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Optimal Reporting Systems in Bank Runs*

Gaoqing Zhang     Ronghuo Zheng

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Abstract

We study the role of reporting systems in the context of bank runs. In our model, a bank receives an early but imprecise estimate of its investment performance, and its financial reporting system generates a report. We find that, from a financial-stability standpoint, the optimal reporting system requires full disclosure when the bank’s early estimate is below a certain threshold, but no disclosure otherwise. Importantly, such optimal reporting threshold should be tailored to the bank’s exposure to bank-run risk. In particular, the threshold is non-monotonic and U-shaped in the bank-run risk. We also relate our results to current accounting standards and discuss their implications for policy-making and empirical research.

Keywords: Reporting systems; banks; bank runs; financial stability.

JEL Codes: G21, G28, M41, M48.

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1 Introduction

Fair value accounting rules are often blamed for contributing to financial instability. One most notable instance is during the 2007-2009 financial crisis, many critics attributed fair value accounting as a cause for the financial meltdown and the panic-based runs on banks, and more broadly, financial institutions.\(^1\) However, it is probably less recognized that most of banks' assets are not reported at fair value in its purest form; instead, financial reporting of these assets often entails asymmetric recognition of unrealized fair value gains and losses (Laux and Leuz, 2010). For example, loans and leases, which account for a majority of banks' assets, are initially measured at historical cost and later subject to impairment testing for fair value losses: “An investment is impaired if its fair value is less than its cost.” (Financial Accounting Standards Board, ASC 320-10-35-21). When the fair value exceeds the cost, however, recognition of fair value gains is usually prohibited.

This paper seeks to understand the role of reporting systems in maintaining financial stability. In a canonical bank-run model (Morris and Shin, 2000), we show that the optimal reporting system resembles some aspects of the current accounting standards as it requires full disclosure of all unfavorable news, but disclosure of favorable news only below some threshold. This result thus implies that, in contrast to the criticism of fair value measurement for destabilizing financial markets, the implementation of fair value measurement that entails asymmetric treatments of unrealized gains and losses may play a stabilizing role. We also characterize the optimal threshold below which banks should report, which generates implications for how reporting systems should be designed from a financial-stability standpoint.

\(^1\)As a perhaps more extreme example, Steve Forbes, chairman of Forbes Media, alleged that mark-to-market accounting was “the principal reason” that the U.S. financial system melted down. See https://hbr.org/2009/11/is-it-fair-to-blame-fair-value-accounting-for-the-financial-crisis. The 2007-2009 financial turmoil also reveals ample evidence on the fragility of banks to risk of runs, especially in the “shadow banking” markets that are not covered within traditional government safety nets such as the federal deposit insurance (e.g., Pozsar et al., 2010). More recently, amid the 2023 regional banking crisis, financial reporting of unrealized losses has drawn considerable attention for its role in the runs on regional banks, including Silicon Valley Bank (e.g., Jiang et al., 2023).
We consider a model that consists of a representative bank that finances its project (e.g., a loan) by attracting investments from a group of investors. The bank is vulnerable to runs because its investors can withdraw their investments even before the return from the institution’s project is realized. Such early withdrawals are costly as they force the bank to liquidate its project prematurely to meet the withdrawal requests, thus resulting in liquidation losses. Prior to the investors’ withdrawal decisions, the bank observes an early but imprecise estimate of its project return, and its financial reporting system produces a report about the project return. We do not restrict the choice of the reporting system but instead focus on the ex ante optimal choice of a financial reporting system (e.g., by a bank or its board of directors whose incentives are aligned with its investors’) that maximizes the total investors’ surplus. Upon receiving the report, investors then use it in conjunction with their own private information in determining whether to withdraw from the bank.

We first examine a benchmark in which the bank discloses its early estimate fully. While full disclosure eliminates inefficient continuation of insolvent banks, this does not achieve full efficiency as it sustains inefficient runs on solvent banks. These runs are panic-based, triggered by investors’ strategic uncertainty about each other’s decision to withdraw.

Our analysis next characterizes the optimal reporting system, which exhibits two salient features. First, the optimal reporting system mandates full disclosure only when the bank’s early estimate of its investment outcome falls below a threshold; otherwise, the bank should not recognize the estimate in its report. This reporting system is consistent with current accounting standards that treat unrealized gains and losses asymmetrically and thus lends some support to the value of these standards in maintaining financial stability. Second, the optimal reporting system ensures no runs on the set of banks that do not disclose, even though continuing those banks may sometimes be inefficient. Note that this feature of the optimal system stands in stark contrast to that of full disclosure. While full disclosure avoids all inefficient continuations at the expense of sustaining inefficient runs, the optimal system aims at achieving a better balance between the two types of inefficiency, tolerating some
inefficient continuations in exchange for fewer inefficient runs.

With the optimal reporting system characterized, we further derive and discuss some additional implications for designing financial reporting systems from a financial-stability standpoint. An implication from our analysis is that, to maintain financial stability, a bank should tailor its reporting threshold to its vulnerability to the bank-run risk as well as its relative exposure to systemic shocks. Interestingly, while the optimal threshold increases monotonically in the size of systemic shocks (i.e., banks should report more often when they have greater exposure to systemic shocks), the threshold is actually non-monotonic and U-shaped in the bank-run risk. The intuition is as follows. When the bank-run risk increases from a relatively low level, it is optimal to lower the reporting threshold and expand the no-disclosure set to mitigate the increase in inefficient runs. However, when the bank-run risk rises above a certain cutoff, the reporting threshold should be raised to induce more disclosure. This adjustment is optimal under the circumstances of heightened bank-run risk as it improves investors’ assessment of the quality of the banks in the no-disclosure set such that these banks continue to suffer no runs.

Lastly, to the extent that under the optimal reporting system, the bank should report all unrealized losses but not necessarily all unrealized gains, this optimal system is a compromise between a fair value rule (full disclosure of all unrealized gains and losses) and an impairment rule (disclosure of unrealized losses but not gains). Both the fair value and impairment rules are commonly employed in the accounting practice of reporting banks’ assets. Therefore, to broaden the implications of our model, we consider and compare the economic consequences of the fair value and the impairment rules. Our analysis suggests the impairment rule dominates the fair value rule in maintaining stability when the bank-run risk is relatively high and banks are more exposed to their idiosyncratic shocks than systemic shocks.
1.1 Related Literature

This paper is related to the accounting and economics literature on the role of information disclosure in bank runs. Banking institutions, such as commercial banks, investment banks, and investment funds, are exposed to the risk of runs due to the liquidity and duration mismatch between their assets and liabilities (Diamond and Dybvig, 1983). Opacity is often attributed as a key contributor to financial instability (Morgan, 2002). Toward that end, the seminal paper by Morris and Shin (2000) develops a tractable framework of bank runs suitable for examining the role of information disclosure in bank runs, which has then been widely employed in the literature. Prior papers have mostly focused on examining specific information structure in a Morris-Shin setting, and thus do not directly address the issue of \textit{ex ante} optimal reporting systems, which is a key focus of our paper. For instance, Bouvard, Chaigneau, and Motta (2015) consider a setting with a continuum of banks, whose fundamental consists of a uniformly-distributed aggregate component and a binary idiosyncratic component. A regulator chooses to either disclose the idiosyncratic components of all banks fully or none. Their main result is that the regulator should disclose if the aggregate shock is sufficiently bad. Liang and Zhang (2019) employ a structure of correlated private information to represent the classic accounting trade-off of objectivity and accuracy and study the value of accounting objectivity in mitigating inefficient runs. Our paper contributes to this literature by characterizing the optimal reporting system within the Morris-Shin framework. The optimal system is consistent with the accounting practice that treats unrealized losses and gains asymmetrically and thus lends some support to such practice on the grounds of financial stability. Our analysis also sheds light on how financial reporting systems should be designed from a stability standpoint and, in particular, how the optimal design choice should be tailored to various characteristics of banks.

This paper is also related to the literature on the information environment of banks. This literature is vast and thoroughly reviewed in three surveys by Beatty and Liao (2014), Goldstein and Sapra (2014), and Acharya and Ryan (2016). Some recent papers have
focused on examining the use of accounting information in prudential regulations of banks (e.g., Corona, Nan, and Zhang, 2015, 2019a, 2019b; Bertomeu, Mahieux, and Sapra, 2023; Mahieux, Sapra, and Zhang, 2023). Perhaps more related to our paper, Goldstein and Leitner (2018) consider a model in which disclosure may hinder risk-sharing among banks but under other circumstances, some partial disclosure is necessary for risk sharing to occur. In contrast to our paper, the optimal disclosure rule in Goldstein and Leitner (2018) is inconsistent with the single-cutoff reporting system that we have derived as the one in Goldstein and Leitner can take a more complicated form (e.g., multiple cutoffs or non-monotone rules).

More broadly, this paper is related to the extensive disclosure literature. Due to the size of this literature, we refer interested readers to several recent reviews by Beyer et al. (2010), Ewert and Wagenhofer (2012), Stocken (2013), and Armstrong et al. (2016). In particular, recent accounting literature studies the issue of ex ante information design applying the framework of Bayesian persuasion following Kamenica and Gentzkow (2011) (e.g., Jiang and Yang, 2017; Michaeli, 2017; Huang, 2019; Friedman, Hughes, and Michaeli, 2020, 2022; Dordzhieva, Laux, and Zheng, 2022; Göx and Michaeli, 2023; Laux and Zheng, 2023). In addition, to the extent that the optimal reporting system in our model requires asymmetric disclosure of favorable and unfavorable news, our paper is related to the accounting literature on conservatism (e.g., Gigler et al., 2009; Göx and Wagenhofer, 2009; Gao, 2013; Caskey and Laux, 2017). Ewert and Wagenhofer (2012) also survey some of this literature. However, this literature has not explored the role of conservative reporting in bank runs.

Mostly related to our paper, Göx and Wagenhofer (2009) examine the optimal accounting policy in a setting in which a financially-constrained firm raises debts from a lender. The key friction in their setting is that the firm’s management may lack incentive to exert effort, which creates a need for the firm to pledge its assets to the lender to mitigate the moral hazard problem.2 Accordingly, the measurement rule of the asset value plays an essential

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2 More recently, Bertomeu and Cheynel (2015) and Jiang and Yang (2021) also study the optimal measurement rules considering frictions of both inefficient continuation and liquidation in a setting of corporate finance related to Göx and Wagenhofer (2009).
role in determining whether the firm’s project can be successfully financed. Related to our paper, Göx and Wagenhofer also find an impairment rule that reports news below a cutoff is optimal in the debt-financing setting. Pooling firms with high asset values increases the probability of financing and efficient investment. Nonetheless, there are several key differences between our paper and Göx and Wagenhofer. First, the results in our paper are driven by the friction of panic-based runs and inefficient continuations whereas the main friction in Göx and Wagenhofer is the moral hazard problem of the manager seeking debt financing. Accordingly, in our model, the optimal reporting system depends on the level of bank-run risk, banks’ exposure to systemic risk, etc., which Göx and Wagenhofer do not study. Second, the optimal impairment rule in Göx and Wagenhofer only requires disclosing whether the news is above or below the impairment threshold, rather than its exact value. This is because, a firm reporting below the impairment threshold does not get financed in Göx and Wagenhofer so it does not matter whether the exact news is revealed. In contrast, revealing the exact realization of the news below the reporting threshold is necessary and strictly optimal in our setting as it facilitates efficient liquidation *ex post.*

In another closely related paper, Gao and Jiang (2018) examine the role of banks’ *ex post* reporting discretion in affecting runs. Similar to our paper, Gao and Jiang show that the optimal degree of reporting discretion depends on the trade-off between inefficient runs and inefficient continuation. Allowing more reporting discretion mitigates inefficient runs at the expense of inefficient continuation. However, there are some important differences between the two studies. First, while Gao and Jiang consider banks’ *ex post* reporting discretion and abstract away from the *ex ante* choice of optimal reporting systems, we focus on the optimal design of financial reporting systems from an *ex ante* perspective. Second, the exact implications of the two studies differ. In particular, Gao and Jiang show that the optimal degree of reporting discretion increases monotonically in the bank-run risk whereas a key insight from our study is that the relationship between the optimal reporting threshold and the bank-run risk is non-monotone.
Figure 1: Timeline of the model.

The rest of the paper proceeds as follows. Section 2 describes our model. Section 3 analyzes the model. Section 4 discusses some implications and extensions of the model and Section 5 concludes. An Appendix contains the proofs of our results.

2 The Model

We examine an environment that consists of four dates \( t = 0, 1, 2, 3 \), a representative bank, and a continuum of investors.\(^3\) Figure 1 summarizes the timing of events.

At \( t = 0 \), the bank is endowed with an investment project that yields a stochastic gross rate of return \( 1 + \tilde{r} \) realized at \( t = 3 \). Similar to Morris and Shin (2000), Gao and Jiang (2018), and Liang and Zhang (2019), the bank’s project is illiquid and the net rate of return obtainable at \( t = 3 \) is decreasing in the proportion of investments withdrawn at \( t = 2 \), as denoted by \( l \in [0, 1] \). Specifically, we assume that at \( t = 3 \), the net rate of return is \( 1 + \tilde{r} - \delta l \). The term \( \delta l \) represents the cost of liquidating the illiquid project (or premature liquidation of assets) to meet the investors’ withdrawals. The parameter \( \delta > 0 \) captures the extent to which the withdrawals impair the investment return.\(^4\) In other words, for 1 unit

\(^3\)We do not model the financial structure of the bank and its investors can be broadly interpreted as investors/creditors financing the bank.

\(^4\)An interpretation of \( \delta \) can be the degree of liquidity mismatch between banks’ assets and liabilities, which is then related to the vulnerability of banks to the risk of runs and financial instability. For example, a bank with relatively more liquid assets (e.g., short-term marketable securities, short-term notes) is likely to have a smaller \( \delta \) as liquidating these assets to meet investors’ withdrawal requests causes fewer disruptions and is less costly. Conversely, a bank with relatively more illiquid assets (e.g., subprime mortgages, long-term securities and notes) may have a larger \( \delta \) as liquidating these assets often entails significant liquidity discounts and can severely hurt the bank’s terminal return. For another example, as observed in the 2023 regional banking crisis, larger banks with access to stable funding sources and under stronger coverage of the federal safety nets face smaller risk of runs (e.g., JP Morgan Chase), relative to smaller regional banks that rely on less stable uninsured funding (e.g., Silicon Valley Bank).
of investment at \( t = 0 \) and early withdrawal \( l \) at \( t = 2 \), the investment yields \( l \) unit at \( t = 2 \) and \((1 - l)(1 + \tilde{r} - \delta l)\) at \( t = 3 \).

The bank finances the project by attracting investments from a group of investors, with unit mass indexed by the unit interval \([0, 1]\), each of whom contributes 1 unit of the consumption good.\(^5\) All investors are risk neutral and earn a payoff of \( u_i = c_{i2} + c_{i3} \), where \( c_{i2} \) and \( c_{i3} \) denote investor \( i \)’s consumption at \( t = 2 \) and at \( t = 3 \) respectively. The bank invests all the investments attracted in its project.\(^6\)

The investment return \( \tilde{r} \) depends on two independent random variables \( \tilde{\lambda} \) and \( \tilde{\theta} \) such that

\[
\tilde{r} = \tilde{\lambda} + \tilde{\theta}. \tag{1}
\]

\( \tilde{\lambda} \) has a general distribution \( F(.) \) and a density \( f(.) \) with full support on \([ -\bar{\lambda}, \bar{\lambda} ]\). \( \tilde{\theta} \) has a general distribution \( G(.) \) and a density \( g(.) \) with full support on \([ -\bar{\theta}, \bar{\theta} ]\). Without loss of generality, we normalize the mean of \( \theta \) to 0. We assume that the density of \( \theta \) does not increase too fast in \( \theta \), that is,

\[
\frac{\partial \ln g(\theta)}{\partial \theta} < \frac{2}{\delta}. \tag{2}
\]

For example, the condition is always met if \( \theta \) follows a uniform distribution, the density of which is a constant. It is also met if \( \theta \) follows a truncated normal distribution and the variance of \( \theta \) is not too small, that is, the density of \( \theta \) is relatively flat.\(^7\) As it will be clear later, the condition regarding the distribution of \( \theta \) helps to rule out an uninteresting scenario in which it is optimal to trigger more runs on stronger banks in order to mitigate runs on weaker banks. For tractability and ease of exposition, we will use the uniform distribution of \( \theta \) as a continuing example.

\(^5\)Note that investors always choose to fund the bank at \( t = 0 \) because, as we will discuss later, they are allowed to redeem their original investments at the face value at \( t = 2 \).

\(^6\)Alternatively, Morris and Shin (2000) assume that the gross return \( e^r \) and investors have log utilities \( u_i = \log (c_{i2} + c_{i3}) \). Adopting either of the two structure yields the same optimal reporting system.

\(^7\)More precisely, if \( \tilde{\theta} \) is normally distributed with mean 0, variance \( \sigma^2 \) and truncated in the interval \([ -\bar{\theta}, \bar{\theta} ]\),

\[
\frac{\partial \ln g(\theta)}{\partial \theta} < \frac{2}{\delta} \text{ for all } \theta \in [ -\bar{\theta}, \bar{\theta} ] \text{ if and only if } \sigma > \sqrt{\frac{30}{2}}.
\]
The bank observes $\lambda$ at $t = 1$, which represents some early, albeit imperfect, information about the investment return before the return is realized. The random variable $\tilde{\theta}$ thus captures the residual uncertainty regarding the investment return after the bank learns $\lambda$. The two components of $\tilde{r}$ can be interpreted in different ways. For instance, the first component $\tilde{\lambda}$ represents the bank’s proprietary information about the expected return of the project (i.e., $E[\tilde{r} | \lambda] = \lambda$), which the bank acquires in the due diligence process of investigating its project and can be communicated to outside investors in financial reporting. The actual realization of the project return $r$ depends on not only the expected return $\tilde{\lambda}$ but also some market-wide shocks $\tilde{\theta}$ outside of the bank’s control, e.g., business cycles, government policies, etc. For another example, to the extent that banks are more likely to have inside information about the idiosyncratic component in their returns, $\tilde{\lambda}$ represents the idiosyncratic shock in the return, whereas the orthogonal component $\tilde{\theta}$ represents the systemic shock.\footnote{In Section 4.6, we consider an extension of our model with multiple banks, where $\tilde{\lambda}$ indeed captures the idiosyncratic component in each bank’s return, and $\tilde{\theta}$ represents the common systemic shock.}

In our subsequent discussions, we adopt these two interpretations interchangeably as long as no confusion arises. Under these interpretations, the bounds $\{\bar{\lambda}, \bar{\theta}\}$ on $\lambda$ and $\theta$ then measure the relative magnitudes of the bank’s known information/idiosyncratic shocks to the residual uncertainty/systemic shocks.\footnote{A bank’s exposure to idiosyncratic/systemic shocks is likely to depend on its characteristics. For instance, a larger national bank’s return is more likely driven by economy-wide systemic shocks such as business cycles than that of a smaller regional bank. For another example, a bank investing in a well diversified portfolio of assets is more subject to systemic shocks than one that holds specific assets. Lastly, there is empirical evidence that investment returns are more subject to systemic (idiosyncratic) shocks in economic downturns (expansions) (e.g., Brockman, Liebenberg, and Schutte, 2010).}

More generally, this dichotomy of $\theta$ and $\lambda$ in $r$ captures the property of financial reporting systems in which not all information is amenable to inclusion in financial reports (e.g., Ijiri, 1975). Accordingly, to capture that $\theta$ represents the portion of information that is not amenable to be included in financial reports, we assume that $\lambda$ and $\theta$ are orthogonal and independently distributed.

At $t = 0$, an \textit{ex ante} reporting system is prescribed and governs how the bank issues a report $x$ about $\lambda$ at $t = 1$. In the baseline model, we do not place any restrictions on how
the reporting system is set, but rather derive the optimal reporting system that maximizes the expected total surplus. For instance, the reporting system may be perfectly informative about all \( \lambda \), partially informative about only some values of \( \lambda \), or not informative at all. One way to implement the optimal system is to consider the \textit{ex ante} optimal choice of financial reporting systems by the bank or the board of the bank, whose incentive is aligned with its investors’.\(^{10}\) After characterizing the optimal reporting system, we also consider and compare several accounting rules in practice to enrich the model implications, in particular, an impairment rule and a fair value rule.\(^{11}\)

At \( t = 1 \), the bank observes the early information \( \lambda \) and its financial reporting system generates a report \( x \) about \( \lambda \).\(^{12}\) Along with the bank’s report \( x \), each investor \( i \) observes a private signal \( s_i \) about the random variable \( \tilde{\theta} \):

\[
s_i = \tilde{\theta} + \tilde{\varepsilon}_i, \tag{3}\]

where \( \tilde{\varepsilon}_i \sim N\left(0, \frac{1}{\beta}\right) \).\(^{13}\) Following the global games literature (Morris and Shin, 2000; Gao and Jiang, 2018), we consider in our main analysis a limiting case in which \( \beta \) approaches infinity such that each investor observes \( \theta \) (almost) perfectly.\(^{14}\) We resort to the global games

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\(^{10}\)We thank an anonymous reviewer for suggesting us consider the \textit{ex ante} optimal choice of financial reporting systems by the bank. Note that our assumption that the bank maximizes the total investors’ surplus is commonly made in the bank-run literature, dating back to the seminal work of Diamond and Dybvig (1983). In addition, our baseline model does not consider banks’ decision to voluntarily disclose \( \lambda \), in addition to the disclosure generated by its reporting system. In Section 4.4, we show that banks have no incentive to make additional voluntary disclosure.

\(^{11}\)It is noteworthy that banks often apply some mixture of impairment and fair value rules to measure their assets, e.g., held-to-maturity (HTM) debt securities such as loans (ASC 320), where banks must evaluate whether to report losses when the fair value of their HTM assets is below the amortization cost. However, banks are usually not allowed to report gains when the fair value exceeds the amortization cost.

\(^{12}\)In cases that the bank does not report \( \lambda \), investors may have incentives to acquire private information about \( \lambda \). In Section 4.5, we consider investors’ private information acquisition decisions and show that such learning should be limited on grounds of maintaining financial stability.

\(^{13}\)It is plausible that the information events of observing \( \lambda \) and \( s_i \) are sequential so that investors do not need to wait for the arrivals of both pieces of information to make their withdrawal decisions, especially given that the literature has shown that the timing of information events can be consequential in various settings (e.g., Guttmann, Kremer, and Skrzypacz, 2014; Friedman, Hughes, and Michaeli, 2020; Menon, 2020; Menon and Nan, 2023). We show that the sequence does not matter in our setting. This is because, investors can always postpone their withdrawal decisions until they observe both signals, which would at least lead to the same payoff had investors withdrawn earlier.

\(^{14}\)Consistent with the results in the global games literature (e.g., Morris and Shin, 2000; Liang and Zhang,
technique to obtain a unique equilibrium outcome in bank-run models that would otherwise feature multiple equilibria, and the equilibrium uniqueness is instrumental in deriving the optimal reporting system.\textsuperscript{15} At $t = 2$, investor $i$ decides whether to withdraw her investment given her information set $\{x, s_i\}$. At $t = 3$, the net rate of return is realized and distributed.

Several assumptions of our model merits some discussions. First, we assume that investors are equipped with sufficiently precise private information. Two examples of bank runs may help to motivate this assumption. During the 2007-2009 financial crisis, investors in shadow banking markets such as markets of repo (Brunnermeier, 2009), asset-backed commercial papers (Acharya, Schnabl, and Suarez, 2013), money market mutual funds (Schmidt, Timmermann, and Wermers, 2016), among others (Pozsar et al., 2010), refuse to rollover their short-term lending or withdraw their short-term claims, which leads to severe liquidity dry-ups and runs on banking institutions. Interestingly, investors participating in such runs are primarily constituted of sophisticated institutional investors, such as mutual funds, investment banks, and hedge funds (Shin, 2009; Pozsar et al., 2010), who are likely to be equipped with private knowledge about the banks they have funded (Pozsar et al., 2010). More recently, there is a wave of runs on regional banks such as Silicon Valley Bank, Signature Bank, etc. These runs are mostly triggered by large uninsured depositors, who are likely to possess relatively precise private information (e.g., Jiang et al., 2023).

Alternatively, our model can also be recasted as capturing the coordination among creditors in rolling over their short-term debts to (non-financial) firms. In fact, Morris and Shin (2004) have employed a similar structure to ours to model creditors’ rollover decisions, where they also assume that creditors possess private information about the firm. Accordingly, our analysis may generate implications for designing reporting systems in debt-

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\textsuperscript{15}The global games technique was first introduced by Carlsson and van Damme (1993) and has been widely used in coordination games that feature multiple equilibria, including bank-run models, to obtain uniqueness. The equilibrium selection made by the global games approach is supported by evidence in numerous experimental studies (Anctil et al., 2004; Heinemann, Nagel, and Ockenfels, 2004; Cabrales, Nagel, and Armenter, 2007; Anctil et al., 2010).
financing settings that feature coordination as an important economic tension.\footnote{We thank an anonymous reviewer for helping us connecting our setting to creditors’ rollover decisions.}

Finally, to make the problem interesting, we impose the following assumption:

**Assumption 1:** Conditional on the most favorable report \( x = \lambda \), no investors will withdraw from the bank. As it turns out, this assumption reduces into

\[
\bar{\lambda} > \bar{\theta} + \frac{\delta}{2}. \tag{4}
\]

Assumption 1 ensures that there exists a reporting system that can at least stabilize runs in some cases. Otherwise, runs can always occur regardless of the reporting system.

## 3 Analysis

### 3.1 The bank-run equilibrium

We solve the model by backward induction and derive first the investors’ withdrawal decisions at \( t = 2 \). As shown in Morris and Shin (2000), it is without loss of generality to consider only the switching strategy in which an investor chooses to withdraw if and only if her private signal \( s_i \) is below some threshold \( \hat{s} \). To derive the equilibrium threshold \( \hat{s} \), consider a marginal investor whose signal \( s_i \) is exactly equal to \( \hat{s} \). If she withdraws, she receives 1 unit of consumption goods. If she chooses not to withdraw, her expected utility is

\[
E[1 + \tilde{r} - \delta l | \hat{s}, x],
\]

which is equal to

\[
1 + E[r | x, \hat{s}] - \delta E[l | x, \hat{s}] = 1 + E[\lambda | x] + E[\theta - \delta \Phi(\sqrt{\beta}(\hat{s} - \theta)) | \hat{s}] , \tag{5}
\]

\( \Phi(\cdot) \) stands for the cumulative standard normal distribution function. To understand (5), note that the investor’s updated belief of \( \tilde{r} \) conditional upon her information set \( \{x, \hat{s}\} \) is \( E[r | x, \hat{s}] = E[\lambda | x] + E[\theta | \hat{s}] \). In addition, by the law of large numbers, the marginal investor’s expectation of the portion of investors who withdraw \( E[l | x, \hat{s}] \) is equal to the probability
that a particular investor \( j \) withdraws. Because investor \( j \) also follows the same switching strategy, she withdraws if and only if her signal \( s_j \leq \hat{s} \), i.e.,

\[
\mathbb{E}[l|x, \hat{s}] = \mathbb{E} [\Pr(s_j \leq \hat{s}|\theta)|\hat{s}] = \mathbb{E} \left[ \Phi \left( \sqrt{\beta} (\hat{s} - \theta) \right) \right] \tag{6}
\]

In equilibrium, the investor who observes a \( s_i \) equal to \( \hat{s} \) is indifferent between staying and withdrawing. This in turn gives:

\[
\mathbb{E} [\lambda|x] + \mathbb{E} \left[ \theta - \delta \Phi \left( \sqrt{\beta} (\hat{s} - \theta) \right) \right] = 0 \tag{7}
\]

When \( \beta \) approaches infinity, the private signal \( \hat{s} \) approaches \( \theta \), and (7) gives \( \hat{s} = \frac{\delta}{2} - \mathbb{E} [\lambda|x] \). Therefore, an investor withdraws if and only if

\[
s_i = \theta \leq \hat{s} = \frac{\delta}{2} - \mathbb{E} [\lambda|x]. \tag{8}
\]

We summarize the investors’ equilibrium decisions in the following lemma.

**Lemma 1** Investors withdraw from the bank if and only if

\[
\mathbb{E} [\lambda|x] + \theta \leq \frac{\delta}{2}.
\]

An important message from Lemma 1 is that the investors’ withdrawal decisions *in equilibrium* may not necessarily be socially optimal. To elaborate, we first derive the socially optimal withdrawal decision that maximizes the total surplus. Given that the bank always breaks even, the total surplus is equal to the sum of all investors’ utilities, \( \int_0^1 u_i \, di \). The total surplus is \( \int_0^1 1 \, di = 1 \) if all investors withdraw, whereas it is \( \int_0^1 (1 + \lambda + \theta) \, di = 1 + \lambda + \theta \) if all investors stay. It is therefore socially optimal to withdraw from the bank if and only if \( \lambda + \theta \leq 0 \). Compared to the socially optimal decisions, investors may withdraw either too much or too little. For our convenience, we define the two types of inefficiency as follows.

**Definition 1** Inefficient run occurs when \( \lambda + \theta > 0 \) and investors withdraw, whereas inefficient continuation occurs when \( \lambda + \theta < 0 \) and investors stay.
3.2 Optimal reporting system

We now turn to our main research question of, from a financial-stability perspective, how the reporting systems should be set optimally to maximize the total investors’ surplus, or equivalently, how a bank that maximizes the surplus of its investors should *ex ante* design its financial reporting systems. Lemma 1 suggests that the optimal reporting systems must jointly minimize both inefficient runs on the bank and inefficient continuations of the bank. Before we characterize the optimal reporting systems, it is instructive to examine a case of full disclosure. From an accounting standpoint, full disclosure can be interpreted as a fair value accounting rule in which the bank always reports the expected value of its project conditional on its private information, which reveals $\lambda$ perfectly. The following proposition shows that full disclosure does not always achieve full efficiency.

**Proposition 1** Under a fair value rule that leads to full disclosure, i.e., $x = \lambda$, the equilibrium is as follows:

1. if $\lambda \geq \bar{\theta} + \frac{\delta}{2}$, no investors withdraw and the equilibrium is efficient;

2. if $\lambda \in \left[\bar{\theta}, \bar{\theta} + \frac{\delta}{2}\right)$, investors withdraw when $\theta < -\lambda + \frac{\delta}{2}$ and inefficient run occurs when $\theta \in (-\bar{\theta}, -\lambda + \frac{\delta}{2})$;

3. if $\lambda \in \left[-\bar{\theta} + \frac{\delta}{2}, \bar{\theta}\right)$, investors withdraw when $\theta < -\lambda + \frac{\delta}{2}$ and inefficient run occurs when $\theta \in (-\lambda, -\lambda + \frac{\delta}{2})$;

4. if $\lambda \in \left[-\lambda, -\bar{\theta} + \frac{\delta}{2}\right)$, investors always withdraw and inefficient run occurs when $\theta \in (-\lambda, -\bar{\theta} + \frac{\delta}{2})$;

5. if $\lambda < -\bar{\theta}$, investors always withdraw and the equilibrium is efficient.

Proposition 1 highlights both the benefit and the cost of implementing full disclosure. As shown in Figure 2, while full disclosure eliminates inefficient continuation, it does so at the expense of sustaining some inefficient runs when the disclosure $\lambda$ is of some intermediate
Figure 2: An illustration of Proposition 1. \( \lambda \in [-\bar{\lambda}, \bar{\lambda}] \) represents the bank’s private information, where \( \bar{\lambda} \) represents the bound of \( \lambda \), \( \theta \in [-\bar{\theta}, \bar{\theta}] \) represents the residual uncertainty about the bank’s investment return conditional on \( \lambda \), where \( \bar{\theta} \) represents the bound of \( \theta \), and \( \delta \) represents the cost of liquidation/the risk of bank runs.

An implication of Proposition 1 is that, it may be surplus-enhancing to tolerate some inefficient continuations in exchange for fewer inefficient runs. Apparently, this move requires a departure from full disclosure. Toward that end, we characterize in Propositions 2 and 3 some properties of the partial-disclosure systems that aim at achieving a better balance between inefficient runs and inefficient continuations. Proposition 2 first provides a set of necessary conditions for the optimal reporting system.

**Proposition 2** The optimal reporting system must satisfy the following necessary conditions:
1. if \( \lambda < -\bar{\theta} \), the bank shall always report \( \lambda \) perfectly;

2. if \( \lambda > \bar{\theta} + \frac{\delta}{2} \), the bank shall always disclose the same report \( x_H \);

3. if \( \lambda \in [-\bar{\theta}, \bar{\theta} + \frac{\delta}{2}] \), the bank shall either report \( x_H \) or report \( \lambda \) perfectly.

The first two parts of Proposition 2 are intuitive. Part 1 states that it is never optimal to withhold any information when \( \lambda < -\bar{\theta} \). The reason is that, when \( \lambda < -\bar{\theta} \), the liquidation payoff of the bank’s project always exceeds the continuation payoff for any value of \( \theta \) (i.e., \( \lambda + \theta < 0 \)) so withdrawing from the bank is the socially optimal decision. From Lemma 1, disclosing that \( \lambda < -\bar{\theta} \) indeed triggers withdrawals by the investors, thus achieving efficiency. Part 2 of Proposition 2 states that it is optimal to pool all banks with \( \lambda > \bar{\theta} + \frac{\delta}{2} \) together. This is because investors will never withdraw inefficiently from the group of banks with high \( \lambda \), regardless of whether they disclose fully or are pooled. Furthermore, pooling yields an additional benefit as it allows some banks that receive a lower \( \lambda \in [-\bar{\theta}, \bar{\theta} + \frac{\delta}{2}] \) to be mixed with the group of banks with \( \lambda > \bar{\theta} + \frac{\delta}{2} \). This improves the investors’ assessment of \( \lambda \) for the pooled banks with \( \lambda \in [-\bar{\theta}, \bar{\theta} + \frac{\delta}{2}] \), which helps to avoid inefficient runs on them.

Part 3 of Proposition 2 merits more discussion. It states that the banks with \( \lambda \in [-\bar{\theta}, \bar{\theta} + \frac{\delta}{2}] \) shall either be pooled with the banks with \( \lambda > \bar{\theta} + \frac{\delta}{2} \) (via reporting the same \( x_H \)) or fully separated from others (via full disclosure), but never pooled among themselves. This result reflects a consideration to optimally mitigate runs across different values of \( \lambda \in [-\bar{\theta}, \bar{\theta} + \frac{\delta}{2}] \). To elaborate, consider a simple example with two banks each of whom learns \( \lambda = \lambda_1 \) and \( \lambda = \lambda_2 \), respectively, and \( -\bar{\theta} < \lambda_2 < \lambda_1 < \bar{\theta} + \frac{\delta}{2} \), i.e., Bank-\( \lambda_2 \) is financially weaker than Bank-\( \lambda_1 \). We compare two cases, a separating case in which the two banks issue different reports, and a pooling case in which two banks issue the same report. Figure 3 suggests that, relative to separating, pooling with the stronger Bank-\( \lambda_1 \) helps to reduce runs on the weaker Bank-\( \lambda_2 \) because, under pooling, the Bank-\( \lambda_2 \) will no longer be liquidated if \( \theta \in \left[ \frac{\delta}{2} - \frac{\lambda_1 + \lambda_2}{2}, \frac{\delta}{2} - \lambda_2 \right) \). However, pooling also triggers more runs on the stronger Bank-\( \lambda_1 \) when \( \theta \in \left( \frac{\delta}{2} - \lambda_1, \frac{\delta}{2} - \frac{\lambda_1 + \lambda_2}{2} \right] \). In other words, pooling essentially substitutes runs on weak
Figure 3: An illustration of Proposition 2. $\lambda \in [-\bar{\lambda}, \bar{\lambda}]$ represents the bank’s private information, where $\bar{\lambda}$ represents the bound of $\lambda$, $\{\lambda_1, \lambda_2\}$ represents the private information of two banks, each observing $\lambda = \lambda_1$ and $\lambda = \lambda_2$, respectively, where $\lambda_1 > \lambda_2$, $\theta \in [-\bar{\theta}, \bar{\theta}]$ represents the residual uncertainty about the bank’s investment return conditional on $\lambda$, where $\bar{\theta}$ represents the bound of $\theta$, and $\delta$ represents the cost of liquidation/the risk of bank runs. In Panel A, when $\lambda_2$ is disclosed, investors withdraw if and only if $\theta < \frac{\delta}{2} - \lambda_2$, while when the two banks are pooled, $\mathbb{E}[\lambda|x] = \frac{\lambda_1 + \lambda_2}{2}$, and investors withdraw if and only if $\theta < \frac{\delta}{2} - \frac{\lambda_1 + \lambda_2}{2}$ (Lemma 1). Therefore, the Bank-$\lambda_2$ is no longer liquidated when $\theta \in [\frac{\delta}{2} - \frac{\lambda_1 + \lambda_2}{2}, \frac{\delta}{2} - \lambda_2)$ under pooling. Similarly in Panel B, when $\lambda_1$ is disclosed, investors withdraw if and only if $\theta < \frac{\delta}{2} - \lambda_1$. The Bank-$\lambda_1$, therefore, is liquidated more often when $\theta \in [\frac{\delta}{2} - \lambda_1, \frac{\delta}{2} - \frac{\lambda_1 + \lambda_2}{2})$ under pooling.
banks with runs on strong ones. Note that under an optimal reporting system that maximizes the total surplus, one should never make such substitution because, *ceteris paribus*, it is never worth saving the less valuable weak banks (e.g., Bank-$\lambda_2$) at the expense of failing the more valuable strong banks (e.g., Bank-$\lambda_1$), as long as the mass of runs on weak banks is not too large relative to that on strong banks. For instance, consider an example that $\theta$ is uniformly distributed. In this case, the amount of runs on the weak bank reduced by pooling is the same as the amount of increases in the runs on the strong bank, i.e.,

$$\Pr \left( \theta \in \left[ \frac{\delta}{2} - \frac{\lambda_1 + \lambda_2}{2}, \frac{\delta}{2} - \lambda_2 \right] \right) = \Pr \left( \theta \in \left[ \frac{\delta}{2} - \lambda_1, \frac{\delta}{2} - \frac{\lambda_1 + \lambda_2}{2} \right] \right) = \frac{\lambda_1 - \lambda_2}{4\theta}. \quad (9)$$

It is thus never optimal to exchange the same amount of runs on the weak bank for the same amount of runs on the strong bank.\(^{17}\) Given a general distribution of $\theta$, the changes of runs on the weak bank and on the strong bank can be different, depending on the distribution of $\theta$ across different values. When the density of $\theta$ is relatively flat in $\theta$ (i.e., condition (2) holds), the magnitude of the reduced runs on the weak bank, i.e.,

$$\Pr \left( \theta \in \left[ \frac{\delta}{2} - \frac{\lambda_1 + \lambda_2}{2}, \frac{\delta}{2} - \lambda_2 \right] \right),$$

does not dramatically exceed that of the increased runs on the strong bank, i.e.,

$$\Pr \left( \theta \in \left[ \frac{\delta}{2} - \lambda_1, \frac{\delta}{2} - \frac{\lambda_1 + \lambda_2}{2} \right] \right),$$

since there is enough probability at smaller values of $\theta \in \left[ \frac{\delta}{2} - \lambda_1, \frac{\delta}{2} - \frac{\lambda_1 + \lambda_2}{2} \right)$. As long as the amount of runs on strong banks that pooling increases is not too small, it continues to be true that substituting runs on weak banks with runs on strong ones is suboptimal. This explains why the banks with $\lambda \in \left[ -\bar{\theta}, \bar{\theta} + \frac{\delta}{2} \right]$ should not be pooled among themselves.

A key message of Proposition 2 is that the optimal reporting system is a dichotomy that mandates full disclosure at some values of $\lambda$ but a report insensitive to $\lambda$ elsewhere. Relative to full disclosure, the partial-pooling system can be desirable because mixing weak banks with stronger ones helps to reduce runs on the weak banks. However, this benefit of pooling

\(^{17}\)Mathematically, the expected surplus loss from pooling is $EL \equiv \frac{1}{2\bar{\theta}} \int_{\frac{\delta}{2} - \lambda_1}^{\frac{\delta}{2} - \lambda_2} (\lambda_1 + \theta) \, d\theta$ whereas the expected gain from pooling is $EG \equiv \frac{1}{2\bar{\theta}} \int_{\frac{\delta}{2} - \lambda_1}^{\frac{\delta}{2} - \frac{\lambda_1 + \lambda_2}{2}} (\lambda_2 + \theta) \, d\theta$. Note that the surplus loss always outweighs the surplus gain, i.e., $EL - EG = \frac{(\lambda_1 - \lambda_2)^2}{8\bar{\theta}} > 0$. 

Electronic copy available at: https://ssrn.com/abstract=3740168
is limited by a downside. Intuitively, as more weak banks are added to the pooling group, it can lower investors’ assessment of the quality of the pooled banks and thus put those pooled banks at higher risk for suffering runs. Proposition 3 shows that this downside is a dominant consideration. In fact, weak banks should be added only when this does not trigger any runs on the pooled banks.

**Proposition 3** Under the optimal reporting system, investors never withdraw from the pooling group that reports \( x_H \) for any realization of \( \theta \), i.e., \( \mathbb{E} (\lambda | x_H) \geq \bar{\theta} + \frac{\delta}{2} \).

The intuition for Proposition 3 can be gleaned similarly as in part 3 of Proposition 2. Since the surplus generated from the pooled high-\( \lambda \) banks are always greater than that from the low-\( \lambda \) banks to be added, it hurts the total surplus to trade runs on the weak banks for runs on the strong ones. This no-run-on-pooled-banks requirement naturally sets a bound on the mass of weak banks that can be included in the pooling group.

The final step toward deriving the optimal reporting system is to determine which of the banks with moderate \( \lambda \in [\bar{\theta}, \bar{\theta} + \frac{\delta}{2}] \) shall be pooled with the strong banks with \( \lambda > \bar{\theta} + \frac{\delta}{2} \), while respecting the pooling constraint outlined in Proposition 3. Denote by \( \Lambda \) the set of the banks that learn \( \lambda \in [\bar{\theta}, \bar{\theta} + \frac{\delta}{2}] \) but are pooled with the banks with \( \lambda > \bar{\theta} + \frac{\delta}{2} \). For the set of banks that are included in the pooling group (i.e., \( \lambda \in \Lambda \cup (\frac{\delta}{2} + \bar{\theta}, \bar{\lambda}] \)), they never experience runs for any realization of \( \theta \) (Proposition 3). For the set of banks that are excluded (i.e., \( \lambda \in [\bar{\theta}, \frac{\delta}{2} + \bar{\theta}] \setminus \Lambda \)), they are required to disclose fully and thus survive runs only upon favorable realizations of \( \theta > \frac{\delta}{2} - \lambda \) (part 3 of Proposition 2). A pooled bank’s project generates an expected surplus of \( 1 + \mathbb{E} [\lambda + \theta] = 1 + \lambda \) whereas a non-pooled bank’s project generates an expected surplus of

\[
W (\lambda) \equiv 1 + \int_{\frac{\delta}{2} - \lambda}^{\theta} (\lambda + \theta) g(\theta) d\theta.
\]

\(^{18}\)The notation \( \Lambda \cup [\frac{\delta}{2} + \bar{\theta}, \bar{\lambda}] \) stands for the pooling set that is the union of the set of \( \lambda \in \Lambda \) and \( \lambda \in [\frac{\delta}{2} + \bar{\theta}, \bar{\lambda}] \), whereas \( [\bar{\theta}, \frac{\delta}{2} + \bar{\theta}] \setminus \Lambda \) stands for the set of banks that learn \( \lambda \in [\bar{\theta}, \frac{\delta}{2} + \bar{\theta}] \) but \( \lambda \notin \Lambda \).
Figure 4: An illustration of Lemma 2. \( \lambda \in [-\bar{\lambda}, \bar{\lambda}] \) represents the bank’s private information, where \( \bar{\lambda} \) represents the bound of \( \lambda \), \( \delta \) represents the cost of liquidation/the risk of bank runs, and \( W(\lambda) \) represents, for a given \( \lambda \), the expected surplus when \( \lambda \) is revealed. In this numerical example with \( \bar{\lambda} = 5, \bar{\theta} = 1 \) and both \( \lambda \) and \( \theta \) being uniformly distributed, the colored lines represents \( W(\lambda) \) evaluated at \( \delta = 1, 2, 3 \), respectively, whereas the dashed 45-degree line represents \( 1 + \lambda \), that is, the net expected surplus for a given \( \lambda \) when \( \lambda \) is pooled. Observe that (1) when \( \delta = 1 \), \( 1 + \lambda \geq W(\lambda) \) if and only if \( \lambda \geq 0.5 \); (2) when \( \delta = 2 \) or 3, \( 1 + \lambda \geq W(\lambda) \) if and only if \( \lambda \geq 0 \).

The optimal \( \Lambda \) is set by solving the following program:

\[
\begin{align*}
\max_{\Lambda \subseteq [-\bar{\theta}, \frac{1}{2} + \bar{\theta}]} & \int_{\lambda \in \Lambda \cup (\frac{1}{2} + \bar{\theta}, \bar{\lambda})} (1 + \lambda) f(\lambda) \, d\lambda + \int_{\lambda \in [-\bar{\theta}, \frac{1}{2} + \bar{\theta}] \setminus \Lambda} W(\lambda) f(\lambda) \, d\lambda, \\
\text{s.t.,} & \quad \mathbb{E} \left[ \lambda \mid \lambda \in \Lambda \cup \left[ \frac{\delta}{2} + \bar{\theta}, \bar{\lambda} \right] \right] \geq \frac{\delta}{2} + \bar{\theta}.
\end{align*}
\]

The constraint of the optimization program stems from the requirement of Proposition 3 that runs on the pooling group must be avoided. The objective function of the program reflects the consequences of pooling.

More specifically, the objective function (11) represents a non-trivial trade-off in the choice of the pooling set. Including a bank in the pooling set avoids runs on the bank but entails inefficient continuation if \( \lambda + \theta < 0 \), while excluding a bank from the pooling set risks inefficient runs because, upon disclosure, investors withdraw excessively even if \( \lambda + \theta > 0 \).
A bank, therefore, should be added to the pooling set only when the expected surplus under pooling, $1 + \lambda$, exceeds that under separating, $W(\lambda)$. The following lemma establishes an intuitive result that $1 + \lambda \geq W(\lambda)$ only when the bank’s early information $\lambda$ is above some threshold (as illustrated in Figure 4).

**Lemma 2** $1 + \lambda \geq W(\lambda)$ if and only if $\lambda \geq \lambda^z \in [0, \bar{\theta} + \frac{\delta}{2})$, where $\lambda^z \in (0, \bar{\theta} + \frac{\delta}{2})$ is the unique solution to $1 + \lambda = W(\lambda)$ if and only if $-\bar{\theta} + \frac{\delta}{2} < 0$ and $\lambda^z = 0$ otherwise.

A direct implication of Lemma 2 is that, all banks with $\lambda \geq \lambda^z \in [0, \bar{\theta} + \frac{\delta}{2})$ should be in the pooling set as long as this does not trigger runs on the pooled banks (i.e., the constraint (12) does not bind when $\Lambda = [\lambda^z, \bar{\theta} + \frac{\delta}{2}]$). In particular, when $\theta$ is uniformly distributed, $\lambda^z = \max\{0, \bar{\theta} - \frac{\delta}{2}\}$. Otherwise, the pooling set must be curtailed at a higher threshold for $\lambda$ to ensure the satisfaction of (12). Combining the two cases, we summarize the full characterization of the optimal reporting system in the following proposition.

**Proposition 4** The optimal reporting system is that if $\lambda \geq \lambda^*$, the bank reports $x_H$ while if $\lambda < \lambda^*$, the bank reports $\lambda$ perfectly. There exists a unique cutoff $\hat{\delta}$, as characterized in the appendix, such that, the optimal reporting threshold $\lambda^*$ satisfies:

1. if the liquidation cost $\delta < \hat{\delta}$, $\lambda^* = \lambda^z \in [0, \bar{\theta} + \frac{\delta}{2})$, where $\lambda^z \in (0, \bar{\theta} + \frac{\delta}{2})$ is given in Lemma 2;

2. if $\delta \geq \hat{\delta}$, $\lambda^*$ is the unique solution to the equation $\mathbb{E}(\lambda|\lambda \geq \lambda^*) = \frac{\delta}{2} + \bar{\theta}$.

Proposition 4 states that the optimal reporting system mandates full disclosure only when the bank receives some early information below an optimal threshold. Upon learning sufficiently favorable news, however, the bank should not recognize any changes in the financial report. In this light, Proposition 4 lends some support to the practice of treating unrealized gains and losses asymmetrically (e.g., the lower of cost or market (LCM) accounting rules, or the treatment of held-to-maturity debt securities) on the grounds of
Figure 5: An illustration of Proposition 4 when $\delta \geq \hat{\delta}$. $\lambda \in [-\bar{\lambda}, \bar{\lambda}]$ represents the bank’s private information, where $\bar{\lambda}$ represents the bound of $\lambda$, $\theta \in [-\bar{\theta}, \bar{\theta}]$ represents the residual uncertainty about the bank’s investment return conditional on $\lambda$, where $\bar{\theta}$ represents the bound of $\theta$, $\delta$ represents the cost of liquidation/the risk of bank runs, and $\lambda^*$ represents the optimal reporting threshold, i.e., the bank reports if and only if $\lambda < \lambda^*$.

financial stability. As illustrated in Figure 5, compared to full disclosure, asymmetric disclosure rules like LCM result in higher surplus by achieving a better balance between inefficient runs and inefficient continuations. Under the optimal system, some inefficient continuations are sustained among the set of banks that are pooled but this is necessary to reduce inefficient runs.

Conversely, International Financial Reporting Standards (IFRS) 7 requires that “an entity shall disclose the fair value of that class of assets and liabilities in a way that permits it to be compared with its carrying amount.” (Para. 25-30 Fair value) Accordingly, under IFRS 7, banks effectively disclose all unrealized gains and losses (i.e., the difference between the fair value and the historical cost of their financial assets). Our analysis suggests that, from a financial-stability standpoint, while recognizing unrealized losses may help to curb
inefficient continuation, recognizing all unrealized gains (especially the sufficiently large ones with \( \lambda \geq \lambda^* \)) can be detrimental as it may sustain some inefficient runs. Accordingly, a policy implication of our study is that, to improve financial stability, standard-setters may consider prohibiting banks from reporting fair value on the financial assets whose fair value exceeds the historical cost by a wide margin.

4 Model Implications and Extensions

In this section, we derive and discuss additional implications for empirical studies and regulatory policies.

4.1 Information systems design of banks

The properties of the optimal reporting system may shed light on banks’ \textit{ex ante} design of their financial reporting systems from a financial-stability standpoint. If banks design their information systems optimally to maximize their investors’ surplus, they should prefer to obtain very precise information about their asset values when the asset values are low but imprecise information when the asset values are high. For instance, such information systems can be implemented in a two-step procedure. First, the information system generates a binary signal indicating whether the estimated asset value \( \lambda \) is above the optimal reporting threshold \( \lambda^* \). Second, if \( \lambda \geq \lambda^* \), the information system simply reports that \( \lambda \geq \lambda^* \) but not the precise value of \( \lambda \). On the other hand, if \( \lambda < \lambda^* \), the information system generates and reports the precise value of \( \lambda \). Note that under such an information system, banks always report any decline in asset values but optimally choose not to report all gains in asset values. Accordingly, from an \textit{ex ante} perspective, banks’ reported asset values will be downward biased, which stands in contrast to the common belief that banks’ reported asset values would be upward biased if they had the option to design their financial reporting systems.\(^{19}\)

\(^{19}\)We thank an anonymous reviewer for suggesting these implications.
4.2 Comparative statics of the optimal threshold

We next derive how the optimal reporting threshold $\lambda^*$ varies with model parameters to generate additional empirical and policy implications. Such analysis of comparative statics requires us to work with more specific distributions of $\lambda$ and $\theta$; accordingly, we assume that both $\lambda$ and $\theta$ are uniformly distributed throughout the remainder of the paper. We summarize the comparative statics of the optimal threshold $\lambda^*$ in Proposition 5.

**Proposition 5** When both $\lambda$ and $\theta$ are uniformly distributed,

1. the optimal threshold $\lambda^* = \max \{0, \bar{\theta} - \frac{\delta}{2}\}$ if the liquidation cost $\delta < \hat{\delta} \equiv \max\{\frac{3}{4}(\bar{\lambda} - \bar{\theta}), \bar{\lambda} - 2\bar{\theta}\}$, and $\lambda^* = \delta + 2\bar{\theta} - \bar{\lambda}$ if $\delta \geq \hat{\delta}$;

2. $\lambda^*$ decreases in $\delta$ if $\delta < \hat{\delta}$ and increases in $\delta$ if $\delta \geq \hat{\delta}$;

3. $\lambda^*$ increases in $\bar{\theta}$ and decreases in $\bar{\lambda}$.

The first part of Proposition 5 provides the expression of the optimal reporting threshold $\lambda^*$ under the uniform-distribution specification. The second part of the proposition characterizes how $\lambda^*$ varies with the liquidation cost $\delta$. Recall that as discussed in the model setup, an interpretation of $\delta$ can be the degree of liquidity mismatch between banks’ assets and liabilities, which is then related to the vulnerability of banks to the risk of runs and financial instability. Interestingly, we find a non-monotonic relationship between the optimal reporting threshold and the risk of runs/liquidity mismatch, as illustrated in Figure 6. A key implication is that when banks design their information systems, they should tailor their choices to the vulnerability to the risk of runs. In particular, they should report less when the risk of runs rises but remains not too severe; however, when the risk of runs continues to increase and passes some critical level $\hat{\delta}$, banks should report more often.

The intuition for Part 2 of Proposition 5 is as follows. When the bank is relatively more resilient to runs (i.e., $\delta < \hat{\delta}$), it is easy to satisfy (12), i.e., the no-run-on-pooled-banks requirement. Absent the constraint, all banks with $\lambda \geq \lambda^*$ should be pooled together to
minimize inefficient runs. As the run risk rises and banks face more inefficient runs, the reporting threshold $\lambda^*$ should be lowered further in order to include more banks in the no-disclosure set. However, as the bank’s exposure to runs becomes sufficiently large (i.e., $\delta \geq \hat{\delta}$), meeting (12) is no longer guaranteed. In other words, facing a significant run risk, one cannot reduce inefficient runs by simply pooling banks together. This, in turn, requires some weaker banks to disclose their $\lambda$s in order to trim the no-disclosure set. The higher the risk of runs, the harder to safeguard the pooled banks from runs, and the more disclosure that are needed.

The result regarding how the bank-run risk affects the reporting threshold may generate some interesting cross-sectional as well as time-series implications. For instance, our model predicts that banks with both low and high risk of runs may implement higher reporting thresholds (i.e., more likely to disclose favorable news) than banks with intermediate levels of bank-run risk (as illustrated in Figure 6). Similarly, our model also predicts that a spike in the risk of runs $\delta$ (e.g., the liquidity dry-up during the 2007-2009 financial crisis) may
cause banks to either reduce or increase disclosure of favorable news.\footnote{It is noteworthy that, there are significant ambiguities in linking a phenomenon as broad as the 2007-2009 liquidity crisis to model parameters. For instance, one may wonder whether the liquidity dry-up should be interpreted as either a higher risk of runs $\delta$, or simply a lower realization of the bank’s fundamentals $\theta$. We believe that the liquidity dry-up is more closely tied to an increase in $\delta$. The reason is that, in our model, a lower $\theta$ reduces banks’ terminal returns at all levels of liquidity (i.e., all levels of $l$), whereas a higher $\delta$ has greater negative impacts on banks with lower levels of liquidity (i.e., those with higher $l$). In this light, there is evidence that, amid the liquidity dry-up, banks with less liquidity got significantly greater damages than those with more liquidity (Shin, 2009), which might suggest that the liquidity dry-up is more closely tied to an increase in $\delta$.}

For another example, consider the passage of the Dodd-Frank Act, a major policy response to the 2007-2009 financial crisis. A primary goal of this act is to enhance the stability of the banking system, which should help to mitigate the risk of runs (i.e., reduce $\delta$). Proposition 5 then predicts a non-monotonic shift in banks’ reporting policies post passage of the Dodd-Frank Act: banks facing high risk of runs will implement lower reporting thresholds (disclose less) whereas banks that are more resilient to the risk of runs will implement higher reporting thresholds (disclose more).

The last part of Proposition 5 characterizes how the optimal threshold $\lambda^*$ varies with the bounds $\{\bar{\lambda}, \bar{\theta}\}$ on $\lambda$ and $\theta$, respectively. Recall that $\lambda$ can be interpreted as the bank’s private information of the expected investment return (or idiosyncratic shocks) and $\theta$ can be interpreted as the residual uncertainty conditional on the bank’s expectation or some systemic shocks outside the bank’s control. Accordingly, the bounds $\{\bar{\lambda}, \bar{\theta}\}$ then measure how idiosyncratic shocks versus systemic shocks contribute to total return volatility. For example, if $\bar{\lambda}$ is close to 0 and $\bar{\theta}$ is sufficiently large, the bank’s investment return varies primarily with systemic shocks.

We find that banks should report more often (i.e., $\lambda^*$ increases) when systematic shocks contribute more to total return volatility (i.e., a higher $\bar{\theta}$) and report less when idiosyncratic shocks contribute more to total return volatility (i.e., a higher $\bar{\lambda}$). The intuition is as follows. Under the optimal reporting system, moderately weak banks with $\lambda \geq \lambda^*$ are pooled with strong banks to mitigate inefficient runs. Higher systematic and idiosyncratic volatility has opposing effects on investors’ withdrawal incentives. Higher systematic return volatility
makes inefficient withdrawal decisions more likely (i.e., \( \Pr(\theta < \frac{\theta}{2} - \lambda) \) is increasing in \( \bar{\theta} \)), which in turn requires less pooling to increase expected returns of the pooled set. Higher idiosyncratic return volatility increases the expected returns for any pooled set (i.e., \( \mathbb{E}[\lambda|\lambda > \lambda^*] \) is increasing in \( \bar{\lambda} \)) and hence, lowers investors’ incentive to withdraw from the pooled set. This, in turn, allows more banks to be pooled together to improve efficiency.

### 4.3 Fair value v.s. impairment rules

In our baseline model, we consider the optimal reporting system that maximizes the total investors’ surplus. Under the optimal system, the bank should report all unrealized losses but not necessarily all unrealized gains. In some sense, this optimal system is a compromise between a fair value rule (full disclosure of all unrealized gains and losses) and an impairment rule (disclosure of unrealized losses but not gains). In practice, both fair value and impairment rules are prevalent in accounting standards and there are debates on which is preferred from a financial-stability point (e.g., Laux and Leuz, 2010; Goldstein and Sapra, 2014; Khan, 2019). Therefore, to generate additional policy implications, we now compare the economic consequences of the fair value and the impairment rules in our model.

We interpret the case of full disclosure (Proposition 1) as a fair value rule, and the impairment rule as one that reports only unrealized losses. Since \( \lambda \) is uniformly distributed with \( \mathbb{E}[\lambda] = 0 \), the impairment rule requires the bank to report \( \lambda \) only when \( \lambda < 0 \).

\[ \text{(21)} \]

We first summarize the equilibrium outcome under the impairment rule in the following lemma.

**Lemma 3** Under the impairment rule that reports \( \lambda \) only when \( \lambda < 0 \), the equilibrium outcome is the same as the full disclosure case in Proposition 1 if the bank observes and reports \( \lambda < 0 \). If the bank observes \( \lambda \geq 0 \) and reports \( x_H \), the equilibrium is as follows:

\[ \text{(28)} \]

\[ \text{In practice, an impairment rule requires disclosure of the true asset value if it is below the cost but reports the asset at its cost otherwise. Our modeling of the impairment rule can be related to the fair value and the cost of assets as follows. When } \lambda \text{ is uniformly distributed, the bank’s project yields an expected return of } 1 + \mathbb{E}[\tilde{r}] = 1, \text{ which equals the cost of investment } 1. \text{ Therefore, at the time of investment, the bank reports the cost (i.e., } \lambda = 0). \text{ Upon observing } \lambda, \text{ bank recognizes an impairment and reports } \lambda \text{ if } \lambda < 0 \text{ since the expected investment return falls below the cost, } 1 + \mathbb{E}[\tilde{r}|\lambda] = 1 + \lambda < 1; \text{ if } \lambda \geq 0, \text{ the expected investment return } 1 + \mathbb{E}[\tilde{r}|\lambda] \geq 1 \text{ so that the bank recognizes no impairment and still reports } \lambda = 0. \]
1. If $\bar{\lambda} \geq 2\bar{\theta} + \delta$, no investors withdraw, and inefficient continuation occurs when $\lambda \in [0, \bar{\theta})$ and $\theta < -\lambda$;

2. If $\bar{\lambda} < 2\bar{\theta} + \delta$, investors withdraw when $\theta < \frac{\delta - \lambda}{2}$. Inefficient run occurs when $\lambda \in [\max \{0, \bar{\lambda} - \delta\}, \bar{\lambda}]$ and $\theta \in (\max \{-\lambda, -\bar{\theta}\}, \frac{\delta - \lambda}{2})$, whereas inefficient continuation occurs when $\lambda \in [0, \max \{0, \bar{\lambda} - \delta\})$ and $\theta < -\lambda$.

Comparing Lemma 3 with Proposition 1 points to the divergent economic outcomes under the impairment and the fair value rules. When the bank observes $\lambda < 0$, it reports $\lambda$ perfectly under both rules and the equilibrium outcomes are the same across the two. When $\lambda \geq 0$, the bank still reports $\lambda$ perfectly under the fair value rule but reports a commingled report $x_H$ under the impairment rule. The perfect disclosure under fair value helps to eliminate inefficient continuation but sustains some inefficient runs. Conversely, under the impairment rule, pooling all banks with $\lambda > 0$ together helps to reduce inefficient runs at the expense of allowing some inefficient continuation. When $E(\lambda|\lambda \geq 0) \geq \bar{\theta} + \frac{\delta}{2}$ (i.e., $\bar{\lambda} \geq 2\bar{\theta} + \delta$), investors never run on the pooled banks and adopting the impairment rule only entails inefficient continuation. Accordingly, which of the two rules yields higher surplus hinges on the relative importance of inefficient runs and continuation. We characterize the conditions under which the impairment rule dominates the fair value rule in the following proposition.

**Proposition 6** The total investors’ surplus is higher under the impairment rule than under the fair value rule if and only if the liquidation cost is sufficiently high and the bank’s known information (idiosyncratic shock) contributes more to the total return volatility relative to the residual uncertainty (the systemic shock), i.e., $\delta > \bar{\theta}$ and $\bar{\lambda} > 2\bar{\theta} + \kappa\delta$, where $\kappa \in (0, 1)$ is defined in the appendix.

Proposition 6 suggests that switching from the fair value rule to the impairment rule improves financial stability when two conditions are met: 1) the bank is relatively vulnerable to the risk of runs (high $\delta$) and 2) the bank’s known information (the idiosyncratic shock) contributes more to the total return volatility, relative to the residual uncertainty (the
Figure 7: An illustration of the impairment rule. $\lambda \in [-\bar{\lambda}, \bar{\lambda}]$ represents the bank’s private information, where $\bar{\lambda}$ represents the bound of $\lambda$, $\theta \in [-\bar{\theta}, \bar{\theta}]$ represents the residual uncertainty about the bank’s investment return conditional on $\lambda$, where $\bar{\theta}$ represents the bound of $\theta$, and $\delta$ represents the cost of liquidation/the risk of bank runs.
systemic shock). To explain the first condition, recall that the impairment rule dominates the fair value rule if inefficient runs are relatively more costly than inefficient continuation. Inefficient runs yield greater detriments when the liquidation cost $\delta$ is higher. To explain the second condition, note that, under the impairment rule, inefficient continuation occurs only when the bank observes $\lambda \geq 0$ but issues a commingled report, and investors choose to continue but $\lambda + \theta < 0$. Investors’ continuation/withdrawal decisions, again, depend on idiosyncratic and systematic return volatility, $\bar{\lambda}$ and $\bar{\theta}$, as explained earlier in part 3 of Proposition 5. More specifically, higher systematic return volatility $\bar{\theta}$ makes inefficient continuation more likely (i.e., $\Pr(\theta < -\lambda)$ is increasing in $\bar{\theta}$ for any given $\lambda \geq 0$), while higher idiosyncratic return volatility $\bar{\lambda}$ makes inefficient continuation less likely (i.e., $\Pr(\lambda < -\theta|\lambda \geq 0)$ is decreasing in $\bar{\lambda}$ for any given $\theta < 0$). In sum, when both $\delta$ and $\bar{\lambda}$ are sufficiently large relative to $\bar{\theta}$, the downside of inefficient runs is relatively large but that of inefficient continuation is small. Therefore, the impairment rule yields higher surplus than the fair value rule.

Proposition 6 carries empirical and policy implications. For example, consider the introduction of IFRS 7, which mandates a switch from the impairment rule to the fair value rule. Proposition 6 then implies that the capital market effects of introducing IFRS 7 can be ambiguous, with positive effects likely on banks with low bank run risk (low $\delta$), large exposure to systemic shocks (high $\bar{\theta}$) or low exposure to idiosyncratic shocks (low $\bar{\lambda}$), and negative effects otherwise.

4.4 Voluntary disclosure by banks

In the baseline model, we do not consider the bank’s ex post voluntary disclosure incentive in addition to the disclosure required by the ex ante reporting system. Now we consider an extension that the bank can choose to voluntarily disclose $\lambda$ at $t = 1$. We continue to assume that the bank’s incentive is aligned with its investors’ so that it chooses to maximize the total investors’ surplus. If the bank observes $\lambda < \lambda^*$, the bank reports $\lambda$ perfectly given
the optimal reporting system so the additional voluntary disclosure provides no incremental information. If the bank observes \( \lambda \geq \lambda^* \), the bank reveals no information under the optimal reporting system. Furthermore, we argue that, in this case, the bank has no incentive to disclose \( \lambda \) voluntarily. To see this, note that if the bank observes \( \lambda \geq \bar{\theta} + \frac{\delta}{2} \), Proposition 1 implies that the bank does not benefit from disclosing \( \lambda \) as no investors withdraw in either case. On the other hand, if the bank observes \( \lambda < \bar{\theta} + \frac{\delta}{2} \), it faces no runs upon non-disclosure but some runs upon disclosure when \( \theta < \frac{\delta}{2} - \lambda \). Note that, from Proposition 3, triggering runs in the region of \( \lambda \geq \lambda^* \) reduces the total investors’ surplus. Accordingly, to maximize the total investors’ surplus, the bank should not disclose \( \lambda \) voluntarily if it observes \( \lambda \geq \lambda^* \).

It is noteworthy that, in an alternative model with a friction that the bank cares about short-term prices (i.e., investors’ expectation of the bank’s investment return) rather than investors’ terminal payoffs, the bank would have an incentive to voluntarily disclose \( \lambda \) perfectly due to the standard “unraveling” argument. Note that such full voluntary disclosure is detrimental as it triggers inefficient runs. Accordingly, a policy implication is that to implement the optimal reporting system that maximizes the total investors’ surplus, it is necessary to limit voluntary disclosure.\(^{22}\) Furthermore, recall that from Proposition 5, banks with intermediate risk of runs (i.e., a moderate \( \delta \)) would have stronger incentives to limit voluntary disclosure, since they would prefer to implement lower reporting thresholds (thus less disclosure) than banks with extremely high or low risks of runs. To the extent that banks’ voluntary disclosure incentives are driven by the dependence of their payoffs on short-term prices, an empirical implication of our paper is that banks with intermediate risk of runs might prefer to implement incentive contracts that are less tied to short-term prices, relative to banks with either extremely high or low risk of runs, on the grounds of limiting voluntary disclosure and maintaining financial stability.

\(^{22}\)By the same token that excessively precise information may trigger inefficient bank runs, it might also be necessary to limit certain mandatory disclosure, e.g., disclosure of inside information mandated for publicly listed banks.
4.5 Investors’ information acquisition about $\lambda$

In the baseline model, we do not consider investors’ private learning of $\lambda$ when it is not reported by the bank. We offer some discussion of such information acquisition by investors in this section. When the bank reports $\lambda < \lambda^*$, investors have no need to learn $\lambda$. If $\lambda \geq \lambda^*$, the bank makes no disclosure of $\lambda$, and investors, upon observing $\theta$, determine whether to learn $\lambda$. For simplicity, assume that investors can learn $\lambda$ perfectly at no cost. The following proposition characterizes investors’ equilibrium information acquisition decisions.

**Proposition 7** Suppose that investors can learn the bank’s specific information $\lambda$ perfectly at no cost. If the optimal reporting threshold $\lambda^* < \bar{\theta}$, there is a unique equilibrium in which all investors learn $\lambda$, whereas if $\lambda^* \geq \bar{\theta}$, there are two equilibria in which either all investors learn $\lambda$ or none of the investors learn $\lambda$.

Proposition 7 suggests that, interestingly, investors may choose not to acquire information about $\lambda$ upon non-disclosure even though information acquisition is free. This equilibrium prevails when the reporting threshold $\lambda^*$ is larger than the bound of the residual uncertainty $\bar{\theta}$. This is because when the bank observes $\lambda \geq \lambda^*$ but does not disclose, investors rationally infer that $\lambda \geq \lambda^*$, in which case investors’ payoff from continuation $1 + \lambda + \theta$ always exceeds that from withdrawing 1, regardless of the realizations of $\{\lambda, \theta\}$. There is thus no need for investors to acquire information about $\lambda$ and no investors choose to run upon non-disclosure.

Conversely, Proposition 7 shows that investors learning $\lambda$ is always an equilibrium in which case the equilibrium outcome becomes identical with that under full disclosure. Intuitively, anticipating that all other investors choose to learn $\lambda$, an investor infers that, from Proposition 1, runs occur if and only if $\theta + \lambda < \frac{\delta}{2}$ and accordingly, there is an incremental benefit of learning $\lambda$ as its realization affects the equilibrium outcome and the investor’s payoff. Therefore, all investors coordinate to acquire information about $\lambda$, which leads to full disclosure. Recall that the total investors’ surplus is lower under full disclosure than under the optimal reporting system as full disclosure entails excessive inefficient runs. In
this light, an implication from Proposition 7 is that to maintain financial stability, it can be desirable to limit private learning by investors. This point echoes Dang, Gorton, Holmström, and Ordoñez (2017), who argue that “(t)o produce money-like safe liquidity, banks keep detailed information about their loans secret, reducing liquidity if needed to prevent agents from producing costly private information about the banks’ loans.”

4.6 Multiple-bank extension

So far we have analyzed the optimal financial reporting system designed for a single bank, maximizing the total surplus of the investors of the bank. We next extend our model to an economy with multiple banks and investigate whether the main implications pertain. There is a continuum of heterogeneous banks, each indexed by \( i \in [0, 1] \). As in the main model, each bank’s project yields a gross return \( 1 + \tilde{r}_i \), where \( \tilde{r}_i = \tilde{\theta} + \tilde{\lambda}_i \). The common systemic shock \( \tilde{\theta} \) is uniformly distributed in the interval \([−\bar{\theta}, \bar{\theta}]\). In addition, we assume there are cross-sectional variations in banks’ idiosyncratic shocks and exposure to the risk of runs. More specifically, we assume that the idiosyncratic shock \( \tilde{\lambda}_i \) is uniformly distributed in the interval \([-\bar{\lambda}_i, \bar{\lambda}_i]\), where \( \bar{\lambda}_i \in \{\bar{\lambda}_L, \bar{\lambda}_H\} \). Furthermore, \( \tilde{\lambda}_i \) satisfies that \( \Pr(\tilde{\lambda}_i = \bar{\lambda}_H) = \alpha \) and \( \Pr(\tilde{\lambda}_i = \bar{\lambda}_L) = 1 - \alpha \), where \( \alpha \in (0, 1) \) captures the fraction of banks with high idiosyncratic return volatility, \( \tilde{\lambda}_i = \bar{\lambda}_H > \bar{\lambda}_L \). The cost of early liquidation \( \delta_i \in [0, \bar{\delta}] \) and is uniformly distributed. We assume \( \bar{\lambda}_L \) and \( \delta_i \) are all independent of each other. We also assume \( \bar{\lambda}_H \) is sufficiently large, that is, \( \bar{\lambda}_H > 2\bar{\theta} + \bar{\delta} \), and \( \bar{\lambda}_L \) is sufficiently small, that is, \( \bar{\lambda}_L \in (\bar{\theta} + \frac{\bar{\delta}}{2}, 2\bar{\theta}) \).

Note that, we assume \( \bar{\lambda}_L \geq \bar{\theta} + \frac{\bar{\delta}}{2} \) to meet Assumption 1 and \( \bar{\delta} \in [\bar{\theta}, 2\bar{\theta}] \). Each bank’s net return is determined by \( 1 + \theta + \lambda_i - \delta_i l_i \), where \( l_i \) is the fraction of early liquidation.

A main goal of considering the multiple-bank extension is to shed some light on a “one-size-fits-all” problem in the implementation of the optimal financial reporting system. Recall an implication from our main analysis is that, implementing the optimal reporting systems requires tailoring the systems to each individual bank’s characteristics (e.g., the risk of bank runs and the systemic/idiosyncratic return volatility). In practice, however,
regulators are often constrained from tailoring reporting rules to banks’ idiosyncrasies. A complete characterization of the optimal reporting system to fully address the “one-size-fits-all” problem is beyond the scope of our paper, as it requires us to substantially expand our model to consider the constraints regulators likely face in tailoring their rules. Alternatively, we turn to comparing the effects of two prevalent rules that can be implemented universally for all banks, i.e., the fair value and the impairment rules as in Section 4.3, in terms of the aggregate surplus of all banks’ investors. We summarize our result in the following corollary.

**Corollary 1** In an economy with multiple banks, the aggregate surplus is higher under the impairment rule than under the fair value rule if and only if the liquidation cost is sufficiently high and there is a sufficiently large fraction of banks with high idiosyncratic return volatility, i.e., \( \delta > \mu \theta \) and \( \alpha > \tilde{\alpha} \), with \( \mu \in (1, 2) \) and \( \tilde{\alpha} \in (0, 1) \) defined in the appendix.

Corollary 1 follows directly from Proposition 6. Recall that, from Proposition 6, the impairment rule dominates the fair value rule when the risk of bank runs is sufficiently high and banks are more exposed to idiosyncratic shocks. Accordingly, with multiple banks, the impairment rule yields a higher aggregate investors’ surplus when banks overall face high risks of runs (i.e., the upper-bound on the liquidation cost \( \tilde{\delta} \) is large), and most banks are exposed to high idiosyncratic return volatility (i.e., the fraction of banks with high idiosyncratic return volatility \( \alpha \) is high).

5 Conclusion

This paper examines the role of reporting systems in bank runs. We find that the optimal reporting system mandates full disclosure of news below a threshold but no disclosure otherwise. This system is consistent with some aspects of current accounting standards for reporting banks’ assets, which often entail asymmetric treatments of unrealized gains and losses. Our analysis shows that the optimality of such asymmetric reporting lies in striking a balance between two inefficiency, i.e., inefficient panic-based runs on solvent banks and
inefficient continuations of insolvent ones, relative to a system of full disclosure. The optimal reporting system tolerates some inefficient continuations in exchange for fewer inefficient runs, and, in particular, seeks to eliminate any runs on banks that do not report.

Several important caveats regarding the optimality of the reporting system we characterize and the associated implications are in order. We have only derived the optimal reporting system within the bank-run framework developed by Morris and Shin (2000). We employ this framework because it offers great analytical tractability for incorporating information disclosure, and has been widely adopted in the literature that studies information disclosure and bank runs. Employing the Morris-Shin framework thus allows us to better compare our results with prior studies. Nonetheless, it should be noted that the Morris-Shin framework is a parsimonious model that does not capture all important aspects of bank runs, and there are other plausible models of bank runs (e.g., Bryant, 1980; Diamond and Dybvig, 1983; Goldstein and Pauzer, 2005). Exploring the roles of reporting systems in other bank-run settings is an interesting avenue we leave for future research.

In addition, the optimal financial reporting system we derive is bank-specific. Therefore, implementing such optimal reporting systems at an industry/country level can be challenging as it requires regulators to tailor reporting requirements to banks’ individual characteristics. In this light, our paper suggests that regulators consider offering some degrees of flexibility in the reporting requirements for banks.

Lastly, to focus on characterizing the \textit{ex ante} optimal financial reporting system from a stability perspective, we have abstracted away from some important considerations such as the conflicts of interest between banks and their stakeholders (e.g., Mahieux, Sapra, and Zhang, 2023), and banks’ reporting discretions (e.g., Gao and Jiang, 2018). Indeed, an interesting next step is to combine the reporting discretion with the \textit{ex ante} design of reporting systems, and analyze, given the reporting discretion, what the optimal reporting system would be. In spirit, such analysis is in line with Dye (2002), which examines the design of accounting standards considering manipulations by preparers of financial reports.
References


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Appendix: proofs

Proof of Lemma 1: See the main text. We further provide explicit steps showing how we obtain (5) and (6). Since \( \lambda \) and \( \theta \) are independent to each other, we can rewrite \( E[\bar{r} - \delta l|\bar{s}, x] \) as follows and obtain (5):

\[
E[\bar{r} - \delta l|\bar{s}, x] = E[r|x, \bar{s}] - \delta E[l|x, \bar{s}] = E[\lambda + \theta|x, \bar{s}] - \delta E[l|x, \bar{s}]
\]

where we substitute \( l = \Pr(s_j \leq \bar{s}) \) in the last equality. We then can apply the law of iterated expectation to simplify \( E[\Pr(s_j \leq \bar{s})|\bar{s}] \) and obtain (6):

\[
E[\Pr(s_j \leq \bar{s})|\bar{s}] = E[E[\Pr(s_j \leq \bar{s})|\theta]|\bar{s}] = E[\Pr(s_j \leq \bar{s}|\theta)|\bar{s}] = E\left[\Phi\left(\sqrt{\beta}(\bar{s} - \theta)\right)|\bar{s}\right],
\]

where we use \( \Pr(s_j \leq \bar{s}|\theta) = \Phi\left(\sqrt{\beta}(\bar{s} - \theta)\right) \) in the last equality because \( s_i = \bar{\theta} + \varepsilon_i \) and \( \varepsilon_i \sim N\left(0, \frac{1}{\beta}\right) \). Note that (5) and (6) do not depend on the distribution of \( \theta \). When \( \beta \) approaches infinity, the private signal \( \bar{s} \) approaches \( \theta \). Thus, \( E[\theta|\bar{s}] \) approaches \( \theta \) and \( \Phi\left(\sqrt{\beta}(\bar{s} - \theta)\right) \) approaches \( \frac{1}{2} \). The result then follows from (7) and (8). \( \blacksquare \)

Proof of Proposition 1: Note that under full disclosure (i.e., \( x = \lambda \)), \( E[\lambda|x] + \theta = \lambda + \theta \). Therefore, from Lemma 1, investors withdraw if and only if \( \lambda + \theta < \frac{\delta}{2} \) and inefficient run occurs when \( \lambda + \theta \in (0, \frac{\delta}{2}) \). We thus obtain the equilibrium as stated in Proposition 1. \( \blacksquare \)

Proof of Proposition 2: The first two parts of the proposition are obvious so we focus on proving the last part. The proof proceeds in two steps. First, we provide in Lemma 4 a necessary and sufficient condition for the expected surplus given a disclosed \( \lambda \), \( W(\lambda) \) as defined in (10), to be convex in the region of \( \lambda \in [-\bar{\theta}, \bar{\theta} + \frac{\delta}{2}] \). Second, we show that the last part is true if \( W(\lambda) \) is convex for \( \lambda \in [-\bar{\theta}, \bar{\theta} + \frac{\delta}{2}] \).

Lemma 4 \( W(\lambda) \) is a convex function for \( \lambda \in [-\bar{\theta}, \bar{\theta} + \frac{\delta}{2}] \) if and only if \( g'\left(\theta\right) \frac{\delta}{g\left(\theta\right)} < \frac{2}{\delta} \) for \( \theta \in [-\bar{\theta}, \bar{\theta}] \).

Proof. Taking the first and the second derivatives with respect to \( W(\lambda) \) yields:

\[
W'(\lambda) = \int_{\frac{\delta}{2} - \lambda}^{\bar{\theta}} g\left(\theta\right) d\theta + \left(\lambda + \frac{\delta}{2} - \lambda\right) g\left(\frac{\delta}{2} - \lambda\right) = \int_{\frac{\delta}{2} - \lambda}^{\bar{\theta}} g\left(\theta\right) d\theta + \frac{\delta}{2} g\left(\frac{\delta}{2} - \lambda\right) > 0,
\]

and

\[
W''(\lambda) = g\left(\frac{\delta}{2} - \lambda\right) - \frac{\delta}{2} g'\left(\frac{\delta}{2} - \lambda\right),
\]

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which is positive if and only if \( \frac{g'(\frac{\delta}{2} - \lambda)}{g(\frac{\delta}{2} - \lambda)} < \frac{2}{\delta} \). When \( \lambda < -\bar{\theta} + \frac{\delta}{2} \) is disclosed, the bank always suffers runs and the expected surplus is \( W(\lambda) = 1 \). Hence, \( W(\lambda) \) is a convex function for 

\[ \lambda \in [-\bar{\theta}, \bar{\theta} + \frac{\delta}{2}] \]

if and only if \( \frac{g'(\theta)}{g(\theta)} < \frac{2}{\delta} \) for \( \forall \theta \in [-\bar{\theta}, \bar{\theta}] \). Q.E.D.

Note that \( W(\lambda) \) is a convex function for \( \lambda \in [-\bar{\theta}, \bar{\theta} + \frac{\delta}{2}] \). Furthermore, by the law of iterated expectation, the \textit{ex ante} expected surplus of a bank who reports \( x \) is equal to \( W(\mathbb{E}(\lambda|x)) \). The Jensen’s inequality then implies that banks with \( \lambda \in [-\bar{\theta}, \bar{\theta} + \frac{\delta}{2}] \) shall never pool within themselves because \( \mathbb{E}[W(\lambda)|x] > W(\mathbb{E}(\lambda|x)) \).

**Proof. of Proposition 3:** Suppose that \( \mathbb{E}(\lambda|x_H) \in (-\bar{\theta}, \bar{\theta} + \frac{\delta}{2}) \). We prove by contradiction that the total surplus can be improved by dividing those banks reporting \( x_H \) into two groups that report \( x_1 \) and \( x_2 \) correspondingly, such that, \( \mathbb{E}(\lambda|x_1) = \bar{\theta} + \frac{\delta}{2} \), \( \mathbb{E}(\lambda|x_2) \in (-\bar{\theta}, \bar{\theta} + \frac{\delta}{2}) \), and \( x_1 > x_2 \). This statement is true by the Jensen’s inequality because \( W(\lambda) \) is a convex function for \( \lambda \in [-\bar{\theta}, \bar{\theta} + \frac{\delta}{2}] \) as shown in the proof of Proposition 2.

**Proof. of Lemma 2:** The expected surplus of a bank with \( \lambda \) being pooled to survive is \( \lambda + \mathbb{E}[\theta] = \lambda \). From Proposition 2, the expected surplus of being separated is \( W(\lambda) \). Below we show that there exists a unique threshold \( \lambda^x \) such that \( \lambda \geq W(\lambda) \) if and only if \( \lambda \geq \lambda^x \), where \( \lambda^x \in (0, \bar{\theta} + \frac{\delta}{2}) \) if and only if \(-\bar{\theta} + \frac{\delta}{2} < 0 \) and \( \lambda^x = 0 \) otherwise.

Denote \( \Delta(\lambda) \equiv W(\lambda) - (1 + \lambda) \). It is equivalent to show that \( \Delta(\lambda) \leq 0 \) if and only if \( \lambda \geq \lambda^x \). Obviously, when \( \lambda \geq \bar{\theta} + \frac{\delta}{2} \), we obtain \( W(\lambda) = 1 + \lambda \) and thus \( \Delta(\lambda) = 0 \). It is easy to show that \( \Delta''(\lambda) = W''(\lambda) \), so \( \Delta(\lambda) \) is also a convex function for \( \lambda \in [-\bar{\theta}, \bar{\theta} + \frac{\delta}{2}] \).

Moreover, we obtain

\[
\Delta'(\bar{\theta} + \frac{\delta}{2}) = W'(\bar{\theta} + \frac{\delta}{2}) - 1 = \int_{-\bar{\theta}}^{\bar{\theta}} g(\theta) d\theta + \frac{\delta}{2} g(-\bar{\theta}) - 1 = \frac{\delta}{2} g(-\bar{\theta}) > 0.
\]

To further find the region in which \( \Delta(\lambda) \leq 0 \), we discuss the following two cases:

1. If \(-\bar{\theta} + \frac{\delta}{2} \geq 0 \), \( \Delta(\lambda) \leq 0 \) if and only if \( \lambda \geq 0 \). Since \( W(-\bar{\theta} + \frac{\delta}{2}) = 1 \), we obtain

\[
\Delta(-\bar{\theta} + \frac{\delta}{2}) = W(-\bar{\theta} + \frac{\delta}{2}) - \left(1 - \bar{\theta} + \frac{\delta}{2}\right) = -\left(-\bar{\theta} + \frac{\delta}{2}\right) \leq 0.
\]

Due to the convexity of \( \Delta(\lambda) \), combining \( \Delta(-\bar{\theta} + \frac{\delta}{2}) \leq 0 \) and \( \Delta(\bar{\theta} + \frac{\delta}{2}) = 0 \) shows that \( \Delta(\lambda) \leq 0 \) for \( \forall \lambda \in (-\bar{\theta} + \frac{\delta}{2}, \bar{\theta} + \frac{\delta}{2}) \). Moreover, if \( \lambda \leq -\bar{\theta} + \frac{\delta}{2} \), we obtain \( W(\lambda) = 1 \) and thus \( \Delta(\lambda) = -\lambda \), which is non-positive if and only if \( \lambda \geq 0 \).

2. If \(-\bar{\theta} + \frac{\delta}{2} < 0 \), there is a unique solution \( \lambda^+ \in (0, \bar{\theta} + \frac{\delta}{2}) \) satisfying \( \Delta(\lambda^+) = 0 \), and \( \Delta(\lambda) \leq 0 \) if and only if \( \lambda \geq \lambda^+ \). Since \( W(\lambda) \) is increasing for \( \lambda \in [-\bar{\theta} + \frac{\delta}{2}, \bar{\theta} + \frac{\delta}{2}] \), we
obtain $\forall \lambda \in (-\bar{\theta} + \frac{\delta}{2}, 0]$, $W(\lambda) > W(-\bar{\theta} + \frac{\delta}{2}) = 1$ and hence,

$$
\Delta(\lambda) = W(\lambda) - 1 - \lambda > -\lambda \geq 0.
$$

Due to the convexity of $\Delta(\lambda)$, there are at most two solutions to $\Delta(\lambda) = 0$ for $\lambda \in [-\bar{\theta} + \frac{\delta}{2}, \bar{\theta} + \frac{\delta}{2}]$. One of the solutions is $\lambda = \bar{\theta} + \frac{\delta}{2}$ as $\Delta(\bar{\theta} + \frac{\delta}{2}) = 0$. Combining $\Delta'(\bar{\theta} + \frac{\delta}{2}) > 0$ and $\Delta(0) > 0$ then shows that the other solution denoted by $\lambda^+$ exists and satisfies that $\lambda^+ \in (0, \bar{\theta} + \frac{\delta}{2})$. In other words, when $\lambda$ increases from 0 to $\bar{\theta} + \frac{\delta}{2}$, $\Delta(\lambda)$ first decreases from above zero to below zero after hitting zero at $\lambda = \lambda^+$, then starts increasing from below zero until hitting zero at $\lambda = \bar{\theta} + \frac{\delta}{2}$. This implies that $\Delta(\lambda) < 0$ for $\lambda \in (\lambda^+, \bar{\theta} + \frac{\delta}{2})$, and $\Delta(\lambda) > 0$ for $\lambda \in (0, \lambda^+)$. Meanwhile, if $\lambda \leq -\bar{\theta} + \frac{\delta}{2} < 0$, we obtain $W(\lambda) = 1$ and thus $\Delta(\lambda) = -\lambda \geq -(-\bar{\theta} + \frac{\delta}{2}) > 0$.

The result follows after combining the above two cases. □

**Proof of Proposition 4:** We prove that the total surplus is maximized when those banks with relatively higher $\lambda$s, that is, $\lambda \in [\lambda^*, \frac{\delta}{2} + \bar{\theta})$, are pooled with banks $\lambda \geq \frac{\delta}{2} + \bar{\theta}$.

We consider two banks with $\lambda_1$ and $\lambda_2$ such that $\lambda_1, \lambda_2 \in [\lambda^*, \bar{\theta} + \frac{\delta}{2})$ and $\lambda_1 > \lambda_2$. One reporting system $S_1$ pools a measure $\pi_1$ of $\lambda_1$ with a certain distribution of banks in $\Lambda$ into reporting $x$, while the other reporting system $S_2$ pools a measure $\pi_2$ of $\lambda_2$ with the same distribution of banks in $\Lambda$ into reporting $x$, such that $E(\lambda|x, S_1) = E(\lambda|x, S_2) = \bar{\theta} + \frac{\delta}{2}$, and all else equal. The conditions on $E(\lambda|x)$ imply

$$
\pi_1 \left( \bar{\theta} + \frac{\delta}{2} - \lambda_1 \right) = \pi_2 \left( \bar{\theta} + \frac{\delta}{2} - \lambda_2 \right). \tag{13}
$$

The surplus difference between $S_1$ and $S_2$ is determined by

$$
\pi_1 \left( 1 + \lambda_1 - W(\lambda_1) \right) - \pi_2 \left( 1 + \lambda_2 - W(\lambda_2) \right).
$$

Due to (13) and $W(\bar{\theta} + \frac{\delta}{2}) = 1 + \bar{\theta} + \frac{\delta}{2}$, it is equivalent to show that

$$
\frac{(1 + \lambda_1 - W(\lambda_1))}{\bar{\theta} + \frac{\delta}{2} - \lambda_1} > \frac{(1 + \lambda_2 - W(\lambda_2))}{\bar{\theta} + \frac{\delta}{2} - \lambda_2} \iff \int_{\lambda_1}^{\bar{\theta} + \frac{\delta}{2}} \frac{W'(\lambda)}{\bar{\theta} + \frac{\delta}{2} - \lambda} d\lambda > \int_{\lambda_2}^{\bar{\theta} + \frac{\delta}{2}} \frac{W'(\lambda)}{\lambda_1 - \lambda} d\lambda,
$$

which is true for $\forall \lambda_1, \lambda_2 \in [-\bar{\theta}, \bar{\theta} + \frac{\delta}{2})$ with $\lambda_1 > \lambda_2$, because $W'(\lambda) > 0$, $W''(\lambda) > 0$ for $\lambda \in [-\bar{\theta} + \frac{\delta}{2}, \bar{\theta} + \frac{\delta}{2}]$ and $W(\lambda) = 1$, $W'(\lambda) = 0$ for $\lambda \in [-\bar{\theta}, -\bar{\theta} + \frac{\delta}{2}]$. In sum, the surplus improves if one prioritizes pooling a bank with higher $\lambda$. Hence, the surplus is maximized when those banks with $\lambda \in [\lambda^*, \frac{\delta}{2} + \bar{\theta})$ are pooled with banks $\lambda \geq \frac{\delta}{2} + \bar{\theta}$.
The objective function (11) and the constraint (12) can be simplified to

\[
\max_{\lambda^* \in [\bar{\lambda} \frac{\lambda}{2} + \bar{\theta}]} \int_{\lambda^*}^{\lambda^* + \bar{\lambda}} (1 + \lambda) f(\lambda) d\lambda + \int_{-\bar{\theta}}^{\lambda^*} W(\lambda) f(\lambda) d\lambda,
\]

s.t., \( \mathbb{E}(\lambda|\lambda \geq \lambda^*) \geq \frac{\delta}{2} + \bar{\theta}. \) (14)

It is then straightforward to show that the optimal threshold satisfies \( \lambda^* = \max \{\lambda^2, \tau^*\} \), where we denote \( \tau^* \) the solution to the equation \( \mathbb{E}(\lambda|\lambda \geq \tau^*) = \frac{\delta}{2} + \bar{\theta} \). Applying the implicit function theorem shows that \( \lambda^2 \) decreases in \( \delta \) (as for \( \lambda^2 \in (0, \bar{\theta} + \frac{\delta}{2}) \), \( \frac{d\lambda^2(\lambda)}{d\lambda} < 0 \) at \( \lambda = \lambda^2 \) and \( \frac{d\lambda(\lambda)}{d\lambda} = \frac{dW(\lambda)}{d\lambda} < 0 \) while \( \tau^* \) increases in \( \delta \). Hence, there exist a threshold \( \hat{\delta} \) such that \( \lambda^* = \lambda^2 \) when \( \delta < \hat{\delta} \) and \( \lambda^* = \tau^* \) when \( \delta \geq \hat{\delta} \). ■

**Proof of Proposition 5**: It is straightforward to verify that \( \lambda^2 = \max \{0, \bar{\theta} - \frac{\delta}{2}\} \) when \( \theta \) is uniformly distributed in \([-\bar{\theta}, \bar{\theta}]\) following the proof of Lemma 2. When \( \lambda \) is uniformly distributed in \([-\bar{\lambda}, \bar{\lambda}]\), the solution to the equation \( \mathbb{E}(\lambda|\lambda \geq \tau^*) = \frac{\delta}{2} + \bar{\theta} \) is \( \tau^* = \delta + 2\bar{\theta} - \bar{\lambda} \).

As a result, \( \lambda^2 > \tau^* \) if and only if the liquidation cost \( \delta < \hat{\delta} \equiv \max\{\frac{\delta}{2}(\bar{\lambda} - \bar{\theta}), \bar{\lambda} - 2\bar{\theta}\} \). Following Proposition 4, the optimal threshold \( \lambda^* = \lambda^2 = \max \{0, \bar{\theta} - \frac{\delta}{2}\} \) if \( \delta < \hat{\delta} \), and \( \lambda^* = \tau^* = \delta + 2\bar{\theta} - \bar{\lambda} \) if \( \delta \geq \hat{\delta} \).

We next derive the comparative statics of \( \lambda^* \) in \( \delta \), \( \bar{\theta} \), and \( \bar{\lambda} \). First, it is easy to see that \( \lambda^2 \) decreases in \( \delta \) while \( \tau^* \) increases in \( \delta \). Hence, \( \lambda^* \) first decreases in \( \delta \) when \( \delta < \hat{\delta} \) and then increases in \( \delta \) when \( \delta \geq \hat{\delta} \). Next, since both \( \lambda^2 = \max \{0, \bar{\theta} - \frac{\delta}{2}\} \) and \( \tau^* \) increase in \( \bar{\theta} \), \( \lambda^* = \max \{\lambda^2, \tau^*\} \) increases with \( \bar{\theta} \). Last, combining that (i) \( \lambda^2 = \max \{0, \bar{\theta} - \frac{\delta}{2}\} \) does not depend on \( \bar{\lambda} \) and (ii) \( \tau^* \) decreases with \( \bar{\lambda} \) shows that \( \lambda^* = \max \{\lambda^2, \tau^*\} \) decreases with \( \bar{\lambda} \). ■

**Proof of Lemma 3**: First, for \( \lambda < 0 \), the impairment rule reports \( \lambda \) and the equilibrium outcome is the same as the full disclosure case in Proposition 1 for \( \lambda < 0 \). Second, if the bank observes \( \lambda \geq 0 \) and reports \( x_H \), from Lemma 1, investors withdraw if and only if \( \mathbb{E}(\lambda|\lambda \geq 0) + \theta < \frac{\delta}{2} \). We thus obtain the equilibrium as stated in Lemma 3. ■

**Proof of Proposition 6**: Denote \( W_{FD} \) the expected social welfare under the fair-value regime with full disclosure, we obtain

\[
W_{FD} = 1 + \int_{-\bar{\theta}}^{\bar{\theta} + \frac{\delta}{2}} \int_{\bar{\theta} + \frac{\delta}{2} - \bar{\lambda}}^{\bar{\theta}} (\lambda + \theta) \frac{1}{2\lambda} \frac{1}{2\theta} d\theta d\lambda + \int_{\bar{\theta} + \frac{\delta}{2}}^{\bar{\lambda}} \frac{1}{2\lambda} d\lambda = 1 + \frac{4\theta^2 - 3\delta^2}{48\lambda} + \frac{\bar{\lambda}}{4}.
\]

We then examine the expected social welfare under the impairment regime with a threshold \( \lambda_0 = 0 \) denoted by \( W_0 \). There are following cases:

1. if \( \bar{\lambda} \geq 2\bar{\theta} + \delta \) so \( \mathbb{E}(\lambda|\lambda \geq 0) \geq \bar{\theta} + \frac{\delta}{2} \), we obtain
(a) if $\tilde{\theta} > \frac{\delta}{2}$ or $\delta < 2\tilde{\theta}$, then
\[
W_0 = 1 + \int_{0}^{\lambda} \frac{1}{2\lambda} d\lambda + \int_{-\theta + \frac{\delta}{2}}^{0} \int_{\frac{\lambda}{2} - \lambda}^{\theta} (\lambda + \theta) \frac{1}{2\lambda} \frac{1}{2\theta} d\theta d\lambda
\]
\[
= 1 + \frac{\lambda}{4} + \frac{1}{96\lambda \theta}[4\tilde{\theta}^3 - 3\delta^2 \tilde{\theta} + \delta^3].
\]
We then can compare the social welfare as follows:
\[
W_0 - W_{FD} = \frac{1}{96\lambda \theta}[4\tilde{\theta}^3 - 3\delta^2 \tilde{\theta} + \delta^3] - \frac{4\tilde{\theta}^2 - 3\delta^2}{48\lambda} = \frac{1}{96\lambda \theta} (\delta - \tilde{\theta})(\delta + 2\tilde{\theta})^2,
\]
which is positive if and only if $\delta > \tilde{\theta}$.

(b) if $\tilde{\theta} \leq \frac{\delta}{2}$ or $\delta \geq 2\tilde{\theta}$, then $W_0 = \frac{\lambda}{4}$ so $W_0 - W_{FD} = -\frac{4\tilde{\theta}^2 - 3\delta^2}{48\lambda} > 0$ given that $\delta \geq 2\tilde{\theta}$.

2. if $\bar{\lambda} < 2\tilde{\theta} + \delta$ so $\mathbb{E}(\lambda|\lambda \geq 0) < \tilde{\theta} + \frac{\delta}{2}$, we obtain

(a) if $\tilde{\theta} > \frac{\delta}{2}$ or $\delta < 2\tilde{\theta}$, then
\[
W_0 = 1 + \int_{0}^{\lambda} \int_{\frac{\lambda}{2} - \lambda}^{\theta} (\lambda + \theta) \frac{1}{2\lambda} \frac{1}{2\theta} d\theta d\lambda + \int_{-\theta + \frac{\delta}{2}}^{0} \int_{\frac{\lambda}{2} - \lambda}^{\theta} (\lambda + \theta) \frac{1}{2\lambda} \frac{1}{2\theta} d\theta d\lambda
\]
\[
= 1 + \frac{(\bar{\lambda} + 2\tilde{\theta})^2 - \delta^2}{32\theta} + \frac{1}{96\lambda \theta}[4\tilde{\theta}^3 - 3\delta^2 \tilde{\theta} + \delta^3].
\]
We then can compare the social welfare as follows:
\[
W_0 - W_{FD} = \frac{(\bar{\lambda} - 2\tilde{\theta})^2 - \delta^2}{32\theta} + \frac{1}{96\lambda \theta}[\delta^3 + 3\delta^2 \tilde{\theta} - 4\tilde{\theta}^3],
\]
which is positive if and only if $\delta > \tilde{\theta}$ and $\bar{\lambda} > 2\tilde{\theta} + \kappa \delta$, where $\kappa \in (0, 1)$ is determined by $3(2\tilde{\theta} + \kappa \delta)(1 - \kappa^2)\delta^2 = \delta^3 + 3\delta^2 \tilde{\theta} - 4\tilde{\theta}^3$.

(b) if $\tilde{\theta} \leq \frac{\delta}{2}$ or $\delta \geq 2\tilde{\theta}$, then $W_0 = \frac{(\bar{\lambda} + 2\tilde{\theta})^2 - \delta^2}{32\theta}$ so $W_0 - W_{FD} = \frac{3(\bar{\lambda} - 2\tilde{\theta})^2 - \delta^2}{32\theta} - \frac{4\tilde{\theta}^2 - 3\delta^2}{48\lambda}$, which is positive if and only if $\bar{\lambda} > 2\tilde{\theta} + \kappa \delta$, where $\kappa \in (0, 1)$ is determined by $3(2\tilde{\theta} + \kappa \delta)(1 - \kappa^2)\delta^2 = 6\delta^2 \tilde{\theta} - 8\tilde{\theta}^3$.

Overall, $W_0 > W_{FD}$ if and only if $\delta > \tilde{\theta}$ and $\bar{\lambda} > 2\tilde{\theta} + \kappa \delta$, where $\kappa \in (0, 1)$ is determined by (i) $3(2\tilde{\theta} + \kappa \delta)(1 - \kappa^2)\delta^2 = \delta^3 + 3\delta^2 \tilde{\theta} - 4\tilde{\theta}^3$ if $\delta \in (\tilde{\theta}, 2\tilde{\theta})$; (ii) $3(2\tilde{\theta} + \kappa \delta)(1 - \kappa^2)\delta^2 = 6\delta^2 \tilde{\theta} - 8\tilde{\theta}^3$ if $\delta \geq 2\tilde{\theta}$. ■

**Proof of Proposition 7:** We first show that all investors learning information is always an equilibrium. If all other investors learn $\lambda$, the equilibrium outcome is identical with full
disclosure and runs occur whenever \( \lambda + \theta < \frac{\delta}{2} \) from Proposition 1. Anticipating that, if an investor \( i \) learns \( \lambda \), the investor withdraws if \( \lambda + \theta < \frac{\delta}{2} \), and stays otherwise. As a result, given that the bank discloses the pooled report \( x_H \), her expected payoff is

\[
1 + \mathbb{E}[\max\{\lambda + \theta - \frac{\delta}{2}, 0\} | \lambda > \lambda^*] + \frac{\delta}{2} \Pr(\lambda + \theta \geq \frac{\delta}{2} | \lambda > \lambda^*) > 1. \tag{16}
\]

Otherwise, if an investors \( i \) does not learn \( \lambda \), her expected payoff from staying is

\[
1 + \mathbb{E}[\max\{\lambda + \theta - \frac{\delta}{2}, 0\} | \lambda > \lambda^*] + \frac{\delta}{2} \Pr(\lambda + \theta \geq \frac{\delta}{2} | \lambda > \lambda^*)
\]

\[
+ \mathbb{E}[\min\{\lambda + \theta - \frac{\delta}{2}, 0\} | \lambda > \lambda^*] - \frac{\delta}{2} \Pr(\lambda + \theta < \frac{\delta}{2} | \lambda > \lambda^*),
\]

which is smaller than (16) because the investor incurs a loss of \( \lambda + \theta - \delta \) when \( \lambda + \theta < \frac{\delta}{2} \), and the expected loss is captured by the following term:

\[
\mathbb{E}[\min\{\lambda + \theta - \frac{\delta}{2}, 0\} | \lambda > \lambda^*] - \frac{\delta}{2} \Pr(\lambda + \theta < \frac{\delta}{2} | \lambda > \lambda^*) < 0.
\]

Her expected payoff from withdrawing is 1, which is also smaller than (16). Hence, it is optimal for an investor \( i \) to learn \( \lambda \) given that all other investors learn \( \lambda \).

Next, we show that when \( \lambda^* \geq \bar{\theta} \), there exists an equilibrium that no investors choose to learn \( \lambda \). Suppose that none of the investors learns \( \lambda \). When the bank discloses the pooled report \( x_H \) for \( \lambda \geq \lambda^* \), there is no run. Each investor earns \( \mathbb{E}[\lambda + \theta | \lambda > \lambda^*] \). If an investor \( i \) deviates and chooses to learn \( \lambda \), the investor withdraws if \( \lambda + \theta < 0 \), and stays otherwise. Note that when \( \lambda^* \geq \bar{\theta} \), \( \lambda + \theta \geq 0 \) for any \( \{\lambda, \theta\} \). Hence \( \mathbb{E}[\max\{\lambda + \theta, 0\} | \lambda > \lambda^*] = \mathbb{E}[\lambda + \theta | \lambda > \lambda^*] \), i.e., any investor \( i \) is indifferent between learning and not learning given that none of other investors learns \( \lambda \). Accordingly, no-investor-learning constitutes an equilibrium. \( \blacksquare \)

**Proof. of Corollary 1:** We first show that the expected investor surplus at any banks with \( \lambda = \bar{\lambda}_L \) is lower under impairment rule than under the fair value rule. Proposition 6 shows that if \( \delta \leq \bar{\theta} \), \( W_0 - W_{FD} \leq 0 \). It is also straightforward to verify that \( \bar{\lambda}_L < 2\bar{\theta} < 2\bar{\theta} + \kappa \delta \) if \( \delta > \bar{\delta} \). Hence, \( W_0(\bar{\lambda}_L, \delta) - W_{FD}(\bar{\lambda}_L, \delta) < 0 \) for any \( \delta \in [0, \bar{\delta}] \), so \( \mathbb{E}[W_0(\bar{\lambda}_L, \delta) - W_{FD}(\bar{\lambda}_L, \delta)] < 0 \).

We next show that the expected investor surplus at those banks with high idiosyncratic shock \( \bar{\lambda} = \bar{\lambda}_H \) is higher under impairment rule than under the fair value rule if and only if \( \delta \) is sufficiently high. As \( \bar{\lambda}_H > 2\bar{\theta} + \bar{\delta} \bar{\theta} \) by assumption, following the proof of Proposition 6, we can compare the expected investor surplus as follows:

\[
\mathbb{E}[W_0(\bar{\lambda}_H, \delta) - W_{FD}(\bar{\lambda}_H, \delta)] = \int_0^{\bar{\delta}} \frac{1}{96\lambda_H\bar{\theta}\delta} [\delta^3 + 3\delta^2\bar{\theta} - 4\bar{\theta}^3] d\delta = \frac{1}{96\lambda_H\bar{\theta}} \left[ \frac{\bar{\delta}^3}{4} + \bar{\delta} \bar{\theta} - 4\bar{\theta}^3 \right],
\]

Electronic copy available at: https://ssrn.com/abstract=3740168
which is positive if and only if \( \delta > \mu \bar{\theta} \) where \( \mu \in (1, 2) \) is a root to \( \frac{\mu^3}{4} + \mu^2 - 4 = 0 \).

Overall, the aggregate investor surplus is higher under impairment rule than under the fair value rule if \( \delta > \mu \bar{\theta} \) and \( \alpha \) is sufficiently large, that is, there are sufficient banks with \( \bar{\lambda} = \bar{\lambda}_H \). Specifically, \( \alpha > \bar{\alpha} \in (0, 1) \), where \( \bar{\alpha} = -\frac{E[W_0(\bar{\lambda}_L, \delta) - W_{FD}(\bar{\lambda}_L, \delta)]}{E[W_0(\bar{\lambda}_H, \delta) - W_{FD}(\bar{\lambda}_H, \delta)] - E[W_0(\bar{\lambda}_L, \delta) - W_{FD}(\bar{\lambda}_L, \delta)]} \).