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## Book

# Testing for ambiguity aversion in the Russian stock market

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HIGHER SCHOOL OF ECONOMICS  
NATIONAL RESEARCH UNIVERSITY

*Daniil Esaulov, Maria Davidovich*

**TESTING FOR AMBIGUITY AVERSION  
IN THE RUSSIAN STOCK MARKET**

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This paper represents the first comprehensive empirical study on ambiguity and risk in the Russian stock market. Modern portfolio theory assumes that there is a linear relationship between risk and return and known probabilities of outcomes. This risk-return relationship plays an important role in the financial literature, however, there is no clear empirical evidence on the sign and significance of risk-return tradeoff. Some financial literature argues that risk is not enough to measure the expected returns on the market portfolio. One possible reason for this is an assumption that rational decision makers know the probabilities of outcome that affects asset prices. In this paper the assumption of known probabilities is relaxed. We test the relationship between risk, ambiguity and return of intraday observations on MICEX index for the period 2009-2016 using GARCH-M approach, which allows for heteroscedasticity of returns. Ambiguity is derived from the model developed by Brenner and Izhakian (2011), which is modified to capture the non-normality of the distribution of returns in the Russian equity market. Expected returns appear to have a positive and statistically significant relationship with conditional volatility modeled by GARCH and EGARCH processes. We found out that coefficient of ambiguity is not statistically significant, which can be the evidence of ambiguity-neutral behavior of Russian investors. To the best of our knowledge, this paper is the first empirical work that derives ambiguity, accounting for its specificity, and tests the risk-ambiguity-return relationship in the Russian stock market.

JEL Classification: C51, D81, G12

Keywords: Ambiguity, Ambiguity aversion, Equity premium, Conditional heteroscedasticity

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## 1. Introduction

In the world of finance, decision-makers have to make their choice under ambiguity, the situation in which the probabilities of outcomes are unknown. Modern portfolio theory assumes that there is linear relationship between risk and return and known probabilities of outcome, not accounting for ambiguity. For the last century, it was mainly risk that was analyzed by numerous researchers when testing the influence on expected returns. However, nowadays ambiguity is becoming a crucial concept in the process of decision making in general and in asset pricing in particular. In our paper, we test the fundamental relationship between the excess return on the market portfolio and risk, including ambiguity in this relationship as an exogenous variable. Thus, the usual and restrictive assumption about the precisely known probabilities of outcomes is relaxed. Note that together with ambiguity measure we still need to consider the degree of risk in our model as the measure of ambiguity doesn't account for the magnitude of outcomes, in general, and the magnitude of loss and gain, in particular. Ambiguity considers only probabilities. Changing the outcomes of event doesn't influence its degree of ambiguity but does affect the degree of risk. So both factors jointly play an important role while making choices.

How do investors perceive the ambiguity in the Russian stock market? This paper partly answers this question. In our research we focus on the implication of ambiguity on financial decision-making and test the ambiguity-aversion on more general level by finding the objective degree of ambiguity of Russian investors. We based our paper on the recent approach developed by Brenner and Izhakian (2011) to investigate the attitude of Russian investors toward ambiguity. Briefly (more details in Section 3), their idea is the following: the numerical measure of ambiguity can be found as four times variance of probabilities of loss and tested as exogenous variable additionally to risk on excess return on market.

In addition to Izhakian's approach we test the relationship between risk, return and ambiguity on the Russian stock market using (E)GARCH-M as the base framework because it allows to account for heteroscedasticity of returns. Moreover, we introduce other frameworks in order to validate our results, in particular the specifications of the GARCH model and the approach proposed by Izhakian. Among other things, we test the distribution of the returns and come to the conclusion that they are not normally distributed. For this reason, we use a more suitable distribution for returns, namely non-standardized t-distribution. We use intraday data on MICEX index for the period 2009-2016, which is a proxy for the market portfolio. We use both GKO-OFZ and Mosprime 1 month as a proxy for a risk-free rate. Moreover, to avoid possible endogeneity problem we include lag of ambiguity measure instead of ambiguity measure in current period.

We get the following results: using approach proposed by Izhakian there is no strong statistical evidence for positive relationship between risk and excess return. However, using (E)GARCH-M model, the risk is positively related with ambiguity, which may imply that Russian investors are risk-averse. We have not found the statistically significant effect of ambiguity on the excess return using either Izhakian's approach or E(GARCH)-M approach. This can be the evidence of ambiguity-neutral behavior of Russian investors under certain assumptions. Unfortunately, we cannot compare these results with the results in other emerging markets because, to the best of our knowledge, there is not enough financial literature concerning attitude of investors toward ambiguity in the emerging markets. However, Ivanov (2011) in his experimental study found out that the investors are mainly ambiguity-neutral, which can support our results.

The paper is organized in the following way. Section 2 presents the overview of the literature relevant for testing for ambiguity-aversion in the Russian stock market. Section 3 describes the Shadow Probability theory and risk-ambiguity model, developed by Izhakian. Moreover, this section presents the modified risk-ambiguity model, which accounts for non-normal distribution of Russian returns. GARCH-M model, which we will use to perform empirical analysis, is also described in Section 3. Section 4 shows how this model is implemented in our research, describes the data, states the hypothesis and provides the results of our empirical analysis. Section 5 concludes.

## **2. Literature Review**

The empirical research which is central in this paper is based on Shadow Probability theory, developed by Y. Izhakian (2011), accounting for features of stock market in developing countries. M. Brenner and Y. Izhakian (2011) used this theory to test empirically the basic relationship between risk, ambiguity and return in the U.S. stock market. Our work is closely related to the above-mentioned ones in a way that we find numerical measure of ambiguity for Russian stock market based on Izhakian risk-ambiguity model. Our paper differs from these works in the following way: it incorporates the non-normality of the returns in the Russian stock market and uses different proxies for market returns accounting for its heteroscedasticity. The latter property of returns was proved to be essential while analyzing the relationship between risk and market returns by Merton (1980).

Thus, before proceeding to our empirical research itself, we highlight four different types of literature connected with the topic of our paper. First, we consider theoretical models on ambiguity aversion, which give the basis for the Shadow Probability theory. Then, we describe some empirical research that provides evidence on investors' attitude toward ambiguity.

Moreover, we emphasize some empirical studies that give numerical proxies for the measure of ambiguity. Finally, as we consider risk-ambiguity-return relationship and a great part of this paper is devoted to finding the most suitable proxy for risk in the Russian stock market, it is important to take into account empirical research which tests the fundamental relationship between risk and excess return on the market.

There are several theoretical issues which model ambiguity applying similar to Shadow Probability theory methodology. The model we use in our research is related to primary source of the uncertainty developed by Knight (1921) in a way that it considers ambiguity as the uncertainty about the probabilities of risk. It is supposed that the ambiguity is higher if a rational decision maker is more uncertain about distribution of probabilities of outcome. Neumann-Morgenstern utility theory (1953) takes a significant place in decision making under risk and ambiguity.

It states that when rational decision maker faces the unknown probabilities of outcome he will behave in a way to maximize the expected value of function of potential outcomes. However, the assumptions of the central model of this paper differ from those of Neumann- Morgenstern utility theory in a way that the investor acts like he does not know the probabilities of outcomes and considers these probabilities random. In reality it is very unlikely that distribution of probabilities is exactly known, especially if we are talking about stock market.

Lately, subjective utility theory was proposed by L.J. Savage (1972) based on the subjective utility function and subjective utility distribution. Similar to Izhakian's approach, Ellsberg (1961) states that the measure of probabilities is sub-additive, meaning that sum of probabilities is not equal to unity, which is considered as the violation of subjective utility theory. Numerous experiments have identified that decision makers' behavior is controversial to Savage's axioms of subjective expected utility due to ambiguity. One of the most popular papers on violation of subjective utility theory is proposed by Schmeidler (1989) based on Choquet utility theory. He introduced the Choquet integral, which allows integration of non-additive probabilities. The central model of our research is based on main theories of ambiguity, assuming that probabilities of outcomes are random.

Izhakian's model considers a risk free rate as the reference point which determines whether the investor perceives his return as gain or loss. "Prospect theory" of Daniel Kahneman and Amos Tversky (1979) also assumes that there is a reference point. The difference is that in prospect theory such a reference point is used for showing preferences of risk, while Shadow probability theory uses risk free rate as determinant of attitude to ambiguity by separating between probabilities of gain and loss. Shadow probability theory summarizes theories of non-additive probabilities and reference-dependent beliefs as well as Cumulative Prospect Theory by

Tversky and Kahneman (1992) who in their work applied different risk attitude towards loss and gain and incorporated it into Cumulative Probability Distribution function. However, there is one important distinct between Shadow probability theory and other models described above that ambiguity is applied directly for probabilities but not for utilities.

One obvious advantage of Shadow probability model is that it allows incorporating the measure of ambiguity in empirical research and providing tests for Market Stock Returns in Russia, while model described above are almost purely theoretical and can hardly be applied to real market data. The purpose of this empirical work is to investigate whether the measure of ambiguity is necessary to include in explanation of stock returns in Russian stock market. Moreover, the proposed modification of the Shadow Probability theory allows for non-normality of returns, which is an important feature of emerging markets.

Shadow probability model has similarities not only with existing theoretical models of ambiguity but also with empirical researches in a way that measure of ambiguity from Shadow probability model can be applied to test. There are several empirical researches, which incorporated different models to measure ambiguity. For example, Dow and Werlang (1992) measure ambiguity as the sum of probability of event and the probability of the complement event. Moreover, Panayiotis C. Andreou (2014) proposes the new measure of ambiguity on the stock market which is based on investors' trading activity in the S&P 500 index options market. The author measures the ambiguity as the variance of strike prices of S&P 500 index options. The higher is the dispersion of strike prices, then the higher is the dispersion of investors' believes about future returns, therefore the higher level of ambiguity. Moreover, Panayiotis C. Andreou has found that the higher is ambiguity on stock market, the lower is the expected return, and therefore, ambiguity is negative related to the excess return, which means that investors are ambiguity-lovers. The attitude toward ambiguity shows how much extra premium is required by investor to compensate for the increased ambiguity and in this case investors need less compensation if ambiguity is high, which implies ambiguity loving.

Some empirical research on investors' attitude toward ambiguity have found that the higher is the probability of loss for investors, the higher is the level of ambiguity-loving, while the higher is the probability of gain, the higher is the level of ambiguity aversion. So, if the probability of loss is high, then investors would more likely be ambiguity-lovers as in paper by Viscusi and Chesson (1999). In other behavioral studies, for example, by Asen Ivanov (2011) it is stated that there the percentage of ambiguity-loving, ambiguity-neutral and ambiguity-averse investors is 32%, 46%, 32% correspondingly. M. Brenner and Y. Izhakian (2011) show that investors are ambiguity-lovers in the U.S. stock market, using the S&P 500 index. The Shadow Probability theory was also used to measure Knightian uncertainty in the Chinese Stock market.

In their article, Wang, Yu, Fang, and Zhang (2015) have shown that ambiguity is present in the Chinese market but to a lesser extent in comparison with the USA stock market, but still Chinese investors show a preference for this uncertainty.

Moreover, there are different existing proxies for uncertainty proposed by different studies. There are several common proxies for uncertainty, which are widely used by researchers such as dispersion of analysts/expert opinions and VIX index that stands for volatility index from the option market. When choosing the proxy for ambiguity, it is very important to choose such measure, which better fits the features of investigated market. There are several empirical studies that investigate the investors' behavior under uncertainty using VIX index. For example, Ron Bird and Danny Yeung (2012) use VIX index as proxy for market uncertainty in the USA market. VIX is computed daily by the Chicago Board Options Exchange and is the weighted average of implied 30 day volatility of the S&P 100 stocks as reflected in index option prices. Intuitive explanation for using VIX is that it can give you a protection from uncertainty by allowing selling or buying underlying asset (depending on the nature of option). There is a Russian analogue to VIX index, called RTSVX index. It is calculated basing on the volatility of option prices with futures stands for underlying asset. However, using RTSVX as measure of uncertainty in Russian market is very questionable. One problem of Russian options market that the options are very illiquid and are used rarely by Russian investors. Thus, in comparison to USA market, Russian market of options is not developed enough thus RTSVX index may be a bad proxy for ambiguity. Moreover, one of the commonly used proxies for ambiguity is dispersion in analysts' earnings forecasts. For instance, Diether, Malloy, and Scherbina (2002) explain negative relationship between stock return and uncertainty using dispersion in analysts' earnings forecasts as a proxy for uncertainty. Authors interpret dispersion in analysts' earnings forecasts as a proxy for differences in opinion about a stock. This proxy has an important disadvantage that leads to severe questions when incorporating it into empirical analysis. As an example, dispersion in analysts' forecast is based on forecasts of only a few professional analysts who express their opinions not about stock overall but about earnings. Moreover, the data on analysts' forecasts is updated quite rarely within periods of either 1 or 3 months. This is very restrictive assumption that uncertainty stays the same during some period. Finally, some researchers argue that it is hard to distinguish risk from ambiguity in this context.

The sufficient part of this paper is devoted to testing the risk-return relationship. Our paper uses Merton's idea (1980) of necessity to account for heteroscedasticity of returns. In order to do so, one of the proxies for risk we use in our work is based on conditional volatility of returns estimated from GARCH models (and its specifications such as EGARCH). Following Merton, many empirical studies were conducted in order to test risk-return relationship. For



example, French, Schwert and Stambaugh (1987) investigated this relation using GARCH model and found out that excess return on S&P composite portfolio is positively related to the predictable volatility of stock returns. Moreover, Ser-Huang Poon and Stephen J. Taylor (1992) examined this relationship using UK data. Authors concluded that there is a positive but not statistically significant relationship between expected volatility and excess return on the market portfolio. Enrique Salvador (2012) tested the risk-return relationship in emerging markets and showed that there is positive and significant effect of volatility on excess return on the market. See the rest of corresponding bibliography in these articles.

To sum up, this empirical research contributes to the already existing literature in the following way: basing on important assumptions of commonly used ambiguity models, it provides a reasonable numerical measure for ambiguity, thus allows testing the effect of ambiguity in Russian market, accounting for its specificity. Moreover, it allows determining the degree of ambiguity computed from the Russian stock market. What is also important, it is the first empirical research on the Russian stock market, which combines modeling ambiguity and modeling risk using GARCH models. Thus, this empirical work, which focuses mainly on research of Russian stock market, will try to incorporate the measure of ambiguity into testing and possibly make audience interested in further investigation of this model. What is also important, this is first empirical work that tries to analyze the ambiguity in Russian stock market, obtaining weekly measure of ambiguity, which is different from previous works by allowing of non-normality of returns, which is very essential for emerging markets.

### **3. Model**

The primary aim of our empirical research is to analyze whether ambiguity affects the Russian stock market. There are many theoretical and experimental models on ambiguity aversion. This research provides the measure of ambiguity that can be tested empirically.

In usual models for relationship of risk and return there is known distribution of probabilities, thus these models do not account for ambiguity. It is possible to measure ambiguity using Shadow Probability theory developed by Izhakian (2011). It accounts for ambiguity as it assumes that probabilities of observable events are random. This model allows the total separation of risk and ambiguity measures. First of all, we describe the Izhakian's model and explain how measure of ambiguity can be derived. Second, we propose some modification of the Shadow probability theory for better fit for the Russian stock market. Third, we combine the modified ambiguity measure with a risk measure accounting for heteroscedasticity in returns.

### ***3.1. Preliminaries***

The main assumption of Shadow probability theory is that the probabilities of outcomes are random. It means that not only returns follow particular distribution, but also that there exist special distribution for probabilities of loss. According to Izhakian (2011), it is a uniform distribution. Izhakian proposes the dispersion of probabilities of loss as a proxy for ambiguity (intuitively, as well as the dispersion of outcomes is a proxy for risk). Two measures of uncertainty arise: the first one is risk and the second one is ambiguity. According to this model, if the variance of probabilities is equal to zero, then the ambiguity is absent assuming that investors know the true probabilities of outcome. The question arises: why variance of probabilities is a good measure for ambiguity? We can try to answer this question relying on risk measure. When outcomes are unknown, the smaller dispersion in returns means the greater predictability of outcomes, thus lower risk. The same logic is applied for ambiguity. When probabilities of outcomes are unknown, the smaller dispersion of probabilities means the clearer thoughts about probabilities of particular outcome. Probability of outcomes tends to some known parameter, chasing the ambiguity away. Furthermore, obtaining variance of probabilities of loss means obtaining numerically measure for ambiguity, which can be used for testing the relationship between ambiguity and returns. This estimator has several important advantages over other proxies of ambiguity. First of all, it can be easily obtained from intraday returns. Moreover, the ambiguity is not constant and using Izhakian approach it can be measured every week (in comparison to analysts' forecasts, for example, that provides measure for ambiguity less frequently). Thus, we will use this measure for ambiguity as a proxy of ambiguity in our work.

Due to the presence of ambiguity in this model, the risk premium is not the only thing that stimulates investors to pay for prevalence of expected return on risky asset over the known return on risk-free asset. Izhakian states that ambiguity premium is the premium that investors are willing to pay for replacing ambiguous outcome for an unambiguous one. The total premium accounts both for risk premium and ambiguity premium in Shadow Probability model. The extent to which investors are willing to pay for higher expected return and unambiguous outcome depends on risk aversion and ambiguity aversion coefficients correspondingly. Ambiguity aversion relates to the aversion towards ambiguous outcomes; to be more precise, the uncertainty regarding the probability distribution of outcomes. Ambiguity aversion is different in comparison to risk aversion, because risk aversion only captures the potential of gaining or losing value of the asset. Because both the value of the outcome as well as the probability distribution of the outcome in reality are unknown, it is important to take into account the difference of these two concepts. Note that ambiguity and ambiguity aversion are two different concepts. For instance,

ambiguity can be present on the market, but if Russian investors are neither ambiguity averse nor ambiguity lovers, then stock market will not be affected by ambiguity. It is true under the assumptions of efficient market and inability to reshuffle the market portfolio.

### 3.2. Risk-Ambiguity Model

In this subsection we propose the modified risk-ambiguity model, based on Brenner and Izhakian (2011). In their model they, however, assume that returns on assets are normally distributed, as well as the returns on market portfolio. But in many cases in reality we can observe that returns do not follow normal distribution. Very often returns follow leptokurtic distribution. Leptokurtic distribution has fatter tails and higher peak. In comparison to the normal distribution, fatter tails mean that extreme observations have a great chance to take place and even outlier events can lead to the increased level of risk. Moreover, historical values are concentrated around the mean. We use non-standardized Student's  $t$ -distribution for better fit of returns of Russian stock market. The feature of non-standardized  $t$ -distribution is that it is symmetric and bell-shaped as well as normal distribution, but it has fatter tails. It is assumed that returns are distributed in the following way:

$$r_t \sim \mu + \sigma t(d),$$

where  $\mu$  is a location parameter,  $\sigma$  is a scale parameter,  $t(d)$  is standardized Student's  $t$ -distribution with  $d$  degrees of freedom.

If the risk free rate,  $r_f$ , is used by investor as a reference point, then the probability of loss can be described by cumulative density function of non-standardized student distribution:

$$P_L = P(r_m \leq r_f) \quad (1)$$

where  $t_d$  is the cumulative density function of standardized Student's  $t$ -distribution. Note that  $E(r_t) = \mu$  and  $Var(r_t) = \sigma^2$ . Under these assumptions, ambiguity can be found

in the following way:

$$v_t^2 = 4Var \quad (2)$$

In the section 4 we will estimate  $d$  using MLE (Maximum Likelihood Estimation) from daily data. Then, we will estimate  $\mu$  and  $\sigma^2$  using method of moments for intraday data.

Brenner and Izhakian propose testing the relationship between risk, ambiguity and return in a following way:

$$E(r_t) - r_f = \alpha(\delta_t^2) + \beta(v_t^2) \quad (3)$$

In this formula,  $r_t$  stands for return of the market portfolio,  $r_f$ , is the risk free rate,  $\delta_t^2$  is the risk of the market portfolio and  $\alpha$  is the coefficient of risk aversion (in our case (3) stands for aggregation of the risk-aversion coefficients of the investors). The risk aversion coefficient shows

the attitude of the investor toward risk. The higher is the coefficient of the risk aversion, the higher is the required by investor premium for risk, i.e. the amount of money by which return on risky asset (in our case the return on market portfolio) has to exceed the return on a risk-free asset.  $\beta$  is ambiguity aversion coefficient. The attitude toward ambiguity shows how much extra premium is required by investor in order to induce him to invest in ambiguous asset as compensation for the increased ambiguity.

Contrary to Izhakian's equation (3), we expect that endogeneity problem can arise: there is no clear evidence that the measure of ambiguity on the right hand-side of this equation is independent from expected return on the market portfolio on the left-hand side. Measure of ambiguity is determined by cumulative probability that can vary much more depending on expected return, so the ambiguity measure can be monotonically dependent with expected return on the market portfolio, therefore there the endogeneity problem may probably arise, leading to biased and inconsistent estimates. To overcome endogeneity problem, we can include the lag of ambiguity variable and perform empirical test with following regression:

$$E(r_t) - r_f = \alpha(\delta_t^2) + \beta(v_{t-1}^2). \quad (4)$$

Intuitively, it also seems plausible to include the lag of ambiguity variable: if ambiguity influences the expected return, then it seems reasonable to assume that investors should firstly observe ambiguity in period  $t - 1$  and then undertake measures that influence expected return on the market portfolio in period  $t$ . If it is so, then this relationship can be applied in order to

provide better forecast of expected returns. We have also performed empirical analysis of regression without lag of ambiguity as proposed by Izhakian. The results can be seen in Appendix. We conclude that there is no statistically significant difference of the results for regression analysis of regression (3) and (4).

In addition to testing the relationship (4), we suggest considering the combined model that incorporates both ambiguity and risk measures as exogenous variables in the following GARCH-in-mean model, thus allowing for heteroscedasticity of returns<sup>1</sup>:

$$r_t = c + \delta\sigma_t^s + \lambda v_{t-1}^2 + \varepsilon_t, \quad (5)$$

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2, \quad (6)$$

$$\ln(\sigma_t^2) = \omega + \alpha_1 \left| \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right| + \beta_1 \ln(\sigma_{t-1}^2) + \gamma \left( \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right) \quad (7)$$

Here  $r_t$  stands for excess return on market portfolio in period  $t$ ,  $\sigma_t^s$ ,  $s = 1, 2$ , is a risk measure modeling either by (6) or (7) and  $v_t^2$  is an ambiguity measure proposed by Izhakian.

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<sup>1</sup> Following Merton (1980), "because the variance of the market return changes significantly over time, estimators which use realized return time series should be adjusted for heteroscedasticity". We will consider the conditional variance of returns as proxy for risk.

If we expect our investors to be risk-averse, then the coefficient  $\delta$  corresponding to  $\sigma_t^2$  has to be positive and statistically significant, if we expect Russian investors to be ambiguity-averse, then coefficient  $A$  corresponding to measure of ambiguity, has to be also positive and statistically-significant.

#### **4. Data and Description of Empirical Research**

This section presents the basic empirical results of the work. We start with data description and proceed with analyzing the properties of our data, particularly, the distribution and volatility dynamics. After this analysis we focus our attention on the relationship between returns and both risk and ambiguity measures. Our goal is to find out if there is some evidence about the significance of the relationship.

##### **4.1. Data**

We are going to use MICEX Index as proxy for the market portfolio. For our analysis we need intraday trading data on MICEX. MICEX Index is cap-weighted composite index calculated based on prices of the 50 most liquid Russian stocks of the largest and dynamically developing Russian issuers presented on the Moscow Exchange with types of economic activity among the main sectors of the economy. It is traded continuously, updated every second and is transparent instrument for investor. The stocks of the most liquid companies such as “Gazprom”, “Sberbank of Russia”, “Lukoil”, “Norilsk Nickel”, “VTB”, “Novatek”, “Rosneft”, “Surgutneftegas” occupies about 80% of the weight in the index. We are going to estimate ambiguity from stock market returns, i.e. the returns on MICEX index. It seems plausible that the MICEX index can reflect to some extent the situation on the Russian stock market. It also seems plausible that the stock market ambiguity captures to a certain extent the overall ambiguity in the economy. Therefore, it is somewhat a good proxy of the market portfolio to measure ambiguity. There are several proxies for risk-free rate used by different authors. For example, it could be MOSPRIME 1 month rate, rate of return on GKO-OFZ. The first one is the average of interest rates estimated by each of the leading banks in Moscow on loans and deposits with a maturity of one month. GKO are short-term zero-coupon Russian Government Treasury Bills. Teplova, T. and E. Shutova (2011) use the rate of return on the Russian treasury bills with one year to its maturity as a proxy for risk-free rate. John Hull (2013) states that LIBOR is often used as a proxy for risk-free rate. MOSPRIME is LIBOR analogue in the Russian stock market. In our work we will use both MOSPRIME 1 month and Russian treasury bill with a maturity of one year as a proxy for risk-free rate. The maturity for GKO-OFZ is 1 year due to the lack of data on short-term OFZ.

Moreover, for further testing of relationship between expected return on market, risk and ambiguity, we need some measure for risk. We will incorporate several measures of risk including one suggested by Izhakian. Intraday data on MICEX index was obtained from Finam database. MOSPRIME1M and GKO-OFZ were obtained from Bloomberg database. The period investigated is from 2009 till 2016.

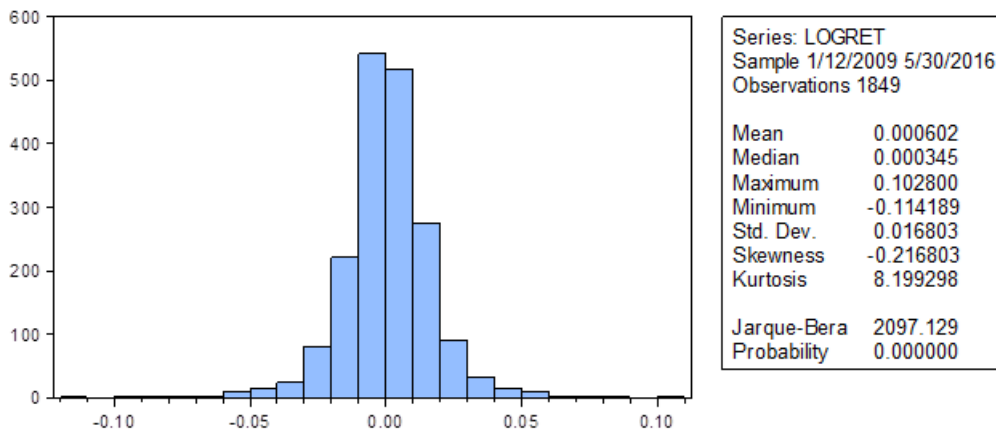
#### 4.2. Descriptive Statistics and Non-Normality of Log Returns

In this subsection we show that log returns on market portfolio constructed using the MICEX index follow non-normal distribution and propose the theoretical distribution that fits our data better.

We compute log returns to incorporate it into regression analysis. Define log return on market portfolio as  $r_t = \log P_t - \log P_{t-1}$ , where  $P_t$  is the value of MICEX index in period  $t$ . Stock market returns were computed in similar way by many researchers. For example, Ser-Huang Poon and Stephen J. Taylor (1992) while testing the relationship between volatility and expected return on market computed return on market portfolio as difference between logarithms of prices in period  $t$  and  $t - 1$  correspondingly. Moreover, Enrique Salvador (2012) tested the relationship between risk and return in emerging markets and also computed log returns on market portfolio.

Figure 1 describes the distribution of the daily log returns. We consider sample of data from 12/01/2009 to 30/05/2016. The histogram and descriptive statistics show the presence of fat tails and high peak (kurtosis = 8,199). Moreover, distribution is negatively skewed (skewness = -0.216). These facts indicate the non-normality of returns.

Figure 1: Distribution of the daily log returns



Both Jarque-Bera and Shapiro-Wilk tests reject the hypothesis of returns being normally distributed.

Using EViews we fit  $t$ -distribution into our data. Figure 2 describes the kernel density estimate together with theoretical normal distribution and  $t$ -distribution that fit the given sample in the best possible way. It can be seen from the graph that  $t$ -distribution works better for our data than normal distribution does. Using maximum-likelihood estimation we obtain the estimate for the degrees of freedom  $d$  (equal to 3.273).

Figure 3 describes the distribution of corresponding weekly data (383 observations). In order to normalize weekly returns to the daily basis we compute the mean of daily returns for each week. We see that by analogy with daily data the distribution of weekly returns is not normal.

Figure 2: Kernel and theoretical distribution

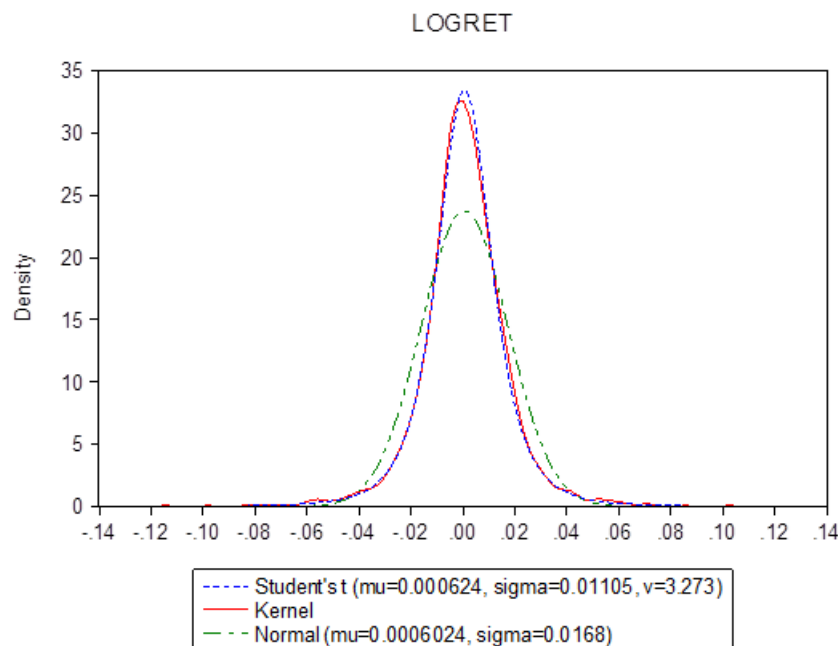
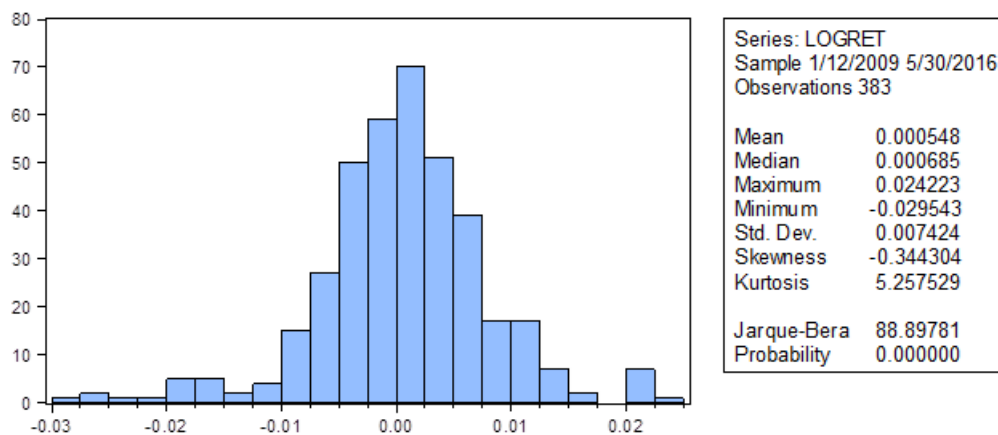


Figure 3: Distribution of the weekly log returns



### 4.3. Heteroscedasticity of Returns and Volatility Dynamics

Figure 4 describes the average weekly excess returns on MICEX index for the period from 12/01/2009 to 30/05/2016 (only sample of excess returns computed using one risk free rate presented, another one follows the same dynamics). Over this period, one can observe the presence of volatility clustering.

Figure 4: Average weekly excess return on the MICEX index

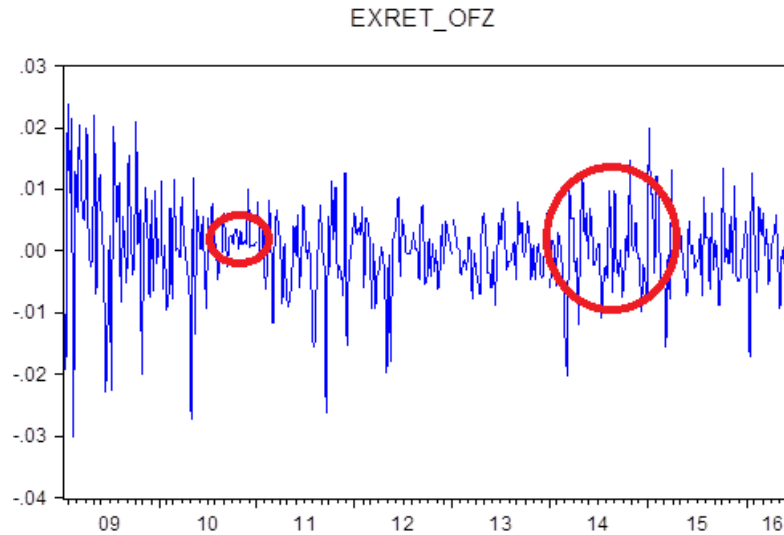


Table 1 presents the Ljung-Box autocorrelation test results for log returns, excess returns constructed using two different risk free rates (OFZ-GKO and Mosprime 1 month) and their squares. We provide the values of Q-statistics (including 6 and 12 lags for the series in levels) and p-values for the corresponding tests. Results show absence of autocorrelations for returns. This fact motivates us not to include return lags in the mean equation. Also, we see the presence of autocorrelations in the squares of returns which is the signal of heteroscedasticity. This is supported by ARCH-LM heteroscedasticity tests. These results motivate us to use GARCH model for conditional variance of returns. In addition to GARCH(1,1) model we estimate EGARCH(1,1,1) that captures “leverage effect”. The corresponding estimations are provided in subsection 4.6.

Table 1: Ljung-Box autocorrelation test

	Q(6)	Q(12)	Q(6) squares	Q(12) squares
Log returns	8.48 (0.205)	20.9 (0.170)	71.08 (0.000)	96.16 (0.000)
Excess returns (Mosprime)	8.51 (0.282)	22.26 (0.203)	69.51 (0.000)	94.20 (0.000)
Excess returns (GKO-OFZ)	8.32 (0.216)	22.04 (0.178)	70.73 (0.000)	95.6 (0.000)

P-values in parentheses



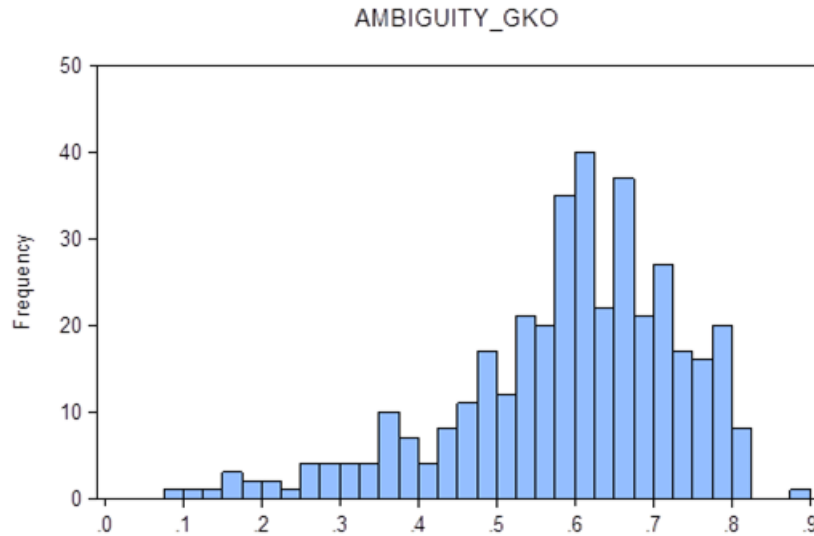
#### 4.4. Ambiguity Measure

Firstly, we compute the measure of ambiguity using equation (1). In order to do so we compute 15 minute log returns  $r_t$  for the MICEX index, from 34 to 54 each day (depending on the length of trading session). That leads to approximately 170-270 return observations each week. The time horizon we consider is from 12/01/2009 to 30/05/2016.

This interval is chosen due to the lack of intraday data for earlier period. 15 minutes intervals are chosen for simplicity.

We use daily data to compute the mean and variance of  $r_t$  for each day using method of moments. Results are normalized to the daily basis (are multiplied by the corresponding number of 15 minute intervals). Using formula (1) and estimate of the degrees of freedom for corresponding t-distribution from previous subsection we compute probabilities of loss for each day. Using vector of probabilities for each week the variance and consequently the degree of ambiguity for each week can be calculated. Figure 5 represents<sup>2</sup> distribution of ambiguity measure W for GKO-OFZ reference point<sup>3</sup>.

Figure 5: Distribution of ambiguity with GKO reference point



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<sup>2</sup> Distribution with Mosprime and zero reference points have been computed and are similar

<sup>3</sup> Ambiguity measures using normal distribution (as Brenner et al. (2011) suggested) have also been computed. We provide the corresponding results in Appendix.

Table 2 shows the descriptive statistics of ambiguity measure constructed using OFZ-GKO as the risk-free interest rate.

Table 2: Descriptive statistics of ambiguity measure (GKO)

Mean	Median	Maximum	Minimum	Std. Dev.	Skewness	Kurtosis
0.591252	0.609855	0.888745	0.084559	0.143668	-0.929260	3.865690

#### 4.5. Hypothesis

Main hypothesis is the following: excess return on Russian stock market depends on risk and probably on ambiguity. Following Merton (1973), we expect that the effect of risk on excess return is positive. The effect of ambiguity on excess return is less evident. Brenner and Izhakian (2011) show ambiguity-loving preferences of the US investors using S&P 500 index as proxy for the market portfolio. However, the results of our research can be different from this study since in our work as we consider emerging market in comparison to the developed US market. It would be interesting to compare the results in the Russian stock market with the results in other emerging markets. We investigated the sources of articles and we consulted with authoritative sources but we did not find such articles in the available language. Therefore, using the existing empirical base, we are not able to make predictions about the attitude of Russian investors toward ambiguity.

#### 4.6. Estimation Output

In this subsection we focus our attention on the relationship between excess returns, risk and ambiguity measure. We consider weekly excess returns as MICEX,  $r_m$ , which is the proxy for the return for market portfolio, minus the risk free rate,  $r_f$ . We provide results both for MOSPRIME1M and GKO-OFZ as for the proxy for risk free rate. The weekly market return is obtained by taking log return on monthly prices from Bloomberg Database.

First, we test the following relationship:

$$r_t = \alpha + \beta_1 \text{MVAR}_t + \lambda u_{t-1}^2 + \varepsilon_t,$$

$r_t$  is an excess return on the market portfolio, is Izhakian's measure of ambiguity obtained as described in section 4.4, is a risk measure proposed by Izhakian. This variable stands for mean variance of returns. The daily variance is computed from intraday data (discussed in subsection 4.4) and multiplied by number of 15 minutes intervals for each day. Then, to turn the variance in weekly basis we will find the average of daily variances. The results of this regression are describes in Table 3<sup>4</sup>.

<sup>4</sup> VIF test confirms the absence of multicollinearity.

Table 3: Testing with Izhakian's risk measure

Reference point	$\alpha(x 10^{-5})$	$\beta_1$	$\lambda$	AIC
OFZ-GKO	1.09 (0.011)	1.71 (1.10)	0.0003 (0.11)	-6.982
Mosprime 1 month	-7.19 (-0.071)	1.53 (0.98)	0.0005 (0.20)	-6.984
Zero	37.1 (0.231)	1.73 (2.38)	-0.00070 (-0.59)	-6.982

T-statistics in parentheses

The estimated coefficients turn out to be insignificant. This result motivates us to use another model for risk that incorporates heteroscedasticity of returns.

Table 4 represents the estimated parameters for the mean and variance equations using the GARCH-in-mean framework (5)–(6) for the Russian stock market. Mean equations differ by inclusion of risk and ambiguity trade-offs.

The main conclusion of these results is that this framework shows evidence of the risk-return trade-off in Russian stock market. Coefficient 5 (responsible for risk-return tradeoff) appears to be positive and significant. This is true both for excess returns  $r_t^{mspr}$  and  $r_t^{gko}$ . Moreover, inclusion of risk measure into mean equation reduces the AIC information criterion, which is the sign of GARCH-in-mean model being qualitatively better than usual GARCH. However, there is no evidence about significance or sign of the coefficient  $\lambda$  that represents the ambiguity-aversion parameter.

Table 4: Log-likelihood estimates for models with conditional variance following GARCH(1,1) process.

Variance equation: $\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2$							
Mean equation	c	$\omega(x10^{-6})$	$\alpha$	$\beta$	$\delta$	$\lambda$	AIC
$r_t^{mspr} = c + \varepsilon_t$	0.0001 (0.416)	1.63 (1.88)	0.06* (1.97)	0.9* (24.72)			-7.194
$r_t^{mspr} = c + \delta \sigma_t + \varepsilon_t$	-0.004* (-2.64)	2.03* (2.09)	0.06* (1.99)	0.89* (25.27)	0.7* (2.86)		-7.206
$r_t^{mspr} = c + \delta \sigma_t + \lambda v_{t-1}^2 + \varepsilon_t$	-0.004* (-2.37)	2.25* (2.22)	0.06* (1.99)	0.89* (25.34)	0.74* (2.91)	0.0007 (0.35)	-7.207

T-statistic in parentheses. Significance: \* p<0.05  
 $r_t^{mspr}$  is weekly excess return on the market portfolio with Mosprime 1 month as corresponding risk-free rate, normalized to one day

In order to test for asymmetry (“leverage”) effect we perform a test on standardized residual estimated from GARCH(1,1) model. The result of the test indicates the possible presence of asymmetric effects. Therefore we find it necessary to provide the estimation for EGARCH framework (5)–(7) accounting for “leverage effect”. Table 5 shows the corresponding results.

Basic conclusion from Table 5 is similar to the one from Table 4: coefficient  $\delta$  of risk-aversion is significant and positive, coefficient  $\lambda$  of ambiguity-aversion is statistically insignificant. Note that coefficient  $\gamma$ , which is responsible for asymmetry, is significantly negative. It is consistent with Nelson’s (1991) results.

Excess returns on market portfolio are shown to have a positive, statistically significant relationship with conditional volatility following both GARCH(1,1) and EGARCH(1,1) processes. The relationship between ambiguity measure for market stock returns and excess return on the market portfolio is less clear: we have not found statistically significant effect of ambiguity on excess return using either GARCH(1,1) or EGARCH(1,1) for conditional volatility. It can be the evidence of risk-neutrality of Russian investors.

#### 4.7. Interpretation of Results

Excess returns on market portfolio are shown to have a positive, statistically significant relationship with conditional volatility following both GARCH(1,1) and EGARCH(1,1) processes. However, using MVAR as a proxy for risk we did not find any statistical evidence for significance of risk coefficient.

Table 5: Log-likelihood estimates for models with conditional variance following EGARCH(1,1) process.

Variance equation:								
$\ln(\sigma_t^2) = \omega + \alpha_1 \left  \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right  + \beta_1 \ln(\sigma_{t-1}^2) + \gamma \left( \frac{\varepsilon_{t-1}}{\sigma_{t-1}} \right)$								
Mean equation	c	$\mu$	$\alpha$	$\beta$	$\gamma$	$\delta$	$\lambda$	AIC
$r_t^{\text{mspr}} = c + \varepsilon_t$	0.000	-0.37*	0.12*	0.97*	-0.07*			-7.186
	(0.30)	(-1.97)	(1.99)	(60.92)	(-1.80)			
$r_t^{\text{mspr}} = c + \delta\sigma_t + \varepsilon_t$	-0.002*	-0.53*	0.09*	0.95*	-0.08*	51.9*		-7.208
	(-3.02)	(-3.43)	(2.03)	(63.14)	(-2.34)	(3.45)		
$r_t^{\text{mspr}} = c + \delta\sigma_t + \lambda v_{t-1}^2 + \varepsilon_t$	-0.002*	-0.56*	0.08*	0.95*	-0.08*	52.01*	0.0005	-7.212
	(-1.98)	(-3.52)	(1.97)	(60.56)	(-2.47)	(3.31)	(0.27)	

T-statistic in Parentheses. Significance: \* p<0.05  
 $r_t^{\text{mspr}}$  is weekly excess return on the market portfolio with Mosprime 1 month as corresponding risk-free rate, normalized to one day

The relationship between ambiguity measure for market stock returns and excess return on the market portfolio is less clear: we did not find statistically significant effect of ambiguity on excess return using either GARCH(1,1) or EGARCH(1,1) for conditional volatility. Moreover, we did not find any significant relationship for ambiguity aversion when we were testing basic OLS relationship with MVAR as a proxy for risk and Izhakian's measure of ambiguity. We can interpret the insignificance of the coefficient as the evidence for ambiguity-neutral behavior of the Russian investors. Thus, we argue that the insignificance of the coefficient leads to ambiguity-neutrality. This is equivalent to the following statement: if investors are not ambiguity-neutral (for example, ambiguity averse), then the coefficient should be significant in order to lead to the rise in the expected return. However, it can be true under several crucial assumptions. First of all, we assume that there is no option outside the market for investors. Second, we assume that the market is efficient, therefore, it incorporates all available information in prices (1970). Let us suppose that ambiguity rises and investors are ambiguity averse. This information should be incorporated into prices, thus the prices change with the increased ambiguity. As investors do not have the option to rebalance their portfolio (their portfolio itself consists only from the market portfolio), then they are willing to leave the market. The expected return on the market portfolio should increase in order to induce investors to stay in the market.

It can be the intuitive explanation why increased ambiguity leads to increase in the expected return on market when investors are ambiguity-averse. Therefore, we can argue that the insignificance of the coefficient can be the evidence of ambiguity-neutral behavior of the Russian investors. Ambiguity-neutral investors ignore ambiguity while making financial decisions. Ivanov (2011) in his experimental study discovered that agents are mainly ambiguity-neutral, which supports our results.

## **5. Conclusion**

The ambiguity issue has been analyzed by various researchers in theoretical and experimental works for over half a century. However, the academic literature lacks the empirical analysis on ambiguity. Moreover, no comprehensive empirical study has been provided on the topic of ambiguity in the Russian stock market. To the best of our knowledge, our work is the first empirical analysis on the attitude of investors toward ambiguity in the Russian equity market. We concentrate on the implication of ambiguity in financial- decision making and fill this gap by analyzing the objective degree of ambiguity of Russian investors. In our research, the standard assumption about the precisely known probabilities of outcome is relaxed, that is more essential for the financial world. The analysis of attitude of Russian investors toward ambiguity can be the foundation for future studies.

We use Shadow Probability theory in order to find the measure of ambiguity numerically using intraday data on MICEX index from 2009 to 2016. Then, we test the relationship between risk, ambiguity and excess return on market portfolio, performing OLS regression analysis. The risk-ambiguity model, proposed by Brenner and Izhakian is extended in different directions to validate the result. First, we prove that returns on MICEX index, which is considered as the market portfolio, are not normally distributed. Therefore, we use non-standardized t-distribution. Moreover, we combine modeling of ambiguity using Brenner and Izhakian's approach with modeling of conditional volatility using GARCH models.

First, we perform the empirical analysis not accounting for non-normal distribution of returns and its heteroscedasticity. We obtain statistically insignificant results in terms of coefficients both for ambiguity and risk. Then, we perform the test which rejects the normal distribution of returns in the Russian stock market. Therefore, we use non-standardized distribution. Empirical tests were performed both for ambiguity and the first lag of ambiguity to avoid possible endogeneity problem. However, we have not found statistical significance either for risk or ambiguity in both cases. To account for heteroscedasticity of returns, in the equation for excess return we combine both ambiguity and the modeling of conditional volatility using the GARCH approach. We also perform the test for both ambiguity in the current period and the first lag of ambiguity. These tests give similar results about the relationship between risk, ambiguity and excess return. Excess returns on market portfolio appear to have a positive, statistically significant relationship with conditional volatility following both GARCH(1,1) and EGARCH(1,1) processes. The relationship between ambiguity measure for market stock returns and excess return on the market portfolio is less evident: we have not found statistically significant effect of ambiguity on excess return using either GARCH(1,1) or EGARCH(1,1) for conditional volatility. It may be the evidence of risk-neutrality of Russian investors under assumptions of efficient market and inability of investors to rebalance their portfolio.

Thus, we conducted the first analysis of the attitude of investors to ambiguity in the Russian stock market. As for the further analysis, one might implement more sophisticated models for measuring ambiguity. Moreover, it is possible to measure ambiguity-aversion not only for stocks, but also for other types of assets including bonds, mutual funds and Exchange traded funds. Furthermore, one might attempt to investigate how the ambiguity aversion can be used to explain the Equity Premium Puzzle.

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## 6. Appendix

Table 8: Ambiguity constructed using the assumption that

Mean equation	$r_t \sim N(\mu, \sigma^2),$ GARCH						AIC
	c	$\omega(\times 10^{-6})$	$\alpha$	$\beta$	$\delta$	$\lambda$	
$r_t^{\text{mspr}} = c + \delta\sigma_t + \lambda u_{t-1}^2 + \varepsilon_t$	-0.003*	1.76*	0.038*	0.91*	0.59*	0.0025	-7.144
	(-1.73)	(3.29)	(2.09)	(47.00)	(2.28)	(1.04)	
$r_t^{\text{gko}} = c + \delta\sigma_t + \lambda u_{t-1}^2 + \varepsilon_t$	-0.003*	1.77*	0.038*	0.91*	0.61*	0.002	-7.144
	(-1.75)	(3.30)	(2.09)	(47.00)	(2.32)	(1.06)	
$r_t^{\text{gko}} = c + \delta\sigma_t + \lambda u_{0t-1}^2 + \varepsilon_t$	-0.003*	1.77*	0.038*	0.91*	0.61*	0.002	-7.144
	(-1.75)	(3.30)	(2.09)	(47.00)	(2.32)	(1.04)	

T-statistic in parentheses.

Table 9: Ambiguity constructed using the assumption that

Mean equation	$r_t \sim N(\mu, \sigma^2),$ EGARCH						AIC
	c	$\mu$	$\alpha$	$\beta$	$\gamma$	$\delta$	
$r_t^{\text{mspr}} = c + \delta\sigma_t + \lambda u_{t-1}^2 + \varepsilon_t$	-0.002*	-0.43*	0.07*	0.96*	-0.08*	41.27*	-7.159
	(-1.765)	(-4.61)	(1.96)	(104.4)	(-3.22)	(2.94)	(0.57)
$r_t^{\text{gko}} = c + \delta\sigma_t + \lambda u_{t-1}^2 + \varepsilon_t$	-0.001*	-0.44*	0.07*	0.96*	-0.08*	42.67*	-7.159
	(-1.74)	(-4.62)	(2.04)	(103.0)	(-3.26)	(3.02)	(0.63)
$r_t^{\text{gko}} = c + \delta\sigma_t + \lambda u_{0t-1}^2 + \varepsilon_t$	-0.001*	-0.44*	0.08*	0.96*	-0.08*	42.67*	-7.160
	(-1.75)	(-4.62)	(2.04)	(102.9)	(-3.26)	(3.03)	(0.62)

T-statistic in parentheses.

$r_{\text{mspr}}$  is weekly excess return on the market portfolio with Mosprime 1 month as corresponding risk-free rate, normalized to one day;  $r_{\text{gko}}$  is weekly excess return on the market portfolio with GKO-OFZ as corresponding risk-free rate, normalized to one day;  $u_t^2$  is daily ambiguity measure calculated using Mosprime 1 month/GKO-OFZ as risk-free reference point;  $u_t^2$  is daily ambiguity measure calculated using zero as reference point.

**Есаулов, Д. М., Давидович, М. С.**

Тестирование неприятия неопределенности на российском рынке ценных бумаг : препринт WP1/2016/04 [Электронный ресурс] / Д. М. Есаулов, М. С. Давидович ; Нац. исслед. ун-т «Высшая школа экономики». – Электрон. текст. дан. (500 Кб). – М. : Изд. дом Высшей школы экономики, 2016. – (Серия WP1 «Институциональные проблемы экономики»). – 26 с. (на англ. яз.).

Данная работа является первым полноценным эмпирическим исследованием, посвященным теме неопределенности и риска на российском рынке ценных бумаг. Современная портфельная теория предполагает линейную взаимосвязь между риском и доходностью, при этом предполагается, что вероятности исхода являются известными. Финансовая литература построена на фундаментальном соотношении между риском и доходностью. Несмотря на это, свидетельство о знаке и значимости риска подтверждается не всеми эмпирическими работами. Некоторые финансовые статьи утверждают, что одного риска недостаточно для нахождения ожидаемой доходности рыночного портфеля. Это может происходить, потому что вероятности исходов считаются известными, в то время как в действительности они не известны, что является свидетельством неопределенности на рынке ценных бумаг.

В данной статье предполагается, что вероятности исходов не являются известными величинами. Эта работа тестирует взаимосвязь между ожидаемой доходностью, неопределенностью и риском, используя внутрисдневные данные по индексу ММВБ в период с 2009 по 2016 г. Мера неопределенности выводится из модели, предложенной Бреннером и Ицхакианом (2011). В данной работе эта модель модифицируется для учета ненормальности распределения доходности. Условная волатильность, оцениваемая моделью GARCH, оказывает значимое и положительное влияние на доходность рыночного портфеля. Коэффициент неопределенности не является статистически значимым, что может свидетельствовать о нейтральном отношении инвесторов к неопределенности.

Ключевые слова: неопределенность, неприятие неопределенности, премия по акциям, условная гетероскедастичность

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*Препринт WP1/2016/04*  
*Серия WP1*  
*Институциональные проблемы российской экономики*

Есаулов Даниил Михайлович, Давидович Мария Сергеевна

**Тестирование неприятия неопределенности  
на российском рынке ценных бумаг**  
*(на английском языке)*

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