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Lotto as Options: Measuring the Silver Lining Effect

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Lotto tickets normally have negative expected value that sometimes turns positive with carryover or promotions. The purchase of lotto tickets by professional bettors has been shown to be rational economic behavior during these periods. The typical player is a reliably intransitive gambler known to play continually and is subject to a regressive tax when a professional bettor wins the jackpot. We perform an out-of-sample experiment that provides countervailing evidence of an economically significant silver lining for the typical player from easier-to-win small prizes and social benefits making lotto one type of investment for this demographic. We show the linkage between jackpot size and economic boom and bust in Puerto Rico. Lotto participation by the typical player is, if not rational, less irrational in terms of prospect, strain, networking, and consumption theories.

JEL Codes: D12; D46; D91.

Keywords: Lotto; Prospect Theory; Portfolio Theory; Strain Theory; Networking Theory; Consumption Theory.

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I. Introduction

A lotto ticket offers the bearer an opportunity to win a highly publicized pari-mutuel jackpot and numerous small prizes. Pari-mutuel is a betting pool in which the players holding winning lotto tickets divide the total amount bet, less a management fee and taxes. The pool is distributed among all winning bets. Lottery and horse racing are the two most commonly played parimutuel games. Lotto ticket sales in the United States exceed seventy billion dollars annually, more than four times the sales of professional sports tickets, box office movie theater passes, music media, video games, or books (Isidore, 2015). Forty percent of lower-income and fifty three percent of upper-income United States citizens polled reported buying a state lottery ticket in the past year (Auter, 2016). Lotoland (2022) reports that 70 percent of the United Kingdom's population over age 18 plays the national lottery regularly. These players also have access to EuroMillions. This represents 45 million British players engaged in continuous play including infrequent periods in which the expected value of a lotto ticket is negative. Lower income players are primarily responsible for the largest portion of lottery play and shoulder a greater proportion of loss than higher income households (Beckert and Lutter, 2013). However, we show that small-fixed prizes represent a greater proportion of cost recovery for these lower income players than for a wealthy ticket holder.

We perform an economic experiment with two different strategies showing that fixed payout small prizes are a silver lining that have a disproportionately positive impact on the well-being of the lower class player than for the wealthy. This counterbalances the impact of indirect regressive taxation of lower income players from Oster (2004) and helps explain the vigor of play of the typical player. Our data allows us to offer rational explanations for the seemingly irrational consistent demand among lower income players for lottery tickets of negative expected value. We further extend prospect theory, as well as sociological strain, consumption, and network theories. Finally we show that the small Puerto Rico LotoPlus lotto game is tightly related to economic boom and bust while large multi-state national lotteries are not.

II. The expected value of lotto

The expected value of a lotto ticket is the sum of the probabilities of winning a prize times the value of each prize. In our notation, the expected value of a random variable *X* is denoted by *E*[*X*]. The expected value of a small prize (*E*[*S*]) is a simple calculation for fixed prize payouts where the probability of a win is p, and the dollar amount of the small prize is *S*.

$$E\left[S\right]=\Pi\left(p\right)S \tag{1}$$

The jackpot prize varies from game to game as the pari-mutuel pool changes. The dollar amount of the payout of this grand prize is random but we can calculate the expected value as the probability of a win (*p*) times the expected share of the jackpot if won (*f*) times the dollar amount of the jackpot (*J*).

$$\text{Expected value of jackpot} = E\left(J\right)=\Pi\left(p\right)\left(f\right)\left(J\right) \tag{2}$$

The probability of winning the jackpot (**J**) is directly impacted by the format of the game in terms of the quantity of numbers a player must select when purchasing a lotto ticket. The probability of winning the jackpot has an inverse relationship with the range of numbers available for selection. The more numbers the player has to select from, the less likely a player will win the jackpot. The expected value of a lotto ticket fluctuates in size from game to game depending upon a number of additional factors identified by Cook and Clotfelter (1993) who utilize a Poisson approximation to the binomial distribution to derive a closed form expression of factors. The payout for a given ticket is dependent upon the probability of winning a particular prize (*p*), the amount wagered by the player (*W*), the rollover from previous drawings (*R*), the fraction of the handle entering the pari-mutuel jackpot pool (*k*), the cost (*B*) for a player to buy a lotto ticket (bet), and the amount bet by other players (*N*). The expression calculates the amount a player can expect to win (*E*[*win*]) from a group of tickets with x winners in n independent random trials each with a probability p of success as follows.

$$\begin{aligned} E\left[win\right] &= \left(pB\right)\left[R+k\left(W+N\right)\right]\sum_{x=0}^{n}\frac{1}{1+x}e^{-p^{N}}\frac{\left(pN\right)^{x}}{x!} \\ &= \frac{B}{N}\left[R+k\left(W+N\right)\right]\left(1-e^{-p^{N}}\right) \end{aligned} \tag{3}$$

Expression (3) shows that the total amount bet by other players, and the fraction of the handle going to the prize pool impact the expected value of a lotto ticket. This function illustrates that splitting the pot is detrimental to the expected return of a lotto ticket. Some combinations are more likely to be split because of conscious selection that results in a preponderance of numbers less than 31. If selected, these would result in a higher frequency of splitting the jackpot with one or more strangers (Ziemba et. al., 1986). Players frequently select a line or lines of numbers on a lotto ticket purchase form in a pattern such as a part or all of a row or a column. Sometimes these patterns are caught. On March 19 of 2008 the Canadian Lotto 6/49 winning numbers rolled out of the cage as 23-40-41-42-44-45 with 43 as bonus (Simon, 1998). This prize should not have been shared by more than 3 winners. A total of 239 winners shared the jackpot from 6,606,690 tickets sold.

The handle is the total revenue from lotto ticket sales. In a pari-mutuel system, all bets are first placed in a pool then the "house-take" or "vigorish" is withdrawn to pay the organizers. Most lotto bets fall short of fair value because the prize pool is about half of the handle. This proportion falls after jackpots are won. Cook and Clotfelter (1993) show that the jackpot increases by $1,255 for every $1,000 rolled over from the previous drawing. The extra $255 is from additional betting attracted into the pool from the rollover; lotto play thus increases as jackpots thus grow. This describes a behavioral feedback loop where rising grand prize values attract new players to the game who further increase the jackpot pari-mutuel pool by increasing ticket sales. Each new player added to the pool increases the expected value of a bet.

But this positive externality is offset by the detriment of a decreased expected share of the jackpot accruing to the winner. Many players withdraw from playing the game after large jackpots are won to sit on the sidelines waiting for sufficiently large grand prizes to once again accrue. These players are more sophisticated in their approach to purchasing a lotto ticket as compared with the typical player who never refrains from play. The mathematical value of a ticket has two components; the expected value of winning a prize (P) which can be either the jackpot (J) or a small prize (S) with a fixed payout.

Mathematical value of a lottery ticket =

$$E[P] = E[J] + E[S] \tag{4}$$

However, the true value of a lotto ticket also includes intangible social benefits (B) derived from ownership of a ticket. The true value of a ticket is the expected value of a jackpot plus the expected value of small prizes and a utility enhancing social benefit.

True value of a lottery ticket =

$$E[P] + E[B] = E[J] + E[S] + E[B]. \quad (5)$$

Breaking apart the value of a lotto ticket into components allows us to identify two types of players. The first, focuses exclusively on winning the jackpot with little or no interest in small prizes or social benefits. The second type of player values social benefits, small prizes, and the jackpot. Expectation is a positive linear function that may be expressed as a Reimman sum of integrals. This piecewise continuous function yields the area between the graph of x and the horizontal axis between the vertical lines $x = a$, $x = b$, $x = c$, and $x = d$. Thus, if $J(x) \geq 0$ for all $x \in [a, b]$, then $\int_a^b J(x)dx \geq 0$ and $S(x) \geq 0$ for all $x \in [b, c]$, then $\int_b^c S(x)dx \geq 0$ and $S(x) \geq 0$ for all $x \in [c, d]$, then $\int_c^d B(x)dx \geq 0.$

Expression (5) can be written as,

$$\text{True value of a lottery ticket} = E[J] + E[S] + E[B] \quad (6)$$

$$= P(x)\int_a^b J(x)dx + P(x)\int_b^c S(x)dx + P(x)\int_c^d B(x)dx \quad (7)$$

This expression of the expected value of a lotto ticket is a piecewise function where $P(J)$, $P(S)$, and $P(B)$ are probabilities and the integrals of $J(x)$, $S(x)$ and $B(x)$ are functions of the random variable x. Prior research we describe below has shown that lotto players focus preferentially on different pieces of either function (6) or (7). These insights allow us to devise two low cost lotto systems that either maximize expected value or expected benefits. Our first system we test is based on the insight that players bet against the crowd by waiting for large jackpots. The second system is based on continual lotto play.

III. Lotto Play Systems

We investigate two strategies to elucidate and quantify the silver lining effect. Mental accounting gives rise to a silver lining effect and is part of prospect theory (Langer and Weber, 2001). Our first strategy of perpetual lotto play is related to a recent survey showing that roughly half of families in the United States have no access to a workplace retirement savings and investment plan (Elkins, 2019). Lotto tickets offer alternative investment opportunities for those people who have no access to investment retirement accounts (IRAs). Some players purchase a ticket for every game throughout the year. They are insensitive to waiting for high carryover to play games with high jackpots. This allows these perpetual players to recoup costs through the winning of small prizes because the distribution of payouts is a convex function offering asymmetric compensation akin to a call option.

Waiting for games with large jackpots means foregoing non-pari-mutuel small prize benefits that arise from the derivative-like aspect of a lottery ticket. Equity call options, for instance, offer convex payoffs with limited liability restricting loss to the premium paid. This is like lotto where the player risks only the ticket price. The bearer shares the upside gain but incurs no loss below the cost of the lotto ticket. Small prize payouts allow lotto tickets to be rationally apportioned to investment portfolios as low-cost external positions like call options in an equity portfolio; albeit with significant market frictions. Far out of the money options have been compared with lotto tickets (Boyer and Vorkink, 2014). We model perpetual players with a strategy of continual play of unpopular numbers that minimizes the risk of splitting the pari-mutuel jackpot. A player can generate a series of unarranged combinations via an online random number generating website. He or she then selects from random combinations with the most numbers over 31. Once a random sequence that includes numbers above 31 is selected the ticket is set making this a passive investment tactic. This ticket is played continually throughout the life of the investor. This strategy maximizes social benefits as well as cash from small prizes.

Proposition 1 – Our first proposition is that small prize wins from continual play extend the length of play, enhance prospects, networking, and consumption while decreasing strain. (P1)

IV. Lotto Tickets as Investments

Any asset offering a positive expected return can be added to an investment portfolio according to the arbitrage theory of capital asset pricing (Ross, 1976). Our second proposition is based on the assertion that a lotto ticket can diversify an investment portfolio if it offers positive expected returns under some conditions. Richard Thaler and William Ziemba (1988) describe these conditions,

"with an expected return of between 40 *and* 60 *cents on the dollar,* [*lotto tickets*] *are usually a poor investment for the rational investor. Even with such low payout rates, it is possible to obtain positive expected value best in lotto games* [*as compared with other pari-mutuel gambles such as horse racing*]. *This occurs because not all numbers are equally popular with the public. Second, if the grand prize is not won in a given drawing, it is carried over to the next week. Thus, prizes can be enormous."*

A winning lotto ticket purchased with cash is a bearer instrument. It is held in a home safe to comprise part of a household portfolio of assets. Investment retirement accounts are tax-efficient but a lotto ticket can neither be traded across a financial exchange nor held in an investment retirement account (IRA). The inefficiency of buying a lotto ticket with a jackpot that is taxed implies that if an eligible taxpayer has not fully contributed to tax-advantaged IRAs he or she is better off maximizing tax-deferred 401(k) or tax-free Roth savings into an exchange-traded equity index fund (ETF) or broad equity indexed fund (Tables 1 and 2, and Siegel, 1994). An investor can also invest savings above IRA contribution limits in an individual investment account in a broad equity index.

But these investments will not have potential lottery-like payouts due to diversification. Only a call option offers a lottery-like payout but requires extensive knowledge and experience to trade and can have drastic tax disadvantages when held outside of an IRA (Crack, 2017). There are times when jackpot carryover and the betting of unpopular numbers increases the expected value of a ticket to as much as $2.25 per $1.00 bet for very large carryovers (Ziemba et al., 1986; Thaler and Ziemba, 1988; Moffitt and Ziemba, 2019a, 2019b). Some players only buy lotto tickets with a positive expected value after waiting for large carryovers. This helps to explain how the massive volume of lotto play increases with jackpot size and is a non-trivial component of the United States enterprise-exchange gaming economy (Auter 2016).

TABLE 1. Roth and Roth 401(k) Contribution Limits (IRC 2018)

	Roth		401(k) / Roth K	
	< 50	> 50	< 50	> 50
Spouse 1	$5,500	$6,500	$18,500	$24,500
Spouse 2	$5,500	$6,500	$18,500	$24,500
	$11,000	$13,000	$37,000	$49,000
Totals			**$48,000**	**$62,000**

TABLE 2. Cost of Lottery Play

Age	Max Contribution	Max Lottery Cost	% Allocation
Young	$49,000	$624	1.26%
Old	$62,000	$624	1.00%

Note: Maximum cost of lottery tickets for investors above (old) and below (young) fifty years of age expressed as a percent of maximum retirement contribution.

Our second system models infrequent lotto play. A rational player must be sensitive to wasting large amounts of time on lotto play because of the low probability of a life-style enhancing win. Perpetual play is more time-consuming than infrequent mathematically optimal play. Many players strike a balance by buying tickets for games with above-average jackpots concomitant with large carryovers. Large jackpots accrue quickly with a high percentage of the United States population playing lotto. The jackpot is carried over to the next game when nobody wins a drawing. This allows large prizes to accrue. Players who wait for large jackpots increase the expected value of their tickets. Among those in optimal play are players who gather into syndicates to attempt to buy the pot (Moffit and Ziemba, 1019a, 2019b). This gives rise to our first hypothesis.

We implement and gather data for a simple semi-passive strategy that refrains from play when the expected value of a lotto ticket is negative. This second system purchases lottery tickets with unpopular numbers only when the jackpot increases enough to offer positive expected values (MacLean et. al., 1992). However, tracking requires discipline to check the jackpot size after each game to ensure that a lotto ticket has positive expected value. Hence this is an active investment tactic. Unlike capital gains in the stock market that can be protected from taxation there is no way to escape taxes on lotto wins. Taxation of lotto gains should push any investor to first fully maximize his or her retirement contributions before purchasing lotto tickets. In other words, positive expected value (EV), infrequent lotto play is most beneficial to investors who have maxed out all tax-advantaged investment opportunities.

Lotto tickets have call value for those savers who are unable to buy and sell equity options, futures contracts, or a single stock that could offer large payouts. Lottery tickets are attractive to rank-and-file employees who save more than the contribution maximums of the workplace 401K, Roth, standard IRA plans, and others. These investors face the inflexibility of menu-driven employee-sponsored defined contribution plans that force investments into a handful of administrator-selected mutual fund choices. But for a lottery ticket, these investors have little hope for an overnight windfall large enough to cause a qualitative change in their social standing. Buying into lotto games is one way the little guy can bet small and win big.

Proposition 2: Infrequent play restricted to periods of high carryover when tickets have positive expected value is less time-consuming but reduces prospects of small wins, diminishes networking, and reduces consumption. (P2)

V. State and Multi-State Lotto

Lotto jackpot size is correlated with state and multi-state populations. Larger states have larger lotto games. Multi-state Mega Millions and Powerball lottos are the largest of all rivaled in size only by the multi-country game EuroMillions. The likelihood of winning the jackpot and state population is inversely related. This is easy to see by comparing the odds between the Puerto Rico LotoPlus and Powerball lottery games listed in Tables 3 and 4 of this study. The correlation of jackpot size and population is thus perplexing. Cook and Clotfelter (1993) explain this seeming paradox via prospect theory. This offers a framework as to why rational people would consistently choose negative expectation bets based on their life situation depicted by a Friedman Savage curve (1948). They also show that games are more successful in larger states despite longer odds. However, they do not discuss how players can choose to play only when the jackpot rises high

TABLE 3. Puerto Rico LotoPlus odds

Ticket Matches	Loto Plus Payout	Odds (1 in …)	Probability
5 of 5 + Bolo Plus	Jack Pot	9,870,120	0.000000101
5 of 5	$10,000	705,009	0.000001418
4 of 5 + Bolo Plus	$1,000	56,401	0.000017730
4 of 5	$150	4,029	0.000248225
3 of 5 + Bolo Plus	$50	1,659	0.000602845
3 of 5	$4	119	0.008438819
2 of 5 + Bolo Plus	$5	151	0.006631300
1 of 5 + Bolo Plus	$2	38	0.026525199
Bolo Plus only	Reintegro	30	0.032894737

Note: This table shows the odds and probabilities associated with the Jackpot and smaller prizes. Probability of win is the inverse of the odds associated with each prize.

enough to offer tickets of positive expected value. Furthermore, these researchers do not discuss how smaller prizes claw-back or cash out ticket costs with much better probabilities of winning.

VI. Powerball Multi-State Lottery

Powerball costs $2 per game with a power-play option for an additional $1 costing $3 for full play. Mega Millions costs $2 per game with a megaplier option for an additional $1. The total cost for full play is $3. The Power play addon bet multiplies small prize payouts by 2, 3, 4, 5, or 10 times. The multiplier number is randomly drawn ex-post by the organizer. The 10X multiplier is allowed only when the jackpot is equal to or less than $150 million. The Power play Match 5 prize with no power ball is a constant $1 million. Small prizes include free play. Syndicated play undermines the integrity of the pool just as insider trading degrades the stock market. Adding free play is the strongest deterrent to syndicates (Moffit and Ziemba, 2018). A free ticket is a free lunch from the perspective of consumption theory. The ticket extends the utility of consuming play by extending the time of daydreaming about a win. Free play blocks syndicates from buying the game and increases payout convexity. The only way a small state lottery can discourage syndicates is to increase the convexity of payouts. The more convex the design, the more often the player gets something back despite losing the jackpot in a particular game. To see the dramatic

TABLE 4. Powerball Odds

Ticket Matches	Powerball Payout	Odds	Probability
5 White + PB	Jackpot	1 in 292,201,338.00	0.000000003422
5 White No PB	$1,000,000	1 in 11,688,053.52	0.00000008555
4 White + PB	$50,000	1 in 913,129.18	0.000001095
4 White No PB	$100	1 in 36,525.17	0.00002738
3 White + PB	$100	1 in 14,494.11	0.00006899
3 White No PB	$7	1 in 579.76	0.001722
2 White + PB	$7	1 in 701.33	0.001424
1 White + PB	$4	1 in 91.98	0.01076
0 White + PB	$4	1 in 38.32	0.02543

Note: Odds and probabilities associated with the Jackpot and smaller prizes from Butler (2018). Probability of win is the inverse of the odds associated with each prize.

difference in convexity between two games, compare the much easier ways (higher convexity game) to win small prizes in Table 3: "Puerto Rico LotoPlus Odds" with the much longer odds of making small wins (less convex format) of Table 4: "PowerBall Odds."

Powerball is one of two multi-state lotto games in the United States. Mega Millions is the other. Not all states are covered by each game. Powerball is played in forty-three states. The only states that do not offer Powerball are small in population; Alabama, Alaska, Hawaii, Mississippi, Nevada, Utah, and Wyoming. The large population of people playing Powerball makes the odds of winning infinitesimally low while simultaneously generating astronomical payoffs for the lucky winner(s). A single play consists of choosing five numbers within a range of 1 and 69. The lotto player must then select one number within a range of 1 and 26 for the Powerball.

The minimum bet is two dollars, for which the bettor receives one entry. Each combination of numbers played is recorded via an internet link with headquarters. The winning combination of 5 numbers is selected at random twice weekly from a discrete numerical range. The Powerball number is selected at random from another discrete numerical range. Powerball prize win probabilities are widely distributed to the public on the web. There are 292,201,338 possible Powerball number combinations. At two dollars per ticket, it would cost $584,402,676 per game to purchase all sequences for a syndicate to buy the jackpot. Winners split the grand prize jackpot paid in cash or in 29 annual installments. Powerball jackpot winner Vinh Nguyen in winning on Sept. 24, 2014, opted for the annuity yielding him the full $228,467,735

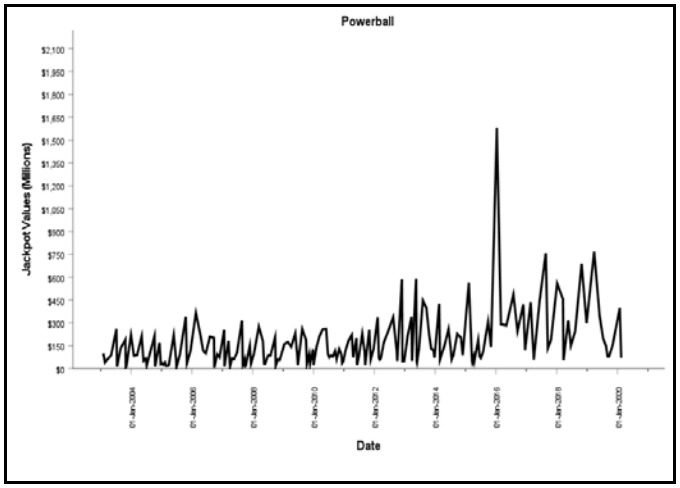

Figure 1—Powerball Jackpot Size Over Time

jackpot paid out over 30 years that would have been a cash lump sum of $134 million. Jackpot annuities (1) pay advertised amounts and (2) protect profligate winners. Small prizes are awarded at every drawing for players who catch a small subset of the winning combination. Forty to fifty percent of the handle (total amount bet) goes to the prize pool in state and multi-state lotteries. About five percent goes to administration. The jackpot is carried over (added) to the next game if a drawing has no grand prize winners.

Table 4 displays the odds of Powerball. These numbers show that the jackpot has 1:292,201,338 odds of winning the jackpot. The November 7, 2022 Estimated Jackpot was $1.90 Billion. This record shattering jackpot increased the value of an unsplit ticket to $6.50 based on these long odds in addition to the expected value of small prizes. Figure 1 depicts fluctuations in the Powerball jackpot from November 1st of 2002 to January 1st of 2020. Notice how dramatically the jackpot slope steepens the higher the jackpot. This shows investors being attracted to the pari-mutuel pool more rapidly when high jackpots generate positive expected values for individual tickets. A player choosing to purchase Powerball tickets during these high jackpots within the collection period from 12/27/17 to 1/06/18 participated in just 21 out of 115 games costing $63. This reduces the dollar cost of

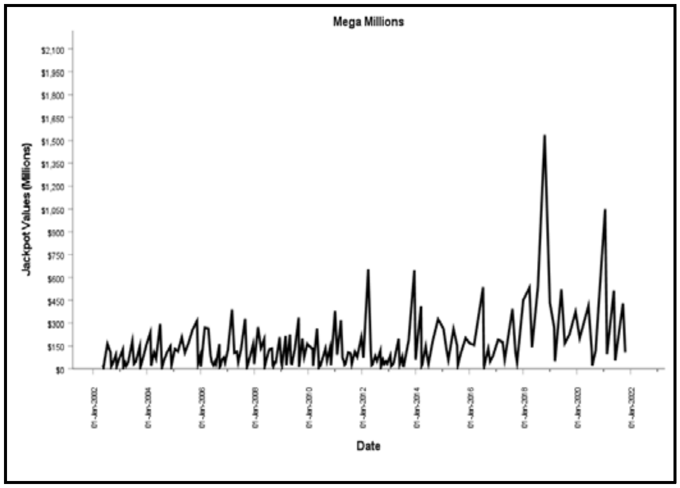

Figure 2—Mega Millions Jackpot Size Over Time

play but the significant additional effort of monitoring the jackpot and resetting the position imposes a high time cost and a greater likelihood of errors through loss, theft, or incorrectly marked lotto play forms.

Notice that the Powerball player has far better odds of winning smaller impartable (non-pari mutuel) prizes ranging from $4 to $1,000,000 than the grand prize jackpot. NBC News reported on November 6 that although the $1.9 billion Powerball jackpot was not won, "More than 10.9 million tickets won cash prizes totaling $102.2 million in Saturday's drawing. Big winners include 16 tickets that matched all five white balls to win a $1 million prize. A ticket in Kentucky won a $2 million prize by matching all five white balls and including the Power Play option for an additional $1 per play. There were also 219 tickets nationwide that won $50,000 prizes, and 51 tickets that won $150,000 prizes (Abbasi, 2022)." These large prizes are large enough to offer the player the opportunity to cash out of a lifetime of the future cost of play generating a costless call option on future lotto payouts. Life expectancy combined with annual cost of play show that these larger small prizes can cash out a lifetime of play creating a free call option on game payouts for the lotto player. Mega Millions rules differ little from Powerball, by one number in the main and the extra ball selection ranges where the highest multipliers are 5X or 10X. There are two games per week nationwide for a total cost of $6 to play Powerball or Mega Millions. Figure 2 shows the fluctuation in Mega Millions jackpots over time that has a pattern similar to that of Powerball.

VII. Puerto Rico LotoPlus

Puerto Rico has a state lottery with a compact format that is much easier to win when compared with Powerball or Mega Millions. It costs $1 per game with an additional $1 double revenge ("doble revancha" branded as "RevanchaX2") and a second add-on multiplier costing another $1 for a total of $3. Double revenge offers four to nine additional ways to win. The first is via a rolling prize that can fluctuate substantially that is bounded above by the LotoPlus handle. For instance the double revenge grand prize of $4,150,000 of August 23rd of 2017 had dropped to $250,000 by June 13th of 2018. An attractive feature of the double revenge grand prize is that it is paid entirely, not discounted as are the LotoPlus or Powerball jackpot lump sum payouts.

A second prize fluctuates on a sliding scale between $2,500 and $10,000. The separate double revenge system selects five pink balls from one cage and a sixth bonus ball from another group of violet balls. Players must catch the five pink balls and the violet ball to win the first prize jackpot. Playing LotoPlus with the Double Revenge and the Multiplier offers a total of 16 ways to win. The 16 prizes of the Puerto Rico state lotto game are far more convex than Powerball that offers just 9 ways to catch a prize. Convexity of payouts offers an array of small prizes that attracts play due to the silver lining effect of Shefrin and Statman (1984). This is because convexity assists players in recovering a portion of the ticket cost. The double revenge further increases convexity in offering two plays, one is automatic, with seven ways to win for each play. Double revenge has a second prize of $2,500. The multiplier increases smaller LotoPlus prizes 2 to 5-fold allowing second prizes to grow up to $50,000.

There are 51 state lotteries including Puerto Rico. Other state and local lotteries with larger formats cost the same to play as Puerto Rico LotoPlus. California is the largest with $278,494,901 in state lottery revenue (NCSL, 2018). For instance, the total cost to play the large California 47/27 format or the much smaller 40/15 formatted Puerto Rico state lottery is $6 per week. The total cost of playing both a state

TABLE 5. Full Cost of Lotto Play Based on Different Starting Ages

Age	Life Expectancy	Full Cost ($624 annually)
20	61.77	$38,544.48
30	52.06	$32,485.44
40	42.49	$26,513.76
50	33.24	$20,741.76
60	24.56	$15,325.44
70	16.53	$10,314.72
80	9.73	$6,071.52

Notes: Based on actuarial life expectancy from the U.S. Social Security Administration.

and a multi-state lottery in California or Puerto Rico is $12 per week. There are 52 weeks per year in the United States offering 104 games of lotto play. Table 5 shows the total cost of adding two lottery tickets in continual play to a retirement portfolio from multi-state and state lotteries is a maximum of $624 per year. Lotto play thus represents a small fraction of between 1.00% and 1.26% of the maximum annual household retirement contribution limits ranging from $49,000 to $62,000 depending on the age of the player. To put this in perspective, seventy-three percent of forty-one million visitors in 2016 vacationing in Las Vegas gambled with an average budget of $578.54. See Las Vegas Sun online "Survey Describes Las Vegas Average Tourist." The amount that the typical individual gambler plays in Las Vegas annually is roughly equivalent to the cost of continuous full play of a lotto ticket. Cook and Clotfelter (1987, 1989, 1990, and 1993) study lotto with respect to scale and implicit taxation. They offer a regression analysis that shows that state lotto sales increase with the size of the jackpot and are highly sensitive to per capita income, payout rate, and population. These variables are thus not a vestige of gambling preferences. The payout rate is the amount of the handle going to a jackpot and small prize winners. The take is the fraction of the handle the lottery deducts from the betting pool to operate the lotto. Multi-state formats run at lower costs because of scale which is why states have eagerly joined Powerball and/or Mega Millions consortiums. For instance, Puerto Rico 2017 income per capita (GNI) is low at $24,020; well below the national average in the United States of $58,030. The island population is 3,337,000 which is about half of the state average of 6,377,141. LotoPlus is the state lotto game of Puerto Rico. It operates under a 5/40 + 1/15 format where the player chooses 5 of 40 and 1 of 15 numbers

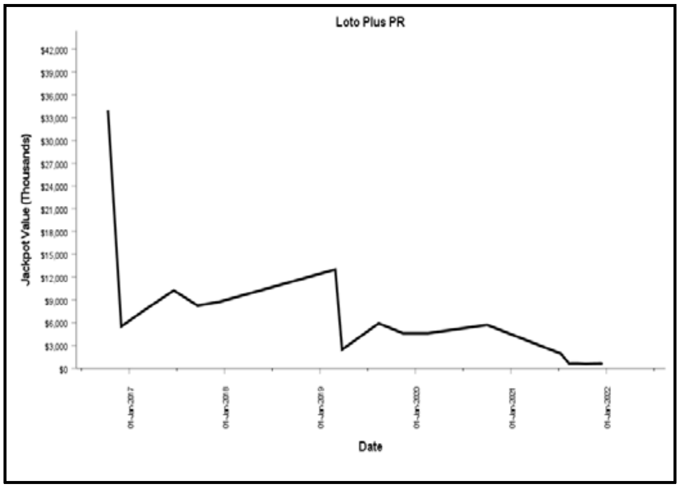

Figure 3—Puerto Rico LotoPlus Jackpot Size Over Time

from two separate panels. Puerto Rico LotoPlus is easier to win than any multi-state or multi-country lotto because of this compact format. Powerball, on the other hand, has a much larger field of numbers and far longer (worse) odds than any state lotto game with a 5/69 + 1/26 format as is the case for the 5/70 + 1/25 selection ranges of Mega Millions.

The best odds of winning a major jackpot are in the lotto games of states with lower populations. The 2017 population of Puerto Rico, for instance, is 3,337,177 million. This is scant when compared with the 38,332,521 population of California with the correspondingly largest state lotto. Local Puerto Rico residents can choose to play multi-state Powerball or LotoPlus. It is far more likely that a player will win LotoPlus because the odds are 1 in 9,870,120 chances. The small state population lotto of Puerto Rico offers far better odds than the much longer 1 in 292,201,338 chances of Powerball. Unlike Powerball, the Puerto Rico LotoPlus probabilities are printed on the instructions on the back of each pre-printed play form. The LotoPlus jackpot with odds of 9,870,120:1, for instance, rose above $9,870,120 just twice during the sample period. Part of the reason for this was the impact of Hurricane Maria which closed Puerto Rico LotoPlus in September and October because there was no power in central headquarters. Note that the slope of the LotoPlus jackpot is much diminished after Hurricane Maria when compared with that of Powerball over the same time. The slope of state lottery jackpot levels over time is an indicator of post-disaster economic recovery in Puerto Rico. Figure 3 graphs the Puerto Rico LotoPlus jackpot size over time which we explore later with OLS regression analysis.

Concerns over which lotto format is easier to win are meaningless with regard to winning a particular jackpot. For example, the base format of the United States lotto system is a game in which players choose 6 numbers out of 49 possibilities (6/49). The probability of winning the jackpot in such a game is 1 in 13,983,816. It would take players 134,360 years to rationally expect to win the jackpot according to Moffit and Ziemba (2019). Furthermore, the player should not expect to recover more than about half of the cost of the ticket with small prizes. Hence, the effort that it takes to purchase tickets just when the jackpot rises enough must be compared with the convenience of simply buying a multi-game ticket for each draw as far into the future as lotto game rules permit. We gain firsthand knowledge of these differences playing out-of-sample with our own money. We describe this out-of-sample data next.

VIII. Data and Analysis

In this section, we report ticket play results in Table 6 for Puerto Rico Powerball, and Puerto Rico LotoPlus. The expected cost of play of $3 per 102 lotto games running for twice a week, 52 weeks per year is $624 for full play annually in multi-state Powerball and Puerto Rico LotoPlus. We show that the more convex format of LotoPlus recovers a portion of the cost of play much more aggressively than Powerball. The gross cost of LotoPlus play is $318 in our sample versus $513 for Powerball. The difference in gross costs between play of the two games is due partly to the closure of LotoPlus from Hurricane Maria. However, our LotoPlus tickets won $190 in small prizes over the 106 games played, reducing the net cost to $128. This is a substantial recovery of 60% of the cost of play in the smaller format lotto game.

Real payoffs won in continual play show that small prizes reduced the cost 13% in the large format game and 60% in the small format lotto. There were just two games in the twelve months in this study in which Puerto Rico LotoPlus tickets had non-negative expected value. The cost of play would have dropped to just $6 to play two games for the optimal play strategy #2 during the sample period. A total of 171

TABLE 6. Lottery Ticket Details

Start Date	End Date	Lottery	Games	Form Cost	Game Cost	Win
7/18/2017	10/6/2017	Loto Plus	24	$ 72.00	$ 3.00	$ 39.00
1/10/2018	2/21/2018	Loto Plus	13	$ 39.00	$ 3.00	–
1/10/2018	3/21/2018	Loto Plus	21	$ 63.00	$ 3.00	$ 87.00
4/21/2017	7/12/2017	Loto Plus	24	$ 72.00	$ 3.00	$ 56.00
1/27/2017	4/18/2017	Loto Plus	24	$ 72.00	$ 3.00	$ 8.00
Totals			**106**	**$ 318.00**	**$ 3.00**	**$ 190.00**
7/18/2017	10/14/2017	PowerBall	26	$ 78.00	$ 3.00	–
1/6/2018	4/4/2018	PowerBall	26	$ 78.00	$ 3.00	–
4/22/2017	7/19/2017	PowerBall	26	$ 78.00	$ 3.00	–
1/11/2017	1/25/2017	PowerBall	15	$ 45.00	$ 3.00	–
1/28/2017	1/26/2017	PowerBall	78	$ 234.00	$ 3.00	$ 68.00
Totals			**171**	**$513.00**	**$3.00**	**$68.00**
Overall			**277**	**$831.00**	**$3.00**	**$258.00**

Note: We play lotto with actual cash for one year in Puerto Rico Lotto Plus and Powerball. The "start" and "end" dates correspond to the length of time each panel of tickets runs in a series of games. "lottery" identifies whether the games played are in LotoPlus or Powerball. "Games" is the number of games played. "Form cost" is the price paid to play a series of games. "Game cost" is the price per game. "Win" lists the amount won for that series of games. "Overall" summarizes results of each column of data for games, form cost, game cost, and win.

TABLE 7. Jockpot Size Regression

	Coefficients	Stand. Error	*t*-statistic	*p*-value
Intercept	0.511	0.145	3.519	0.003
Mega Mill.	1.525E−9	0.000	1.822	0.088
Powerball	1.091E−9	0.000	1.206	0.246
PR Loto+	2.697E−8	0.000	3.134	0.007

Notes: The dependent variable is jackpot size and the independent variable is the Weekly Economic Indicator (WEI). Number of observations is N =759.

games cost us $513 to play Powerball. But unlike LotoPlus just $68 was won in small prizes reducing the net cost of play to $445. These small gains recovered 13% of the cost to play multi-state Powerball lotto. Powerball had jackpot sizes that offered play just 21 times with positive expected values over the total 115 games in the period this study covers. This was the lowest cost strategy of $63 for the year.

Next we wish to know if there is a statistically significant relationship between the dependent (predictor) variable of jackpot values and the independent (response) variable, a weekly economic indicator. To do so we gather bi-weekly values as far back in time to 2002 for Powerball, Mega Millions, and Loto Plus jackpots. We then download values for the Weekly Economic Indicator (WEI) from the Federal Reserve Bank of New York website. The WEI is defined as "an index of ten indicators of real economic activity, scaled to align with the four-quarter GDP growth rate. It represents the common component of a series covering consumer behavior, the labor market, and production." We regress lotto jackpot values as independent variables (y) on the WEI as the dependent variable (X) where N=759.

$$y = X\beta + \varepsilon \tag{8}$$

Table 7 shows the results of this regression. We find a statistically significant relationship between the Jackpot sizes of the Puerto Rico Loto plus and the WEI at the 1% level (p-value = 0.007). However, Mega Millions (p-value = 0.088) and Powerball (p-value = 0.246) jackpots have no statistically significant relationship with the Weekly Economic Indicator (WEI) at or below the 5% level of confidence.

IX. Synthesis

Small prizes increase the value of a ticket by reducing the long-run cost of continual play. This increases prospects (small wins), consumption (hope), and networking (belonging) while reducing the strain (expense) of employing the strategy. Analysis based solely on jackpot odds leads to the narrow behavioral framing of a lottery ticket as a bad investment because cost recovery from small prizes is not considered. This leads to myopic disregard of the social benefits and high likelihood of winning smaller prizes from convex payouts. Our data illustrates how differences in convex format dramatically influence cost recovery, reducing the cost of continual play dramatically in our study by 13% in Powerball and by 60% in Puerto Rico LotoPlus. Optimal lotto play is cheaper than continual play but this strategy forgoes the offset of cost from winning small prizes in continual play generating fewer opportunities for winning large prizes. The continual lotto investment strategy using unpopular numbers we describe is a passive strategy. Continual play is much more rational for the typical player because of strategic passivity and the benefits of cost recovery as well as increased play from the perspective of prospect, strain, network, and consumption theories within the operational constraints of the mechanics of lotto play. We now explain these theories and how our results relate to each.

Sociologists have attempted to explain the persistence of the high volume of lotto play by attributing an intangible social benefit as a factor that enhances demand for lotto tickets but have not linked theoretical relationships between social benefits and either the jackpot or small prizes with regard to either the rich or the typical player (Lutter et. al., 2018). In addition to prospect theory from economics and psychology, three theories from sociology focus on collective social factors that give rise to strain, network, and consumption postulates used to explain persistence and intensity of lotto play based on intangible yet valuable social benefits derived from purchasing and holding a ticket. We fill a gap in the literature by elucidating these linkages between the four major theories used to explain the persistence of lottery play with an experiment with real payoffs. We extend our understanding of these theories by eliciting two strategies designed to allow us to study the pattern of small prize wins over the course of an entire year in a nationwide and a small local lotto system.

Prospect theory

Regarding lottery play, Adam Smith wrote that “the vain hope of gaining some of the great prizes is the sole cause of this demand” for play because of the poor odds of winning the jackpot (Smith, 1776). The cognitive bias hypothesis from psychology frames reliably intransitive gambling behavior as driven by faulty cognition (Rogers, 1998). Reliably intransitive gamblers consistently choose bad gambles with negative expected returns. The Kahneman and Tversky (1979) study used surveys where participants chose imaginary lottery payouts using prospect theory based on the Allias (1953) paradox. Prospect theory is extended to lotto by Cook and Clotfelter (1992) who show that the mathematical expected value of a lotto ticket is based solely on the probability of winning all possible prizes.

Under this paradigm a lottery ticket almost never has positive expected value except when jackpot carryover is unusually high (Ziemba et. al., 1986; Thaler and Ziemba, 1988; and Clotfelter and Cook, 1991). Experimental psychologists researching reliable intransitivity under utility theory such as Kahneman and Tversky are criticized by experimental economists for not paying subjects for choice outcomes (Harrison and List, 2004). Experimental economist Glenn Harrison points out that the Allias paradox is undone when payoffs are actually paid out on the spot, “Conlisk (1989) runs an experiment to test the Allais (1953) Paradox with small, real stakes and finds that virtually no subjects violated the predictions of expected utility theory. Subjects drawn from the same population did violate the ‘original recipe’ version of the Allais Paradox.”

This implies that people are not fools in the aggregate. They buy lotto tickets to add high potential payouts to their holdings rather than to seek action via unbeknownst bad bets the Allais paradox implies. Individual cognitive biases elucidated through prospect theory help explain reliably intransitive gambling behavior (Kahneman and Tversky, 1979). Our cash experiment shows that small prize wins in the aggregate make persistent intransitivity a less likely explanation for persistent lotto play within prospect theory.

Strain Theory

Sociology posits that tension arises from frustration felt by individuals striving to succeed financially when tools for success are not equally distributed in society. This can lead to increased lotto play according to strain theory of Merton, (1938); Bloch, (1951); Devereux, (1980); and Frey, (1984). Work dissatisfaction is conjectured to be correlated with social strain and is measured qualitatively by survey using a 7-point Likert scale with questionnaire selections ranging from strongly agree to strongly disagree. According to this theory, the strain of financial failure is managed by purchasing a lotto ticket. Ownership of a chance to win a large amount of money reduces the strain of the monotony of an unfulfilling job or career (Goffman, 1967). A large lotto win offers hope for material improvement and enhanced social status that cannot be achieved by conventional means for those people who are unconvinced that hard work will change their lot in life (Clottfelter and Cook, 1991).

Strain is quantitatively measured by changes in income (Lutter, et al. 2018). For instance, Icelanders who suffered financial setbacks were substantially more likely to buy lotto tickets (Olason et. al., 2015). Another way lotto play relieves strain is hearing about and identifying with lotto winners when a player owns a ticket. However, increased gambling behavior of all forms including lotto is related to substance abuse and obsession with status issues such as feeling poor (Frehe and Mechtel, 2015; Greco and Curci, 2016). Lotto can destabilize society because ticket purchases have been shown to drain savings away from personal investment in education and reduce time spent in community participation (Beckert and Lutter, 2013).

Women, however, tend to buy fewer tickets with less frequency and work dissatisfaction has no impact on how much females play. A decline in income does not push people to buy tickets who do not already do so, but does increase volume of play among existing lotto players (Lutter et. al., 2018). Strain theory thus has ambiguous support at best. Our result that the Puerto Rico jackpot size decreases (increases) as the United States economy weakens (strengthens) is evidence against strain theory. We would expect jackpot sizes (play) to increase (decrease) as the economy weakens (strengthens) under strain theory. Alternatively, the enjoyment of lotto play drives ticket demand (Downes et. al, 1976; Casey, 2006; Kocher et. al., 2014; and Burger, et. al. 2016). As Haisley et. al., (2008) explain, “Lotteries may be considered a ‘social equalizer’ in that, no matter what your position in society, everyone has an equal chance to win.”

Network Theory

Our results show that the length of time a player can enjoy lotto is substantially increased by small prizes while the jackpot has no impact on extending play. Extending the amount of time a player can enjoy lotto improves networking. Research from sociology supports network effects theory asserting that lotto players derive a social benefit that enhances the utility of lotto play. This utility is derived at the cost of a few dollars spent each week in return for communicating with other players about lotto. Burger et al., (2016) show that lotto players who play for fun and social interaction are happier than "only-for-money" or non-players. Forrest et al., (2002) show that demand for lotto is driven by the desire to buy a dream. This creates value in the form of the emotional joy of enjoying the company of like-minded players who are already friends or become so through networking.

Social networks also exacerbate social contagion that can influence people to buy lotto tickets beyond their means. In this theoretical framework people can either self-select to play for fun and camaraderie or they can be pushed begrudgingly into play because they are poor and desperate. However, players who set budgets and systematize are less likely to get caught up in contagion by planning play. An example of this problem is that friends, spouses, partners, and parents who play lotto increase the frequency that those associated with them will play lotto. Women are more susceptible to contagion effects than men if their parents play lotto (Felsher et. al., 2003). Social contagion is worse for low-income players because poor players are more susceptible to overspending on lotto tickets. However, we show that the largest of the small prizes can be as high as $50,000 in the small single state lottery or $1,000,000 in a large multi-state lottery. The largest of small prizes have a dramatically more positive effect on the wealth of low-income people. We show this trade off in our results.

Studies of networking theory have shown statistically significant increases in both expenditure and frequency of lottery purchase when people play in networks such as syndicates; live alone but play with a small group of friends; or when someone lives in a multi person household with at least one other player. An example is the study of Lutter et. al, (2018) that used a large sample from Germany confirming the results of smaller sample studies from Spain and the United States. See also Humphreys and Perez, (2013). Syndicate play in the US increases the frequency of individual lotto participation but the differences in the amount played for those people who play alone are undetectable (Garvía, 2012). Individuals are most likely to buy more lotto tickets who are in a syndicate than those who are not according to earlier German data from 2006 (Beckert and Lutter, 2013). Those lotto players aged 59 and up play more frequently and with greater expenditure. High income lotto players spend a smaller percentage of income on tickets yet play more frequently ranging from not at all to once weekly, once or twice monthly, or a few times a year. People value, learn from, and are motivated to action by social interaction that can also lead to contagion. This can be related to the lotto play habits of parents.

A number of studies have found that people play lotto to socialize by playing in a syndicate (Rosecrance, 1986; Adams, 1996, 2001; Chesters, 2002; Felsher et. al., 2003; Forrest, Simmons, and Garvía, 2007; Binde, 2009; Guillén and Garvía, 2012; Beckert and Lutter, 2013; Humphreys and Perez, 2013; Kocher et. al., 2014; Friehe and Mechtel, 2015; and Burger et. al., 2016). The literature review of Beckert and Lutter (2013) asserts that these studies support the notion that since [lotto] elicits "these social values, the value of a lottery ticket exceeds the expected monetary return." According to public policy professor Colin Campbell (1987) "in modern societies individuals do not so much seek satisfaction from products, as pleasure from the self-illusory experience which they construct from their associated meanings." This statement captures the essence of non-tangible value in lotto play from social benefits. Puerto Ricans have fewer networking opportunities because the island is much poorer than the poorest state. This partially explains why multi state lotteries are resistant to economic fluctuations but the Puerto Rico lotto is not.

Consumption Theory

Consumption theory postulates that people who play lotto do so because of the consumption value of daydreaming about winning the jackpot. The hypothesis used to test this theory is that lotto players derive consumption benefits in excess of the cost of a ticket from positive emotions arising from fantasizing through daydreams. The subjective utility of consumption is measured indirectly through surveys to attempt to capture differences in individual valuation functions. Support for this theory asserting that a component of the demand for tickets arises from daydreaming was found in analyzing the data from play in the United

Kingdom National Lottery (Forrest et al., 2002). Our results support the notion that small prizes have a more positive impact on daydreaming than the jackpot. Kocher et. al (2014) find support for a consumption theory hypothesis that lotto players prefer delayed game resolution because the anticipated positive emotional thrill is consumed among a substantial minority. Furthermore, this research shows that these same players prefer to purchase a single lotto ticket for two different sequential games rather than two tickets for the same drawing. This is in alignment with a desire to extend the length of play.

Testing of strain, consumption, and network sociology lottery play theories show that social surroundings; a frustrated desire to live a lifestyle beyond financial reach; the span between aspiration and current social position; belief in good luck; and personality traits such as impulsivity are related to lotto play and form a social benefit component of expected value that increases gaming volume (Chiu and Storm, 2010). Desire for more consumption, however, is associated with problem gambling. People 31 to 40 years of age, African Americans, poorly educated people, disadvantaged neighborhoods, convenience of access to gambling, impulsiveness, depression, and having friends who approve of gambling all contribute to problem gaming (Welte et. al., 2002; Welte et. al., 2017).

Operational Inefficiencies of Lotto Play

A player must be sensitive to the time expenditure trade off between infrequent play focused on the optimal expected value from large jackpots that requires little time or focus on cost recovery from small prizes in constant play. Continual play requires time consuming frequent travel to a kiosk, tracking, buying tickets, checking for caught numbers, and renewing replay forms. This effort is easier to justify when expected values are positive. Expected values of lottery tickets can be surprisingly high during record shattering jackpots. Thaler and Ziemba (1988) report that a $1 ticket can have an expected value of between $1 and $2.25 in thousands of combinations because of low coverage from the tendency of lotto players to consciously pick numbers less than thirty-one. This represents an illusion of control players enjoy by picking their own numbers. This is often based on family birth dates rather than using a random number generator (Langer, 1975; Langer and Roth, 1975). But even when tickets offer positive expected value an investor must be sensitive to wasting large amounts of time on lotto play. Operational inefficiencies mount with the amount of tickets played per game. The more tickets played, the longer it takes to fill out lotto purchase forms, gather and manage the cash, haul it all to a kiosk and buy tickets, then carry it back to the home or office where lotto play is managed and tickets are stored.

Following mathematical algorithms that estimate optimal bet sizes increases operational inefficiencies in lotto play. The Kelly formula, for instance, is used to approximate optimal casino bets and investment position sizes (Kelly, 1956). Prospect theory incorporates the Kelly formula. The formula calculates a Kelly optimal size of a series of bets to maximize the logarithm of wealth and was originally developed by a Bell Labs engineer to reduce noise in phone conversations. It is used in financial economics to justify portfolio concentration under convex utility (Friedman and Savage, 1948; Markowitz, 1952; Kahneman and Tversky, 1979, 1984; and Kallberg and Ziemba, 1983). The Kelly Formula is,

$$K = W - \left(1 - W\right)/R \,, \tag{9}$$

where K is the fraction of capital allocated to the next trade, W is the win to total trial ratio, and R is the payoff rate. A fair coin toss has an optimal fixed Kelly bet of 25 percent of capital. Investors are advised to exercise caution with this concept because the Kelly formula produces impractical results akin to false solutions of the internal rate of return. The difficulty in applying this formula to actual cash wagers is that it will spit out bet sizes that are so large as to be impractical to invest. Warren Buffett and Charlie Munger, for instance, make very large bets in the stock market but nothing close to full Kelly betting. Maclean et al. (1992) estimate that a full Kelly formula wager per $10 million of advertised jackpot value is achieved with 65 tickets if the player has an 82.7 percent edge. Contemporary lotto jackpots are much larger than $10 million. Managing 65 or more sequences per $10 million in jackpot value is best obtained via the combinatorial systems of Bluskov (2012).

A high edge bet is achieved by choosing numbers that are known to be unpopular with the public to minimize the splitting of the jackpot. Purchasing 65 tickets for each game requires a total of 130 tickets spanning two games a week. At $3 per ticket, this strategy costs $390 per game. At two games per week, the cost is $780 per week, $40,560 per year, or $405,600 per decade of very time consuming play. These large sums are easier invested in a broad equity index fund or ETF. Playing multiples of 65 tickets is extremely time consuming and tedious manual labor that is highly prone to errors. This became evident as we gathered our data for this single sequence study.

False bet size solutions are not the only barriers players face with the Kelly formula. The transactional costs of playing lotto are different from the costs of stock or bond investing. Completing forms for many tickets is time-consuming and error-prone. Lottery tickets can only be purchased with cash at a kiosk. Carrying large amounts of money in public exposes the lotto player to a greater risk of mugging or losing the cash some other way. It is far more efficient for players to allocate a minimal amount of savings to lotto per year after fully contributing into a Roth IRA, Standard IRA, SIMPLE IRA, SEP IRA, Profit-sharing plan (PSP), Employee stock ownership plan (ESOP), 401(k), 403(b), 457 plan, Individual 401(k), or Individual Roth 401(k).

The Maclean et. al. (1992) strategy requiring 65 tickets is too expensive in time and cost for an individual but not for a lotto club where effort, cost, and payouts are shared among a team of investors. But again, even a small lottery club forces the player to split the jackpot with others and is foiled in games where players win free tickets. A lotto club plays large amounts of tickets but does not try to buy the game as does a syndicate. Large state and multi-state jackpots are too large to buy and also employ measures to discourage syndicates such as free tickets as a small prize. This renders membership useless to the individual player in a large lottery syndicate that is unprofitable today but this was not so in the distant past. Voltaire, for instance, made his fortune not as a writer but rather from buying a French lottery in a syndicate with mathematician Charles-Marie de La Condamine in 1729. The lottery was closed in January of 1730 when authorities caught the error. All members of the Voltaire syndicate were able to keep their gains because they followed flawed rules that the lottery organizer set.

X. Conclusion

The low GNI of $24,020 of Puerto Rico when compared with the much higher $58,030 GNI of the United States implies that Puerto Rican disposable income that can be used to buy lotto tickets is much more restricted. This makes the Puerto Rico LotoPlus jackpot size much more sensitive to fluctuations in the Weekly Economic Indicator than Powerball or Mega Millions.

TABLE 8. International Lotto Statistics

Country	Population	Players	%Play	Lotto Game	Lotto Format	Min. Grd. Prz.	Max Grd. Prz.	Sm. Prz.
Australia	25.69 M	3.60 M	14.01%	Powerball	7/35 + 1/20	$2,568,820.00	$96,330,750.00	Yes
Canada	38.01 M	21.29 M	56.00%	Lotto 6/49	6/49	$3,620,625.00	$46,344,000.00	Yes
Europe	746.40 M	100.00 M	13.39%	EuroMillions	5/50 + 2/12	$16,675,640.00	$218,500,994.29	Yes
New Zealand	5.13 M	1.80 M	35.00%	Powerball	6/40 + 1/40	$2,291,400.00	$25,243,590.19	Yes
South Korea	51.78 M	31.07 M	60.00%	Lotto 6/45	6/45	$752,783.76	$4,133,340.46	Yes
United States	329.50 M	161.46 M	49.00%	Powerball	5/69 +1/26	$20,000,000.00	$1,586,000,000.00	Yes
United States	329.50 M	161.46 M	49.00%	MegaMillions	5/70 + 1/25	$40,000,000.00	$1,537,000,000.00	Yes

Note: Country populations and lotto players are in millions (M). Population is for the entire country. “Players” is the number of players in the country who play lotto. “% Play” is the percentage of the population playing lotto. “Lotto Game” is the name of the country's lotto. The “Lotto Format” describes how many numbers are picked from the field of numbers on the main panel and how many numbers are picked from the bonus panel field of numbers. “Min. Grd. Prz.” is the minimum size of the grand prize jackpot. “Max Grd. Prz.” is the maximum size of the grand prize jackpot. Minimum and maximum grand prize jackpots are converted to US Dollars (USD) at current exchange rates (9/30/2022). “Sm. Prz.” is whether or not the lotto format includes small prize payouts.

This study is based on data from the United States but there is vigorous international lotto activity as well. Table 8 shows that there are substantial populations buying lotto tickets in Australia, Canada, Europe, New Zealand, and South Korea. This ranges from 60% of the population in South Korea countrywide to 13.39% in Europe where not all countries participate in EuroMillions. Maximum grand prize jackpots vary substantially from $4,133,340.46 in South Korea to $1,586,000,000.00 in the United States. Each lotto game in these regions has small prizes that extend and reduce the cost of play. Our research shows that the typical player from any of these regions derives substantial direct and indirect benefits from small prizes with payoffs akin to embedded call options with varied probabilities.

We show that these small prizes increase the social benefits of lotto play for all players by extending and reducing the cost of play. Finally we show that the cost per year of playing a state and national lotto when tickets have positive expected value is low. This implies that high income earners who can max out annual contributions in all available investment retirement accounts (IRAs) are acting rationally when purchasing state and national tickets. This is especially so when jackpots are high enough to offer positive expected values according to the arbitrage pricing theory of Stephen Ross (1976).

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