs at different distances  $R$  between the production and injection wells: *a* – at  $R=30$  m in an absolutely isotropic reservoir; *b* – at  $R=60$  m in an absolutely isotropic formation; *c* – at  $R=30$  m in an isotropic-shifting formation; *d* – at  $R=60$  m in an isotropic shear formation

Obviously, for the optimal placement of production and injection wells in an anisotropic oil-bearing formation, it is necessary to conduct appropriate studies to find the best mutual exchange between them. It is important that the wells are located in such a way that, on the one hand, there is no blocking of oil from the side of reduced permeability, and on the other hand, there is no rapid depletion of the formation from the side of increased permeability.

And also the mutual exchange between the production and injection wells did not stop.

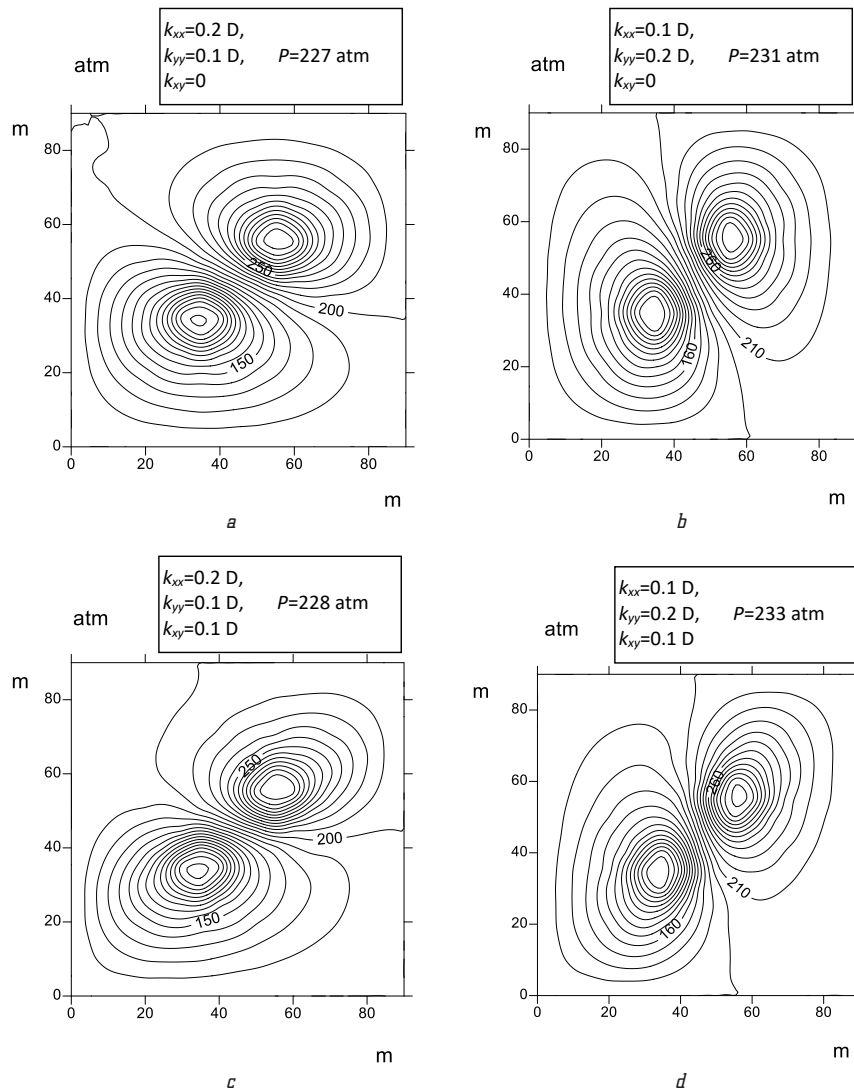
Based on the information obtained, for the effective use of anisotropic low-permeability formations, it is necessary to place production and injection wells in areas with relatively low anisotropy of the formation permeability, especially to avoid places with the presence of landslide permeability of the formation. When placing a system of production and injection wells in anisotropic formations of an oil field, it is necessary to conduct a systematic analysis of the surrounding anisotropy of the formations in order to place them in such a way that would provide an effective dynamics of filtration processes around these wells.

## 7. SWOT analysis of research results

**Strengths.** Based on the study, it is possible to select the optimal location of the «production – injection well» system, taking into account the heterogeneous distribution of permeability over the reservoir area. The study was carried out on the basis of solving the non-stationary anisotropic equation of piezoconductivity using the combined finite element-difference method, which allows combining the advantages of the finite-element method and the finite difference method. Thus, the distribution of reservoir pressure in heterogeneous porous media can be calculated with high accuracy.

**Weaknesses.** The weak side of the studies carried out is their general nature, without reference to a specific field. To confirm the effectiveness of using the combined finite element-difference method for solving the non-stationary anisotropic equation of piezoconductivity, it is advisable to conduct research using the initial data of a particular field.

**Opportunities.** In the future, it is of interest to create a method for calculating well flow rates using the considered numerical method, as well as to create a more complex geometry of the reservoir, which would meet the real conditions of a particular field.



**Fig. 4.** Distribution of the set pressure between the production and injection wells with different permeability of the oil phase in different directions: *a, b* – no shear permeability; *c, d* – shear permeability  $k_{xy}=0.1 D$

*Threats.* This level of research does not require additional costs when implementing the results.

## 8. Conclusions

1. The mathematical formulation of the non-stationary anisotropic filtering problem formulated in the course of the work includes:

- unsteady anisotropic equation of piezoelectric conductivity;
- initial condition (the value of the initial reservoir pressure is set);
- boundary condition that takes into account the infiltration of fluid through the boundaries of the study area.

2. For the decoupling of the non-stationary anisotropic filtration problem, the combined finite-element-difference method of M. Lubkov was applied, which makes it possible to take into account the inhomogeneous distribution of the reservoir characteristics of the reservoir (within the framework of this study, an excellent value of the permeability coefficient was set in the horizontal, vertical and diagonal directions). Using the applied method, it is possible to adequately describe on a quantitative level the

pressure distribution in an anisotropic formation between the production and injection wells.

3. The influence of the non-uniform distribution of the permeability of the porous medium on the distribution of reservoir pressures within the modeled area has been investigated. The simulation results show that the distribution of the pressure field between the production and injection wells significantly depends on their location, both in the isotropic landslide and in the anisotropic oil-bearing reservoir. At  $R > 1$  km, the injection well loses efficiency, at  $R < 100$  m, the average reservoir pressure increases sharply, which indicates active filtration. Moreover, the influence of the permeability of the oil phase in the shear direction dominates over the influence of the permeability in the axial directions (affects the pressure decrease by 4–9.5 %). In the case of a landslide-isotropic reservoir, the wells should be located in the shear (diagonal) direction, which will provide the lowest level of drop in the average reservoir pressure (by 4 %).

Based on the information obtained, for the effective use of anisotropic low-permeability formations, it is necessary to place production and injection wells in areas with relatively low anisotropy of the formation permeability, especially to



avoid places with the presence of landslide permeability of the formation. It is important that the wells are located in such a way that, on the one hand, there is no blocking of oil from the side of reduced permeability, and on the other hand, there is no rapid depletion of the formation from the side of increased permeability. And also the mutual exchange between the production and injection wells did not stop. Obviously, the best conditions for oil production in the corresponding practical case are achieved as a result of the optimal selection of all influential factors of anisotropic filtration. On the other hand, these factors can be estimated using the presented method.

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