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Working Paper

External Information Sources, Investor Beliefs, and Corporate Disclosures

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Abstract: We study a voluntary disclosure model with uncertain information endowment and a possibly relevant external signal (e.g., a peer's financial report). The manager may disclose the firm value before or after the external signal ("early" or "late" disclosure). We show that favorable signals are perceived to be less likely relevant, which reinforces the investors' beliefs that the manager was endowed with unfavorable information. Thus, a favorable external signal may lead to a lower non-disclosure price. In the early scenario, the manager must form an expectation of the nondisclosure price—even though the latter is nonmonotonic in the firm value, a unique early disclosure equilibrium exists. Disclosures of favorable information are clustered before external news and disclosures of unfavorable information after that.

Keywords: voluntary disclosure, dynamic disclosure, external sources of information, uncertain information endowment

1 Introduction

Firm managers can voluntarily disclose value-relevant information that is not amenable to inclusion in mandatory financial reports. If their endowment of private information is uncertain, managers maximizing the price of their firm can withhold unfavorable news (Dye 1985). A key impetus behind the lack of information unraveling in this case is that, when managers remain silent, investors are uncertain whether this is because the managers are not informed or because the managers' private information is not favorable. The literature finds that investor beliefs—and managers' incentives to disclose—can be influenced by external information (e.g., news, analyst forecasts, or peer disclosures) if its arrival is probabilistic and correlated with the managers' information endowment (Dye and Sridhar 1995). Managers are less forthcoming when they expect that a perfectly informative external signal is more likely to arrive—at a later date—if they themselves are informed (Frenkel, Guttman, and Kremer 2020).

In practice, external signals may arrive with certainty (e.g., peers must release mandatory financial reports), but their informativeness may be uncertain (e.g., investors may be unsure if sales reported by peers are informative about the market demand or are driven by idiosyncratic factors). Moreover, managers can disclose their private information not only before but also after the arrival of external news. In this paper, we consider a setting with certain arrival of an external signal, the informativeness of which is uncertain. We ask how does the external signal influence investor beliefs and study the managers' incentives to disclose both before and after the arrival of the signal.

Our model builds on the work of Dye (1985). We consider a manager ("she"), who may be endowed with private information about firm value and can voluntarily and truthfully disclose it to investors. Clearly, a manager who is uninformed has nothing to disclose—thus we focus on an informed manager. The manager's objective is to maximize her firm's market price. The investors price the firm after considering all publicly available information, which consists of the manager's disclosure and information from an external source ("signal"). The signal may be informative about the firm value—we say that, in this case, the signal is "relevant." For example, information about a peer's sales contained in the peer's mandatory financial reports may be informative about the firm's future market demand and sales. However, the information in the peer's report could also be a result of idiosyncratic factors—in this case, the signal is not informative about the firm value and is therefore irrelevant. The investors do not know whether the signal is relevant. An informed manager can assess the relevance only after the signal realization (i.e., after comparing it with her own information).

Although we assume that the manager's information endowment is uncorrelated with the signal's relevance, we find that the investor beliefs about these events are endogenously intertwined and reinforce each other. In particular, an unfavorable signal strengthens the joint beliefs of investors that the signal is relevant and that the nondisclosing manager is informed. Conversely, a favorable signal weakens the investor beliefs—such a signal is unlikely to accurately describe the value of a firm run by an informed nondisclosing manager. At the heart of this effect is the manager's strategic disclosure behavior and the information that can be inferred from it. In an equilibrium where good news are disclosed and bad news are withheld, the investors understand that an informed manager who did not disclose must have observed bad news. Thus, if the external signal is unfavorable, the investors perceive it as more likely relevant, which strengthens their beliefs that the manager is informed. In contrast, it is rationally inconsistent for investors to believe that a favorable signal is relevant and that the manager is informed at the same time. This shift in investor beliefs leads to nonmonotonicity of the nondisclosure price in the external signal. In particular, more favorable external news may paradoxically lead to lower market price. This observation plays a key role for many of our results.

We proceed by considering the case where the manager's disclosure ensues the external signal for exogenous reasons (e.g., because the conference call is scheduled after the release of a peer's financial reports). We refer to such a disclosure as "late." Investors react more sensitively to unfavorable signals than to favorable ones. As one would expect, extremely favorable external news discourages the manager from disclosing, whereas extremely unfavorable news encourages her. We also show that the manager may respond with a disclosure that is less favorable than that already released by the external signal, and vice versa may remain silent following unfavorable external news. The reason for this disclosure of (seemingly) bad news and withholding of (seemingly) good news is that the investors are uncertain about the signal relevance and therefore never price the firm precisely at the signal realization. When the external news leads to a nondisclosure price that is below the one she can achieve by disclosing information, the manager comes forward and voluntarily discloses. Conversely, when the external information leads to a market price that is above the manager's information, the manager prefers silence. Our predictions about the late disclosure regime are consistent with Sletten (2012) who studies firms' voluntary disclosure after a restatement issued by their peers.

Next we consider the case where the corporate disclosure precedes or coincides with the external signal for exogenous reasons (e.g., because the conference call is scheduled before the release of a peer's financial reports). Because an informed manager must decide whether to disclose her information before the realization of the external signal, she must form an expectation about the market price, conditional on the value she observed. Even though her expectation of the nondisclosure price is nonmonotonic in the firm value, we find that a unique disclosure equilibrium nevertheless exists. Notably, the presence of an external signal has a crowding-out effect: the manager is less-likely to disclose. The impetus behind this result is the reinforcement effect of the signal. In particular, favorable external news reinforces the market beliefs that the manager is uninformed. Thus the expected nondisclosure price may exceed the firm value and the manager may benefit from relying on the external signal to reveal the same information that she observed.

We extend our results in several ways. First, we consider dynamic disclosure. In particular, a manager who remained silent before the realization of the external signal can disclose later. We find that the manager at least weakly prefers to delay disclosure until after the external news arrives if there are no costs associated with doing so. However, if delaying is costly (but not prohibitively so), we find that managers observing favorable information prefer to disclose early and avoid the cost of delaying. Thus we predict that disclosures of favorable information are clustered before the arrival of external information and disclosures of unfavorable information after that. Second, we consider a frequent adjustment of market prices and find that it may encourage corporate disclosures.

Broadly speaking, our paper belongs to the analytical literature on voluntary disclosure initiated by Grossman (1981) and Milgrom (1981) and surveyed by Beyer, Cohen, Lys, and Walther (2010).[1,2] Similar to the analytical models of Dye (1985) and Jung and Kwon (1988), we allow managers to be endowed with information about firm value with some probability. However, unlike those authors, we consider additional sources of information. The effect of external sources of information on voluntary disclosure is also the focus of several other studies. Dye and Sridhar (1995) consider a model with multiple firms where voluntary disclosure by one of them causes an update in the investor beliefs about the information endowment of the other managers. This happens because the managers' endowments in their model are assumed to be correlated. Frenkel, Guttman, and Kremer

[1] Unlike Dye (1983), Gigler and Hemmer (1998, 2001), and Bonham (2021), we take the manager's objective to maximize firm price as given and focus on the disclosure game and the interaction with the external signal.

[2] The literature on verifiable disclosure assumes that the manager decides whether to disclose only after she observes the random variable. In contrast, the literature on Bayesian persuasion assumes that the manager, before observing the value of the firm, commits to a reporting system that maps between firm value and a public report. This literature was initiated by Kamenica and Gentzkow (2011). For applied models in accounting, see, for example, Göx and Wagenhoffer (2009), Bertomeu and Cheynel (2015), Michaeli (2017), Jiang and Yang (2017), Dordzhieva, Laux, and Zheng (2020), Friedman, Hughes, and Michaeli (2020, 2021), Göx and Michaeli (2020), Gregor and Michaeli (2021), and Cianciaruso, Marinovic, and Smith (2020).

(2020) consider a disclosure problem where a manager faces analyst coverage.[3] In a model where the probability that an analyst observes perfect information is correlated with the probability that the manager is endowed with information, they find that analyst coverage can encourage or discourage early voluntary disclosures, depending on the correlation between the analyst's and the manager's information endowments. In contrast to Dye and Sridhar (1995) and Frenkel, Guttman, and Kremer (2020), we assume that the external signal arrives with certainty (e.g., a peer must release mandatory financial reports), i.e., it is not correlated with the information endowment of the manager. Nevertheless, in our model, this signal affects the investor beliefs about the manager's endowment with information. Furthermore, in our model, the signal is imperfectly informative, and the manager may disclose both before and after the external signal. Our paper also relates to the work of Einhorn (2018), who analyzes voluntary disclosures in the presence of competing information sources and explains a deviation from full disclosure equilibrium to one with partial and selective disclosure.

Our analysis of early disclosure relates to the literature on uncertain investor reactions (e.g., Dutta and Trueman 2002; Suijs 2007). While this literature focuses on uncertainty pertaining to how investors react to disclosure, in our model, the uncertainty pertains to how investors react to nondisclosure. Our analysis of late disclosure relates to the literature studying shifts in the investors' pre-disclosure expectations of the firm value (Jung and Kwon 1988; Acharya, DeMarzo, and Kremer 2011). In addition to affecting the investors' expectation of firm value, the external signal in our model also affects investors' beliefs about the signal's relevance and the manager's information endowment.

Several studies also consider dynamic disclosure models. In a model with a manager who cares about stock prices in two periods, Guttman, Kremer, and Skrzypacz (2014) find that disclosing in the second period is interpreted more favorably. Menon (2020) allows the manager to become informed and an external signal to be realized at either

[3]In a related model, Ebert, Schäfer, and Schneider (2019) study different types of information leaks.

of two periods. If informed, the manager can disclose after the signal realization in each period. In contrast, Acharya, DeMarzo, and Kremer (2011) allow the manager to disclose before or after the realization of the external signal. Like us, Menon (2020) and Acharya, DeMarzo, and Kremer (2011) find that the probability of late disclosure is monotonically decreasing in the value of the external signal. However, Acharya, DeMarzo, and Kremer (2011) find that managers endowed with favorable information prefer to disclose before the realization of external news. In our model, this occurs only if delaying disclosure is costly.

2 Economic setting and preliminaries

2.1 Model setup

We extend the voluntary disclosure framework with uncertain information endowment studied in Dye (1985) and Jung and Kwon (1988) by considering an additional source of public information.

Players and payoffs. The model entails a manager ("she") who runs a firm with terminal value $v \in [v_{min}, v_{max}]$. For simplicity, we normalize $v_{min} = 0$ and $v_{max} = 1$. The manager maximizes the market price $P(\Omega) = \mathbb{E}[v \mid \Omega]$, where Ω is the publicly available information. Investors observe Ω, form beliefs, and price the firm. In the main part of the paper, we assume that the price is set once after all public information Ω is realized. In Section 7, we relax this assumption and allow investors to adjust the market price more frequently.

Information structure. The common prior belief is that v is drawn from a differentiable cumulative distribution function G (with a corresponding probability distribution function g) and has a prior expectation of $\mathbb{E}[v] = \mu$. The publicly available information Ω consists of an exogenous signal and the manager's voluntary disclosure:

1. *Exogenous signal.* There is an exogenous public signal s (e.g., mandatory financial report of a peer) which may or may not be relevant—for example, the information contained in a peer's report may or may not predict the market demand of the manager's firm. In particular, we assume that with probability $q \in (0, 1)$, the signal is perfectly informative about the firm value, $s = v$. We say that the signal in this case is relevant ($\sigma = R$). With probability $1 - q$, the signal is irrelevant ($\sigma = N$) and $s = x \in [0, 1]$ where x is drawn from the same probability distribution as the firm value.[4,5] Neither the manager nor the investors observe σ.

2. *Manager's information endowment and voluntary disclosure.* With probability $p \in (0, 1)$, the manager is informed about the firm value ($\kappa = I$), and otherwise is uninformed ($\kappa = U$).[6] We assume that κ is independent of v and σ and unknown to investors. An uninformed manager cannot credibly communicate the lack of information and has no choice but remain silent ($d = \varnothing$). An informed manager can voluntarily disclose the firm value ($d = v$) to investors at no cost either before the arrival of the external signal (under the "early disclosure" scenario) or after that (under the "late disclosure" scenario). For example, because peers may have different fiscal end of year, disclosures could end up being early or late. Section 4 focuses on late disclosure and Section 5 on early disclosure. In Section 6, we consider dynamic disclosure. Following the voluntary disclosure literature, we assume that any disclosed value is verifiable and thus truthful.

Timeline. The timeline of events is illustrated in Figure 1. At date 1, the manager observes the firm value v with probability p. At date 3, the external signal s is realized.

[4]For simplicity, we assume that relevant signals are perfectly informative and irrelevant ones are pure noise. Our results qualitatively hold in a setting where relevant signals are more informative about the firm value than irrelevant ones.

[5]The support of an irrelevant external signal is the same as that of a relevant one. If this were not the case, the investors could learn whether the external signal is relevant for some signal values.

[6]An informed manager can compare her information with the signal—but only once it is realized—and infer whether the latter is relevant or not.

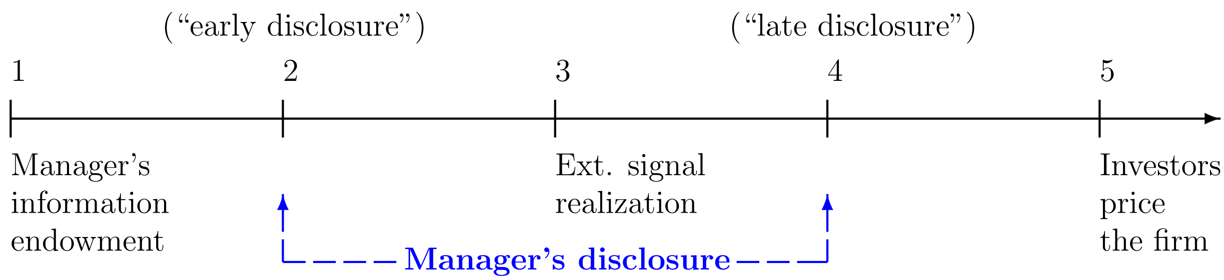

Figure 1: Timeline of events

The manager decides whether to disclose v either at date 2 (early disclosure) or date 4 (late disclosure). At date 5, the investors price the firm.

2.2 Benchmark without external signal

We consider as a benchmark the case in which there is no external signal (Dye 1985; Jung and Kwon 1988) or, equivalently, a case in which the signal is irrelevant with certainty ($q = 0$). We refer to this benchmark as the “Dye model” and use superscript “D.”

The price following disclosure is $P(v) = v$. The price following nondisclosure is

$$P(\varnothing) \quad = \quad \Pr(I \mid \varnothing) \cdot \mathbb{E}[v \mid v \leq v^{\mathrm{D}}] + \Pr(U \mid \varnothing) \cdot \mu \tag{1}$$

when the investors conjecture that an informed manager discloses all values above a threshold v^{D}.[7] In equilibrium, when the manager observes $v = v^{\mathrm{D}}$, she is indifferent between disclosing and withholding her information.[8]

Lemma 0. [**Dye 1985; Jung and Kwon 1988**] *When* $q = 0$*, there exists a unique*

[7]Here, $\Pr(I \mid \varnothing) = \frac{p \cdot G(v^{\mathrm{D}})}{1-p+p \cdot G(v^{\mathrm{D}})}$ and $\Pr(U \mid \varnothing) = 1 - \Pr(I \mid \varnothing)$. Note that these conditional probabilities also depend on the disclosure threshold—but we suppress it to avoid clutter.

[8]From a technical perspective, the unique threshold equilibrium arises because, while the market price following disclosure is equal to the state v (and thus increasing at a rate of 1 with the state), the price following nondisclosure is constant and always between the lowest possible realization of v and the prior mean μ. This ensures that the manager prefers to disclose high values and withhold low ones.

threshold $v^D \in (0, \mu)$*, such that the manager discloses if* $v > v^D$ *and withholds otherwise. The threshold* v^D *is decreasing in the probability of information endowment,* p*.*

The key insight of the Dye model is that informed managers can withhold information and pretend to be uninformed because the market is uncertain about their information endowment. In this paper, we show that potentially relevant (exogenous) signals affect the ability of managers to pretend to be uninformed in two ways. First, because the external signal provides additional information about firm value, it directly affects the market price and the manager's disclosure decision. Second, the signal also affects investor beliefs regarding the manager's information endowment and the signal's relevance. As a result, the signal indirectly affects the market price and the manager's disclosure decision. We dub this the "indirect reinforcement effect."

3 Investor beliefs and market price

We now consider the full-fledged setting with an external signal. Similar to the benchmark case, the market price when the manager discloses ($d = v$) is determined only by the disclosed value,

$$P(s, v) = \mathbb{E}[v \mid v, s] = v. \tag{2}$$

In this case, the investors disregard the external signal because the manager observes the firm value precisely and her disclosure is truthful.

When the manager remains silent ($d = \varnothing$), the market price depends on the investor beliefs about the manager's information endowment and the signal's relevance. Specifically, the investors consider four possible events: (i) the manager is uninformed and the signal is irrelevant; (ii) the manager is uninformed and the signal is relevant; (iii) the manager is informed and the signal is irrelevant; (iv) the manager is informed and the signal is relevant. Therefore the market price can be expressed as:

$$\begin{aligned} P(s,\varnothing) &= \mathbb{E}[v \mid s,\varnothing] \\ &= \Pr(U,N \mid s,\varnothing)\cdot\mathbb{E}[v \mid U,N,s,\varnothing] + \Pr(U,R \mid s,\varnothing)\cdot\mathbb{E}[v \mid U,R,s,\varnothing] \\ &\quad + \Pr(I,N \mid s,\varnothing)\cdot\mathbb{E}[v \mid I,N,s,\varnothing] + \Pr(I,R \mid s,\varnothing)\cdot\mathbb{E}[v \mid I,R,s,\varnothing]. \end{aligned} \tag{3}$$

To simplify (3), consider the case where the investors believe that the signal is irrelevant and the manager is uninformed. Then, because there is nothing to be learned from the signal and the manager's silence, the market expectation about firm value is simply the prior, $\mathbb{E}[v \mid U,N,s,\varnothing] = \mathbb{E}[v] = \mu$. If investors believe the signal is relevant, their expectation of the firm value is simply the signal, $\mathbb{E}[v \mid \kappa,R,s,\varnothing] = s$, regardless of whether they believe the manager is informed ($\kappa = I$) or not ($\kappa = U$). This is because a relevant signal perfectly reflects the firm value.

Lastly, consider the market expectation about v when investors believe the manager is informed and the signal is irrelevant. In this case, there is nothing to be learned from the signal, and so the investors disregard it. The nondisclosure decision in this case, however, indicates that the manager prefers to withhold the observed value. In Sections 5 and 4, we formally establish the existence of a unique equilibrium whereby, under both (late and early) disclosure scenarios, the manager withholds values that are lower than some threshold $\widehat{v} \in [0,1]$ and discloses otherwise.[9] For now, we assume this threshold equilibrium indeed exists. Thus the market expectation about the firm value in this case is $\mathbb{E}[v \mid I,N,s,\varnothing] = \mathbb{E}[v \mid v \le \widehat{v}]$. We can simplify the nondisclosure market price in (3),

$$\begin{aligned} P(s,\varnothing) &= \Pr(U,N \mid s,\varnothing)\cdot\mu + \Pr(U,R \mid s,\varnothing)\cdot s \\ &\quad + \Pr(I,N \mid s,\varnothing)\cdot\mathbb{E}[v \mid v \le \widehat{v}] + \Pr(I,R \mid s,\varnothing)\cdot s. \end{aligned} \tag{4}$$

[9]We use $\widehat{v}$ for a generic threshold. In the following sections, we denote v^{E} and $v^{\mathrm{L}}(s)$ as the thresholds under early and late scenarios, respectively.

Our next result illustrates that, even though the manager's information endowment κ and the signal relevance σ are conditionally independent, the beliefs of the investors about the occurrence of these events are affected by the observed signal in a nontrivial way.

Proposition 1. *Let* $\gamma(s,\widehat{v}) = q(1-\mathbf{1}_{s>\widehat{v}}\cdot p) + (1-q)\big(1-p+pG(\widehat{v})\big)$. *The joint investor beliefs, conditional on* $d=\varnothing$ *and* s, *are given by*

$$\begin{aligned}
\Pr(I,R|s,\varnothing) &= \frac{qp\cdot\mathbf{1}_{s\leq\widehat{v}}}{\gamma(s,\widehat{v})}, \\
\Pr(I,N|s,\varnothing) &= \frac{(1-q)pG(\widehat{v})}{\gamma(s,\widehat{v})}, \\
\Pr(U,R|s,\varnothing) &= \frac{q(1-p)}{\gamma(s,\widehat{v})}, \\
\Pr(U,N|s,\varnothing) &= \frac{(1-q)(1-p)}{\gamma(s,\widehat{v})}.
\end{aligned}$$

At the heart of our result is the manager's disclosure behavior and the information that can be inferred from it about the external signal. In an equilibrium with disclosure threshold $\widehat{v}$, the investors understand that an informed manager who did not disclose must have observed $v\leq\widehat{v}$.[10] Thus, if the external signal exceeds the disclosure threshold ($s>\widehat{v}$), it will be rationally inconsistent for the investors to believe that the signal is relevant and that the manager is informed at the same time. Thus they infer that $\Pr(I,R\mid s>\widehat{v},\varnothing)=0<\Pr(I)\Pr(R)$. In contrast, investors perceive a signal below the disclosure threshold ($s\leq\widehat{v}$) as more likely to be relevant, which strengthens their joint beliefs that the nondisclosing manager is also informed—in this case, $\Pr(I,R\mid s\leq\widehat{v},\varnothing)>\Pr(I)\Pr(R)$.

The preceding discussion implies that unfavorable s strengthens the beliefs of investors that the external information is relevant and that the nondisclosing manager is informed. Conversely, favorable s weakens the investor beliefs—such information is less likely to accurately describe the value of a firm run by an informed, nondisclosing manager. We

[10] We formally prove the existence of the threshold equilibrium below.

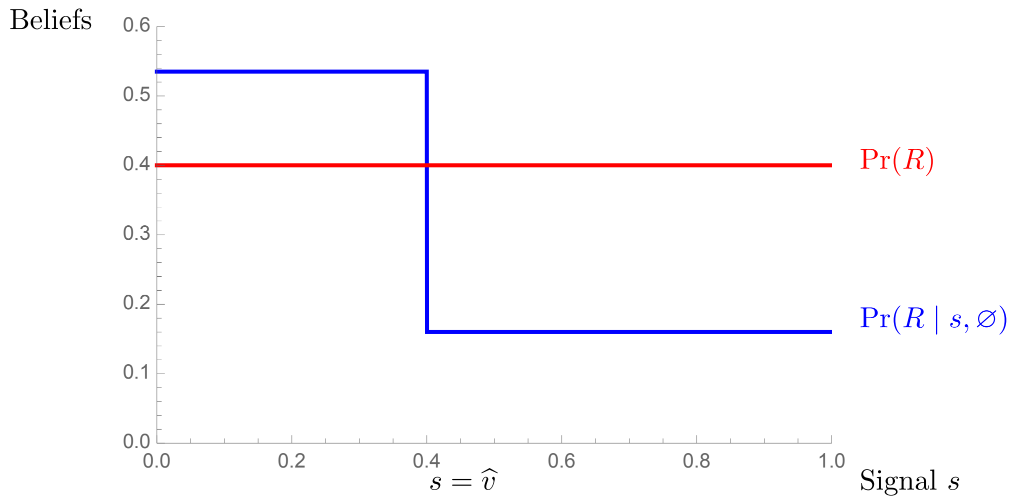

Figure 2: Conditional and unconditional probability that the signal is relevant
Numerical example with uniform distributions of v and x, $p = 0.6$, $q = 0.4$ and $\widehat{v} = 0.4$.

refer to this effect as the signal's "indirect reinforcement" of investor beliefs, which is summarized in the next corollary and illustrated in Figure 2.

Corollary 1. *The conditional probability that the signal is relevant is*

$$\Pr(R|s,\varnothing) = \begin{cases} q\left(\frac{p}{q+(1-q)G(\widehat{v})} + (1-p)\right) > \Pr(R) & \textit{if } s \leq \widehat{v} \\ q(1-p) < \Pr(R) & \textit{if } s > \widehat{v}. \end{cases}$$

We are now ready to formally describe the market price following manager's disclosure d and external signal s.

Proposition 2.

(i) The disclosure price is constant in the external signal: $\frac{\partial}{\partial s} P(s,v) = 0$.

(ii) For a disclosure threshold $\widehat{v}$*, the nondisclosure price is given by*

$$P(s,\varnothing) = \begin{cases} \mathbb{E}[v|s \leq \widehat{v},\varnothing] = \frac{q\cdot s+(1-q)((1-p)\mu+pG(\widehat{v})\mathbb{E}[v|v\leq\widehat{v}]))}{q+(1-q)(1-p+pG(\widehat{v}))} & \textit{if } s \leq \widehat{v}, \\ \mathbb{E}[v|\widehat{v} < s,\varnothing] = \frac{q(1-p)\cdot s+(1-q)((1-p)\mu+pG(\widehat{v})\mathbb{E}[v|v\leq\widehat{v}])}{q(1-p)+(1-q)(1-p+pG(\widehat{v})))} & \textit{if } s > \widehat{v}, \end{cases}$$

where $\frac{\partial P(s,\varnothing|s\leq\widehat{v})}{\partial s} > \frac{\partial P(s,\varnothing|s>\widehat{v})}{\partial s} > 0$ *and* $\lim_{s\to\widehat{v}^-} P(s,\varnothing) \gtreqless \lim_{s\to\widehat{v}^+} P(s,\varnothing)$ *if* $\widehat{v} \gtreqless v^D$.

Three immediate observations arise. First, because investors are uncertain about the relevance of the signal, they never price the firm at s. Second, favorable external information does not necessarily increase stock prices. When the manager shares the firm value with investors, the market price does not depend on the external signal because the manager's information is accurate and her disclosure is truthful. When the manager remains silent, the market price is increasing in the signal but (except in the knife-edge case of $\widehat{v} = v^{\mathrm{D}}$) there is a discontinuity at $s = \widehat{v}$ such that $\lim_{s \to \widehat{v}^-} P(s, \varnothing) \neq \lim_{s \to \widehat{v}^+} P(s, \varnothing)$. This is because the investor beliefs are shifted when the signal reaches the disclosure threshold $\widehat{v}$. As a result, in the neighborhood of the disclosure threshold, better external news lead to a lower nondisclosure price if $\widehat{v} > v^{\mathrm{D}}$. Third, the investors' reaction is more sensitive to unfavorable signals than to favorable ones. The reason is that unfavorable signals are perceived to be more likely relevant than favorable signals (Lemma 1).

4 Late disclosure

We begin with a scenario in which the manager observes the external signal and then decides whether to disclose her information. This could happen when the peer's financial report (or restatement) is released before the manager's conference call.[11] The manager, if informed, can respond by disclosing the value she observed ("late disclosure"). Throughout this section, the superscript "L" denotes late disclosure.

When deciding whether to respond to the signal, the manager compares the disclosure price in (2) with the nondisclosure price in Proposition 2.

Proposition 3. *For any $s \in [0, 1]$, there exists a unique threshold $v^L(s) \in (0, 1)$, such that the informed manager responds if $v > v^L(s)$ and remains silent otherwise. The threshold $v^L(s)$ is increasing in q.*

[11] The manager may also observe the firm value only after the arrival of external information. The results in Section 4 hold qualitatively under such alternative timeline.

Because of their uncertainty about the signal relevance, the investors do not price the firm at s (except for a knife-edge case). Managers observing low v benefit from this uncertainty because (i) relevant signals (that equal the observed v) are given a low weight in the price formation and (ii) irrelevant signals are likely to be more favorable than the true value of the firm. Thus managers observing relatively low v prefer to remain silent. The opposite holds for managers observing high firm value—they do not benefit from the investors' uncertainty and prefer to respond by "correcting" external signals.[12]

Proposition 4.

(i) If $s \lesseqqgtr v^D$ *then* $v^L(s) \gtreqqless s$ *and* $v^L(s) \lesseqqgtr v^D$.

(ii) $v^L(s)$ *is continuous in* s *and* $1 > \frac{\partial}{\partial s} v^L(s \mid s \leq v^D) > \frac{\partial}{\partial s} v^L(s \mid s > v^D) > 0$.

It seems intuitive that managers respond and "correct" unfavorable external signals. Our result, however, shows that this intuition is not always correct. If the external signal is sufficiently low ($s < v^{\mathrm{D}}$), the disclosure threshold exceeds it ($v^{\mathrm{L}}(s) > s$); i.e., the manager withholds values that are *more favorable* than the ones revealed by the external signal. Conversely, if the external signal is sufficiently high ($s > v^{\mathrm{D}}$), the disclosure threshold falls short of it ($v^{\mathrm{L}}(s) < s$); i.e., the manager discloses values that are *less favorable* than the ones revealed by the external signal. Our result may at first seem perplexing: why would anyone withhold favorable news but reveal unfavorable news? The answer is simple: the manager discloses when the observed value exceeds the nondisclosure price. Because the latter is not identical to s (recall that investors are skeptical about the signal's relevance and underreact to it), remaining silent in the face of some unfavorable s and "speaking up" to correct some favorable external news may be beneficial.

[12]From a technical perspective, the equilibrium arises because the nondisclosure price in Proposition 2 is constant in v for a given (fixed) external signal s. In addition, because of the investors' uncertainty regarding the relevance of the external source and the weight put on the prior expectation, the nondisclosure price is always in $(0, 1)$. (It is above 0 even when $s = 0$ and below 1 even when $s = 1$.)

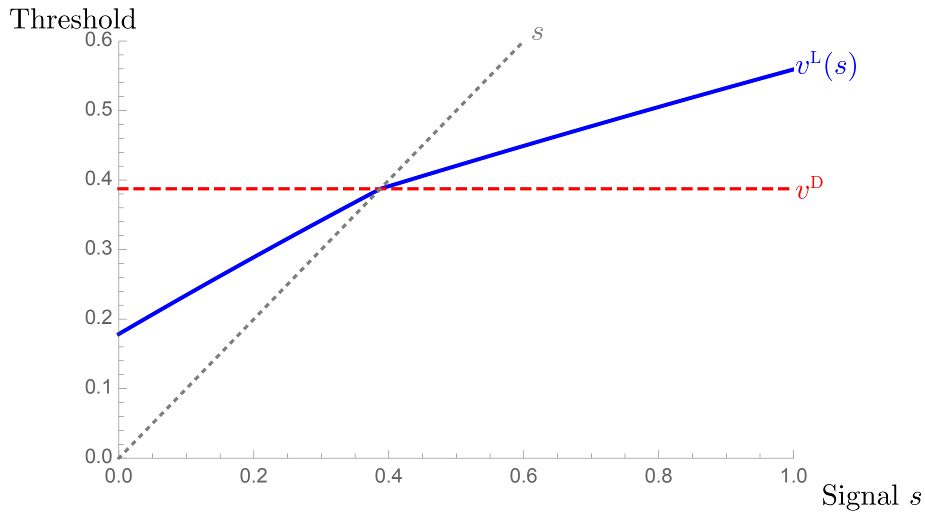

Figure 3: Early disclosure threshold as a function of s

Numerical example with uniform distributions of v and x, $p = 0.6$ and $q = 0.4$.

Our result also implies that the equilibrium threshold is increasing in the external signal and is more sensitive to unfavorable news (Figure 3). Furthermore, the equilibrium threshold is continuous in s even though the nondisclosure price differs for favorable and unfavorable external news. The intuition for this result can be found in part (ii). The exact signal realization that distinguishes between the two pricing options is $s = v^{\mathrm{D}}$. If the external signal is below this value, it is also above the late disclosure threshold. And vice versa, if s is above this value it is below $v^{\mathrm{L}}(s)$. At the critical value, it holds that $s = v^{\mathrm{D}} = v^{\mathrm{L}}(s)$ at which point the two pricing options coincide.

Lastly, whether external news encourage or discourage disclosure depends on s. Given that $v^{\mathrm{L}}(s)$ is an increasing function, a manager facing a sufficiently unfavorable (favorable) signal is less (more) likely to disclose compared with the benchmark case. Only a signal $s = v^{\mathrm{D}}$ has no effect on the probability of disclosure.

5 Early disclosure

We now consider the alternative case of early disclosure where the manager decides whether to disclose her private information before the arrival of the external signal. This could happen when the manager has a scheduled conference call before the information included in financial statements of the firm's peers becomes publicly available.[13] Throughout this section, we use superscript "E" to denote the early disclosure scenario.

A manager who is not endowed with information has no choice but remain silent. An informed manager has to consider the market price following nondisclosure, $P(s, \varnothing)$. Because this price depends on a signal that is not yet available at the time of the early disclosure decision, the manager has to form an expectation about it, conditional on the observed v. Note that v carries information about the signal because the latter may, with some probability, accurately reflect firm value. From the manager's perspective at date 2, the future external signal will be either relevant or irrelevant. Thus a straightforward way to think about the expected price is to present it as

$$\mathbb{E}[P(s, \varnothing) \mid v] = \Pr(R \mid v) \cdot \mathbb{E}[P(s, \varnothing) \mid R, v] + \Pr(N \mid v) \cdot \mathbb{E}[P(s, \varnothing) \mid N, v].$$

Because the signal is not realized yet, the observed value v carries no information about the signal relevance—thus $\Pr(\sigma \mid v) = \Pr(\sigma)$, $\sigma \in \{R, N\}$. The manager expects that a relevant signal equals the observed value v. Hence, with probability $\Pr(R) = q$, the expected nondisclosure price is $\mathbb{E}[P(s, \varnothing) \mid R, v] = P(s = v, \varnothing)$. An irrelevant signal equals x so that, with probability $\Pr(N) = 1 - q$, the expected price is $\mathbb{E}[P(s, \varnothing) \mid N, v] = \mathbb{E}[P(s = x, \varnothing)]$.

When deciding whether to disclose, an informed manager compares her expectation of the nondisclosure price with the price in case of disclosure. Because both prices *depend* on v, it is not immediately obvious that a threshold equilibrium exists. Our next result establishes its existence.

Proposition 5. *There exists a unique threshold $v^E \in (v^D, \mu)$, such that an informed*

[13] Models of early disclosure are also considered by Frenkel, Guttman, and Kremer (2020) and Ebert, Schäfer, and Schneider (2019).

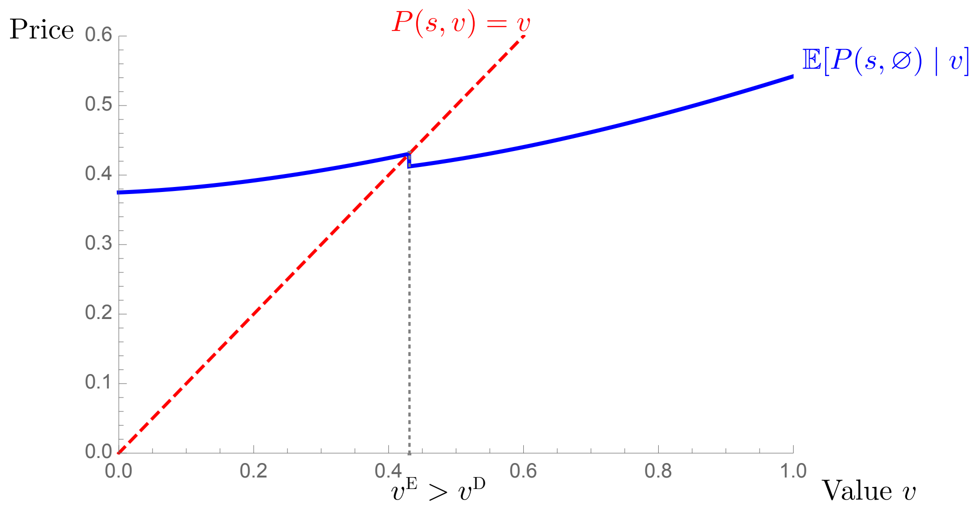

Figure 4: Market price and equilibrium (early) disclosure threshold
Numerical example with uniform distributions of v and x, $p = 0.6$ and $q = 0.4$.
The disclosure price is illustrated with the dashed red line, and the expected nondisclosure price with the solid blue line.

manager discloses the observed firm value if $v > v^E$ *and withholds it otherwise.*

Because investors are uncertain about the manager's information endowment, the firm value does not unravel. Thus, as in the benchmark case, an informed manager can pretend to be uninformed. Moreover, the investors are uncertain about the relevance of the external signal and do not price the firm at s. An informed manager expects that, if she remains silent, the market will overvalue the firm if v is very low but will undervalue it if v is very high. To maximize the market price, the manager prefers to disclose high values but withhold low ones.

Importantly, the expected arrival of external news crowds out managerial disclosure compared with the baseline benchmark case. Managers who withhold under the benchmark continue to withhold in the presence of external information. Moreover, managers observing $v \in [v^{\mathrm{D}}, v^{\mathrm{E}}]$ also withhold their information—we predict that external information crowds-out voluntary disclosure. Another way to interpret our result is to say that some managers are better off when their information is revealed by a potentially relevant external source rather than when they directly disclose it. This is driven by the investors' uncertainty about the relevance of external news and the manager's information endowment. Specifically, if investors believe that the external signal is relevant, they react the same way as they would have if the manager had disclosed the information. However, if they believe that the external source is irrelevant, the investors assign a higher likelihood that the manager is uninformed and hence put more weight on the prior expectation. Thus some managers observing values below μ benefit from relying on the external source to reveal those values.

Lastly, when the manager can only disclose before the external signal (early), the nondisclosure price is nonmonotonic in the external signal in the sense that, even though this price is increasing in s there is a discontinuity at the threshold such that $\lim_{s \to v^{\mathrm{E}-}} P(s, \varnothing) > \lim_{s \to v^{\mathrm{E}-}} P(s, \varnothing)$ (by Proposition 2 and because $v^{\mathrm{E}} > v^{\mathrm{D}}$). In words, in the neighborhood of the early disclosure threshold, the firm's price may decrease after favorable external news. This may explain the empirical evidence of stock price decreases after the release of good news from a peer firm (Sletten 2012).

6 Dynamic disclosure

Until now we assumed that corporate disclosures are either late or early for exogenous reasons. This could happen, for example, when managers respond to unanticipated events, such as restatements of financial reports by a peer firm (exogenous late disclosure), or can announce information only at a conference call scheduled before the release of the peers' financial statements (exogenous early disclosure). In practice, managers who are initially silent may often choose to disclose later, after the arrival of external information. To study such a setting, let $d_t = \{v, \varnothing\}$ be the manager's disclosure at date $t = 2, 4$. When pricing the firm at date 5, the investors consider all past disclosure decisions, $P(s, d_{t=2}, d_{t=4})$. In particular, if the manager discloses on either date, the price is simply v. However, if the manager remains silent on both dates, the price depends on the investors' beliefs about the manager's information endowment and the signal relevance in this case. These beliefs may differ from the ones when the manager can disclose either early or late (studied in the main part of our analysis).

If the manager discloses at $t = 2$, her information remains disclosed at $t = 4$; i.e., she cannot reverse her disclosure. If no disclosure is made at $t = 2$, the disclosure decision at $t = 4$ resembles that in our late disclosure scenario, but, as explained above, the market price considers also the nondisclosure at date 2. When deciding whether to disclose at $t = 2$, the manager compares the price after early disclosure, $P(s, d_{t=2} = v, d_{t=4}) = v$, with her expectation of the price after remaining silent at date 2, $\mathbb{E}_{t=2}[P(s, d_{t=2} = \varnothing, d_{t=4}) \mid v]$.

We begin by considering a scenario where neither delaying nor advancing disclosure is costly. Let $v_1 \equiv \max_s P(s, d_{t=2} = d_{t=4} = \varnothing) = P(s = 1, d_{t=2} = d_{t=4} = \varnothing)$. It is straightforward that $v_1 < 1$ because the market is unsure about the signal relevance. A manager who observes $v > v_1$ knows that, if she does not disclose at date 2, she will certainly disclose at date 4, regardless of the external signal. Thus the manager cannot benefit from delaying disclosure but cannot lose either—in either case, she obtains a market price of v. A manager who observes $v < v_1$ may benefit (but not lose) from delaying her decision if the external signal ends up being favorable—she obtains a market price of v by disclosing immediately and a weakly higher price if she delays. This intuition is summarized in the result below.

Proposition 6. *When delaying and advancing disclosure is cost-free, managers observing $v < v_1$ do not disclose at date 2, whereas managers observing $v > v_1$ are indifferent between disclosing at date 2 or date 4.*

In what follows, we relax the assumption that advancing and delaying disclosure is cost-free. Because only the difference between such costs affects the choice of timing, we focus only on two scenarios: one with cost $c_2 > 0$ for disclosure at date 2 and one with cost $c_4 > 0$ for disclosure at date 4. The analysis in the former case is straightforward. Using the discussion preceding Proposition 6, one can easily see that now all informed managers strictly prefer to delay disclosure. Thus, in equilibrium, there is only disclosure at date 4.

Lastly, we consider the case where managers incur cost for delayed disclosure. At date 2, when deciding whether to delay her disclosure decision, an informed manager compares her payoff from early disclosure, $P(s, d_{t=2} = v, d_{t_4}) = v$, with her expected payoff from delaying the disclosure decision to date 4, $\mathbb{E}_{t=2}[P(s, d_{t=2} = \varnothing, d_{t=4}) \mid v] - \Pr(d_{t=4} = v)c_4$. It is straightforward that, if c_4 is sufficiently high, the manager never delays her disclosure decision.[14] The more interesting case is when c_4 is not prohibitively high.

Proposition 7. *Suppose that the manager incurs cost $c_4 > 0$ for disclosing on date 4. There exist $\underline{c} \in (0, \infty)$ and $v_4 < v_1$ such that, when $c_4 < \underline{c}$, it holds that $d_{t=2} = v$ if $v > v_4$ and $d_{t=2} = \varnothing$ if $v < v_4$.*

Managers observing sufficiently high firm values prefer to disclose early and avoid the cost of delaying. The rest prefer to delay their decision until after the arrival of the external signal. If the signal turns out to be sufficiently favorable, then these managers benefit from remaining silent. Otherwise, they disclose and incur the cost of delay. The presence of cost c_4 makes managers observing $v \in (v_4, v_1)$ reluctant to delay.

Corollary 2. *When the cost for delaying is not prohibitively high, i.e., $c_4 < \underline{c}$, disclosures of favorable information are clustered before the arrival of external news, while disclosures of unfavorable information are clustered after it.*

Our result aligns with recent empirical evidence about clustering of managerial guidance with good news before the release of financial statements by peers and clustering of guidance with bad news after these releases (Sletten 2012).

[14] The early disclosure case in Section 5 can be viewed as a limit case where c_4 is sufficiently high.

7 Frequently adjusted market price

Until now, we have assumed that the market price is formed only once (at date 5), after all the public information is revealed. In capital markets, however, prices are adjusted more frequently—with the arrival of every piece of new information. In this section, we revisit our earlier results in a setting where investors set prices at every date. For simplicity, we return to our assumptions that disclosure is either early or late. Let $P_t^j(\Omega)$ be the price at date t under scenario $j = \{\mathrm{E}, \mathrm{L}\}$ and $\delta \in (0,1)$ be the discount factor faced by the manager.

7.1 Late disclosure with frequently adjusted market price

We begin with late disclosure. It is clear that the starting price is simply the prior expectation: $P_1^{\mathrm{L}} = \mathbb{E}[v] = \mu$. The price at date 2 remains at its date-1 level since no new information arrives. At date 3, immediately after observing the external signal and before any response (or lack thereof) by the manager, the market price is adjusted to $P_3^{\mathrm{L}}(s) = \mathbb{E}[v \mid s] = qs + (1-q)\mu$. As one would expect, favorable external signals increase the price, and unfavorable ones decrease it. Depending on whether the external signal is above or below the prior mean, the price at date 3 may be above or below the date-2 price; i.e., $P_2^{\mathrm{L}} \lesseqqgtr P_3^{\mathrm{L}}(s) \quad \Leftrightarrow \quad s \gtreqqless \mu$.

At date 4, the market price depends on the manager's disclosure: if she discloses, the price is $P_4^{\mathrm{L}}(s,v) = P(s,v) = v$. Otherwise, it is $P_4^{\mathrm{L}}(s,\varnothing) = P(s,\varnothing)$. Because there is no additional information arriving post disclosure, all future payments remain at their date-4 levels. Thus frequent adjustment of the market price only results in scaling of the same payments that the manager had in Section 4 and leaves the manager's decision unaffected.[15]

[15] When deciding whether to disclose, the manager compares the present value of her current and future payments from disclosure, $\Pi^{\mathrm{L}}(v) = P_4^{\mathrm{L}}(s,v) + \delta P_5^{\mathrm{L}}(s,v) = (1+\delta)P(s,v)$, with those from nondisclosure, $\Pi^{\mathrm{L}}(s,\varnothing) = P_4^{\mathrm{L}}(s,\varnothing) + \delta P_5^{\mathrm{L}}(s,\varnothing) = (1+\delta)P(s,\varnothing)$.

Corollary 3. *If at date 4 the manager does not disclose, it holds that* $P_4^L(s,\varnothing) < P_3^L(s)$ *for any* s*. If at date 4 the manager discloses, it holds that* $P_4^L(s,v) \lesseqgtr P_3^L(s)$ *if* $s \gtreqless \frac{v-(1-q)\mu}{q}$.

As one would expect, when the manager does not respond to the external signal, the price decreases (this is consistent with empirical findings in Sletten 2012). This is because the investors conclude that the manager may have observed an even lower value. However, the price may also decrease when the manager responds. This happens when the manager anticipates, after the external signal reveals, a further decrease in the price if she remains silent. To avoid this, the manager discloses values that, while lower than the date-3 price, exceed the price that would have prevailed if she had remained silent.

7.2 Early disclosure with frequently adjusted market price

We continue with the case of early disclosure. It is straightforward that $P_1^{\mathrm{E}} = \mathbb{E}[v] = \mu$ and $P_2^{\mathrm{E}}(v) = v$. If the manager withholds at date 2, the price is $P_2^{\mathrm{E}}(\varnothing)$, which resembles the one defined in (1) but for some conjectured by the investors threshold $\widehat{v}$ instead of v^{D}. (We derive the equilibrium threshold below.) At date 3, if the manager had disclosed at date 2, the market price would remain at its date-2 level, $P_3^{\mathrm{E}}(s,v) = P(s,v) = v$. However, if she had remained silent, the investors would adjust the price after observing the newly arrived external signal. Market prices at date 4 and 5 remain at their date-3 levels.

Do frequent price adjustments affect the manager's disclosure at date 2? An informed manager discloses if the present value of her current and future expected payoffs from disclosure,

$$\begin{aligned}\Pi^{\mathrm{E}}(v) &\equiv \mathbb{E}[P_2^{\mathrm{E}}(v) \mid v] + \delta\mathbb{E}[P_3^{\mathrm{E}}(s,v) \mid v] + \delta^2\mathbb{E}[P_4^{\mathrm{E}}(s,v) \mid v] + \delta^3\mathbb{E}[P_5^{\mathrm{E}}(s,v) \mid v] \\ &= (1+\delta+\delta^2+\delta^3)P(s,v),\end{aligned}$$

exceeds that from nondisclosure,

$$\begin{aligned}\Pi^{\mathrm{E}}(\varnothing) &\equiv \mathbb{E}[P_2^{\mathrm{E}}(\varnothing) \mid v] + \delta\mathbb{E}[P_3^{\mathrm{E}}(s,\varnothing) \mid v] + \delta^2\mathbb{E}[P_4^{\mathrm{E}}(s,\varnothing) \mid v] + \delta^3\mathbb{E}[P_5^{\mathrm{E}}(s,\varnothing)v] \\ &= P_2^{\mathrm{E}}(\varnothing) + (\delta+\delta^2+\delta^3)\mathbb{E}[P(s,\varnothing) \mid v].\end{aligned}$$

Proposition 8. *When the market price is frequently adjusted, there exists a threshold $\widetilde{v}^E \in (v^D, v^E)$ such that the manager discloses (early) if and only if $v > \widetilde{v}^E$. The (early) disclosure threshold $\widetilde{v}^E$ is increasing in the discount factor δ.*

Frequent adjustment of stock prices encourages corporate disclosure, $\widetilde{v}^{\mathrm{E}} < v^{\mathrm{E}}$. The main impetus is that now the manager faces a lower short-term price at date-2, relative to the expected market price after the signal is realized—this makes pretending to be uninformed less beneficial (in present value terms) and encourages disclosure. Nevertheless, external information continues to suppress corporate disclosure even when investors frequently adjust prices, $\widetilde{v}^{\mathrm{E}} > v^{\mathrm{D}}$. Our result that the equilibrium disclosure threshold is increasing in δ suggests that more impatient managers are more likely to disclose their firm value.[16]

It remains to consider the price trend over time. If the manager discloses at date 2, the disclosure price $P_2^{\mathrm{E}}(v) = v$ exceeds the initial price $P_1^{\mathrm{E}} = \mathbb{E}[v] = \mu$ if $v > \mu$. Thus, when the manager discloses $v \in [\widetilde{v}^{\mathrm{E}}, \mu)$, the price can decrease post disclosure. Why does the manager disclose values that are lower than the existing price? The answer is simple—it is because she expects that the present value of the future (expected) market price will be lower if she remains silent. This observation is summarized in the following corollary.[17]

Corollary 4. *The price following disclosure at date 2, $P_2^E(v)$, is lower than the initial price, P_1^E, if the observed and disclosed by the manager value is $v \in [\widetilde{v}^E, \mu)$.*

[16]Notably, extremely impatient managers ($\delta = 0$) disclose only values above v^{D}: they only care about the current period and not about the external information that will reveal in the next period. Extremely patient managers ($\delta = 1$), however, disclose only values above v^{E}.

[17]Following early disclosure, the prices at dates 3-5 remain at their date-2 levels.

If the manager withholds her information, the date-2 market price always decreases from its initial level, $P_2^{\mathrm{E}}(\varnothing) < P_1^{\mathrm{E}}$.[18] The realization of the external signal affects the date-3 market price in a nontrivial way.

Corollary 5. *There exists* $s^{\dagger\dagger} \in (0, \widetilde{v}^E)$ *such that:* $P_2^E(\varnothing) \geq P_3^E(s, \varnothing)$ *if* $s \in [0, s^{\dagger\dagger}]$ *or* $s \in [\widetilde{v}^E, \mu]$. *However,* $P_2^E(\varnothing) \leq P_3^E(s, \varnothing)$ *if* $s \in [s^{\dagger\dagger}, \widetilde{v}^E]$ *or* $s \in [\mu, 1]$.

As one would expect, very favorable external news increases the market price, relative to its date-2 level, and extremely unfavorable news decreases the price. However, Corollary 5 implies that more neutral news (s in the intermediate region) has an ambiguous effect (Figure 10). While the date-2 nondisclosure price $P_2^{\mathrm{E}}(\varnothing) = P(\varnothing)$ does not depend on the signal, the date-3 price, $P_3^{\mathrm{E}}(s, \varnothing) = P(s, \varnothing)$, is nonmonotonic in it (Proposition 2). Because news that exceeds the disclosure threshold is perceived to be less likely to be relevant (Proposition 1), the market price around $s = \widetilde{v}^{\mathrm{E}}$ drops in a sense that $\lim_{s \to \widetilde{v}^{\mathrm{E}-}} P_3^{\mathrm{E}}(s, \varnothing) > \lim_{s \to \widetilde{v}^{\mathrm{E}+}} P_3^{\mathrm{E}}(s, \varnothing)$. A signal $s \to \widetilde{v}^{\mathrm{E}+}$ leads to price decrease from its date-2 level. Conversely, a signal $s \to \widetilde{v}^{\mathrm{E}-}$ leads to price increase.

8 Concluding remarks

We study the effect of possibly relevant external information (e.g., mandatory financial reports of peers) on investor beliefs and managers' incentives to voluntarily disclose private information about their firms. We show that favorable external news is perceived to be less likely relevant, which reinforces the investors' beliefs that managers are endowed with unfavorable information. Thus favorable external news may lead to a lower market price if managers remain silent. As a result, managers may be more likely to disclose after favorable external news. Our results imply that the external information that is more likely relevant may encourage or discourage corporate disclosures, depending on

[18] To see why, note that $P_2^{\mathrm{E}}(\varnothing) = P(\varnothing \mid \widetilde{v}^{\mathrm{E}}) < P(\varnothing \mid v^{\mathrm{D}}) = v^{\mathrm{D}} < \mu = P_1^{\mathrm{E}}$.

their timing. Consistent with empirical evidence (Sletten 2012), we predict clustering of disclosures of favorable information before the arrival of external information and of unfavorable information after that.

Appendix

Proof of Lemma 0: Follows directly from the proof in Dye (1985) and Jung and Kwon (1998) and is omitted.

Proof of Proposition 1: We consider the joint belief $\Pr(\sigma, \kappa | s, \varnothing)$ separately for the cases $s < \widehat{v}$ and $s > \widehat{v}$.

Case 1: Let the observed signal be $s = s' \in [0, \widehat{v})$. Because s is a continuous variable, we use the limit of measurable intervals and express $\Pr(\sigma, \kappa | s = s', \varnothing) = \lim_{\Delta \to 0} \frac{\Pr(s \in [s', s'+\Delta], \varnothing, \sigma, \kappa)}{\Pr(s \in [s', s'+\Delta], \varnothing)}$ for $\sigma \in \{N, R\}$, $\kappa \in \{I, U\}$ and $\Delta > 0$ sufficiently small so that $s' + \Delta \in [0, \widehat{v})$. Let us define $A' \equiv [s', s' + \Delta]$ and focus on the numerator. We note that:

$$
\begin{aligned}
\Pr(s \in A', \varnothing, R, I) &= p \cdot q \cdot 1 \cdot \Pr(s \in A'); \\
\Pr(s \in A', \varnothing, R, U) &= (1-p) \cdot 1 \cdot q \cdot \Pr(s \in A'); \\
\Pr(s \in A', \varnothing, N, I) &= p \cdot G(\widehat{v}) \cdot (1-q) \cdot \Pr(s \in A'); \\
\Pr(s \in A', \varnothing, N, U) &= (1-p) \cdot (1-q) \cdot \Pr(s \in A').
\end{aligned}
$$

For the denominator in $\Pr(\sigma, \kappa | s = s', \varnothing)$, we have $\Pr(s \in A', \varnothing) = \Pr(s \in A', \varnothing, R, I) + \Pr(s \in A', \varnothing, R, U) + \Pr(s \in A', \varnothing, N, I) + \Pr(s \in A', \varnothing, N, U) = \Big(q + (1-q)\big(1 - p + pG(\widehat{v})\big)\Big) \cdot \Pr(s \in A')$. Now we can express:

$$
\begin{aligned}
\Pr(U, R | s = s', \varnothing) &= \lim_{\Delta \to 0} \frac{\Pr(s \in A', \varnothing, R, U)}{\Pr(s \in A', \varnothing)} \\
&= \frac{(1-p)q}{q + (1-q)\big(1 - p + pG(\widehat{v})\big)} \cdot \lim_{\Delta \to 0} \frac{\cancel{\Pr(s \in A')}}{\cancel{\Pr(s \in A')}}; \\
\Pr(U, N | s = s', \varnothing) &= \lim_{\Delta \to 0} \frac{\Pr(s \in A', \varnothing, N, U)}{\Pr(s \in A', \varnothing)} \\
&= \frac{(1-p)(1-q)}{q + (1-q)\big(1 - p + pG(\widehat{v})\big)} \cdot \lim_{\Delta \to 0} \frac{\cancel{\Pr(s \in A')}}{\cancel{\Pr(s \in A')}}; \\
\Pr(I, R | s = s', \varnothing) &= \lim_{\Delta \to 0} \frac{\Pr(s \in A', \varnothing, R, I)}{\Pr(s \in A', \varnothing)} \\
&= \frac{pq}{q + (1-q)\big(1 - p + pG(\widehat{v})\big)} \cdot \lim_{\Delta \to 0} \frac{\cancel{\Pr(s \in A')}}{\cancel{\Pr(s \in A')}}; \\
\Pr(I, N | s = s', \varnothing) &= \lim_{\Delta \to 0} \frac{\Pr(s \in A', \varnothing, N, I)}{\Pr(s \in A', \varnothing)} \\
&= \frac{pG(\widehat{v})(1-q)}{q + (1-q)\big(1 - p + pG(\widehat{v})\big)} \cdot \lim_{\Delta \to 0} \frac{\cancel{\Pr(s \in A')}}{\cancel{\Pr(s \in A')}}.
\end{aligned}
$$

Case 2: Let the observed signal be $s = s'' \in (\widehat{v}, 1]$. Defining $A'' \equiv [s'', s'' + \Delta]$ for $\Delta > 0$

sufficiently small, so that $s'' + \Delta \in (\widehat{v}, 1]$, and following similar steps,

$$\begin{aligned}
\Pr(s \in A'', \varnothing, R, I) &= 0 \cdot p \cdot G(\widehat{v}) \cdot \Pr(s \in A'');\\
\Pr(s \in A'', \varnothing, R, U) &= (1-p) \cdot q \cdot \Pr(s \in A'');\\
\Pr(s \in A'', \varnothing, N, I) &= p \cdot G(\widehat{v}) \cdot (1-q) \cdot \Pr(s \in A'');\\
\Pr(s \in A'', \varnothing, N, U) &= (1-p) \cdot (1-q) \cdot \Pr(s \in A'').
\end{aligned}$$

Summing up, we have $\Pr(s \in A'', \varnothing) = \Big(q(1-p) + (1-q)\big((1-p) + p \cdot G(\widehat{v})\big)\Big) \cdot \Pr(s \in A'')$. We can express

$$\begin{aligned}
\Pr(U, R|s = s'', \varnothing) &= \frac{(1-p)q}{q(1-p) + (1-q)\big((1-p) + p \cdot G(\widehat{v})\big)} \cdot \lim_{\Delta \to 0} \frac{\cancel{\Pr(s \in A'')}}{\cancel{\Pr(s \in A'')}};\\
\Pr(U, N|s = s'', \varnothing) &= \frac{(1-p)(1-q)}{q(1-p) + (1-q)\big((1-p) + p \cdot G(\widehat{v})\big)} \cdot \lim_{\Delta \to 0} \frac{\cancel{\Pr(s \in A'')}}{\cancel{\Pr(s \in A'')}};\\
\Pr(I, R|s = s'', \varnothing) &= 0;\\
\Pr(I, N|s = s'', \varnothing) &= \frac{pG(\widehat{v})(1-q)}{q(1-p) + (1-q)\big((1-p) + p \cdot G(\widehat{v})\big)} \cdot \lim_{\Delta \to 0} \frac{\cancel{\Pr(s \in A'')}}{\cancel{\Pr(s \in A'')}}.
\end{aligned}$$

Summarizing yields our result.

Proof of Corollary 1:

$$\begin{aligned}
\Pr(R|s \leq \widehat{v}, \varnothing) &= \Pr(R|s \leq \widehat{v}, \varnothing, I)\Pr(I) + \Pr(R|s \leq \widehat{v}, \varnothing, U)\Pr(U)\\
&= \tfrac{q}{q+(1-q)G(\widehat{v})} p + q(1-p) = q\big(\tfrac{1}{q+(1-q)G(\widehat{v})} p + (1-p)\big)\\
&= q\big(1 + p\big(\tfrac{1}{q+(1-q)G(\widehat{v})} - 1\big)\big) > q = \Pr(R)
\end{aligned}$$

Note that the last inequality holds because $q + (1-q)G(\widehat{v}) < 1$ for any $\widehat{v} < 1$.

$$\begin{aligned}
\Pr(R|\widehat{v} < s, \varnothing) &= \Pr(R|\widehat{v} < s, \varnothing, I)\Pr(I) + \Pr(R|\widehat{v} < s, \varnothing, U)\Pr(U)\\
&= 0 \cdot p + q(1-p) < q = \Pr(R).
\end{aligned}$$

Proof of Proposition 2: (i) Follows from the notion that the external signal is never more informative than the disclosed firm value. (ii) Follows directly from equation (4) and Proposition 1.

Proof of Proposition 3: Suppose that the investors conjecture that the disclosure threshold is $\widehat{v}$ and let

$$\begin{aligned}
h_b(s \leq \widehat{v}, p, q) \equiv P(s, \varnothing \mid s \leq \widehat{v}) &= \tfrac{q \cdot s + (1-q)((1-p)\mu + pG(\widehat{v})\mathbb{E}[v|v \leq \widehat{v}])}{q+(1-q)(1-p+pG(\widehat{v}))}\\
h_a(\widehat{v} < s, p, q) \equiv P(s, \varnothing \mid s > \widehat{v}) &= \tfrac{q \cdot (1-p) \cdot s + (1-q)((1-p)\mu + pG(\widehat{v})\mathbb{E}[v|v \leq \widehat{v}])}{q(1-p)+(1-q)(1-p+pG(\widehat{v}))}.
\end{aligned}$$

We now show that, for any s, a unique equilibrium exists.

Case (i): Assume that $s < \widehat{v}$:
In equilibrium, it has to hold that $T_b^{\mathrm{L}}(\widehat{v}) = 0$ where $T_b^{\mathrm{L}}(\widehat{v}) \equiv h_b(s \leq \widehat{v}, p, q) - P(s, v = \widehat{v})$. Note that $T_b^{\mathrm{L}}(\widehat{v})$ is decreasing in $\widehat{v}$ and that $\lim\limits_{\widehat{v}\to 0} P(s, v = \widehat{v}) = 0$ whereas $\lim\limits_{\widehat{v}\to 0} h_b(s \leq \widehat{v}, p, q) = \frac{qs+(1-q)(1-p)\mu}{q+(1-q)(1-p)} > 0$. Furthermore, $\lim\limits_{\widehat{v}\to 1} P(s, v = \widehat{v}) = 1$ whereas $\lim\limits_{\widehat{v}\to 1} h_b(s \leq \widehat{v}, p, q) = qs + (1-q)\mu < 1$. Thus, there exists a unique threshold $\widehat{v} = v^{\mathrm{L}}(s) \in (0, 1)$ that satisfies $T_b^{\mathrm{L}}(\widehat{v} = v^{\mathrm{L}}(s)) = 0$.

Case (2) Assume that $\widehat{v} < s$:
In equilibrium, it has to hold that $T_a^{\mathrm{L}}(\widehat{v}) = 0$ where $T_a^{\mathrm{L}}(\widehat{v}) \equiv h_a(s > \widehat{v}, p, q) - P(s, v = \widehat{v})$. Note that $T_a^{\mathrm{L}}(\widehat{v})$ is decreasing in $\widehat{v}$ and that $\lim\limits_{\widehat{v}\to 0} P(s, v = \widehat{v}) = 0$ whereas $\lim\limits_{\widehat{v}\to 0} h_a(\widehat{v} < s, p, q) = qs + (1-q)\mu > 0$. Furthermore, $\lim\limits_{\widehat{v}\to 1} P(s, v = \widehat{v}) = 1$ whereas $\lim\limits_{\widehat{v}\to 1} h_a(\widehat{v} < s, p, q) = \frac{q(1-p)s+(1-q)\mu}{q(1-p)+(1-q)} < 1$. Thus, there exists a unique threshold $\widehat{v} = v^{\mathrm{L}}(s) \in (0, 1)$ that satisfies $T_a^{\mathrm{L}}(\widehat{v} = v^{\mathrm{L}}(s)) = 0$.

Proof of Proposition 4:

(i) We begin by showing $\lim\limits_{s\to\widehat{v}^-} h_b(s \leq \widehat{v}, p, q) = \lim\limits_{s\to\widehat{v}^+} h_a(\widehat{v} < s, p, q) \Leftrightarrow s = v^{\mathrm{D}}$:

$$
\begin{aligned}
&\lim_{s\to\widehat{v}^-} h_b(s \leq \widehat{v}, p, q) = \lim_{s\to\widehat{v}^+} h_a(\widehat{v} < s, p, q) \\
&\Rightarrow \lim_{s\to\widehat{v}^-} \tfrac{q\cdot s+(1-q)((1-p)\mu+pG(\widehat{v})\mathbb{E}[v|v\leq\widehat{v}])}{q+(1-q)(1-p+pG(\widehat{v}))} = \lim_{s\to\widehat{v}^+} \tfrac{q\cdot(1-p)\cdot s+(1-q)((1-p)\mu+pG(\widehat{v})\mathbb{E}[v|v\leq\widehat{v}])}{q(1-p)+(1-q)(1-p+pG(\widehat{v}))} \\
&\Rightarrow \lim_{s\to\widehat{v}^-} \big(q\cdot s+(1-q)((1-p)\mu+pG(\widehat{v})\mathbb{E}[v|v\leq\widehat{v}])\big)(q-qp+(1-q)(1-p+pG(\widehat{v})))= \\
&\qquad \lim_{s\to\widehat{v}^+} \big(q\cdot s-q\cdot p\cdot s+(1-q)((1-p)\mu+pG(\widehat{v})\mathbb{E}[v|v\leq\widehat{v}])\big)(q+(1-q)(1-p+pG(\widehat{v})) \\
&\Rightarrow \lim_{s\to\widehat{v}^-} q\cdot p\big(q\cdot s+(1-q)((1-p)\mu+pG(\widehat{v})\mathbb{E}[v|v\leq\widehat{v}])\big) = \lim_{s\to\widehat{v}^+} qps(q+(1-q)(1-p+pG(\widehat{v})) \\
&\Rightarrow \lim_{s\to\widehat{v}^-} q\cdot s+(1-q)((1-p)\mu+pG(\widehat{v})\mathbb{E}[v|v\leq\widehat{v}]) = \lim_{s\to\widehat{v}^+} q\cdot s+(1-q)(1-p+pG(\widehat{v}))\cdot s \\
&\Rightarrow s = \tfrac{(1-p)\mu+pG(\widehat{v})\mathbb{E}[v|v\leq\widehat{v}]}{(1-p+pG(\widehat{v}))} = v^{\mathrm{D}}.
\end{aligned}
$$

Also, $s < \widehat{v} \Leftrightarrow s < v^{\mathrm{D}}$ because:

$$
\begin{aligned}
&s < \widehat{v} \Leftrightarrow s < \tfrac{q\cdot s+(1-q)((1-p)\mu+pG(\widehat{v})\mathbb{E}[v|v\leq\widehat{v}])}{q+(1-q)(1-p+pG(\widehat{v}))} \\
&\Rightarrow qs+(1-q)(1-p+pG(\widehat{v}))\cdot s < q\cdot s+(1-q)((1-p)\mu+pG(\widehat{v})\mathbb{E}[v|v\leq\widehat{v}]) \\
&\Rightarrow s < \tfrac{(1-p)\mu+pG(\widehat{v})\mathbb{E}[v|v\leq\widehat{v}]}{(1-p+pG(\widehat{v}))} \\
&\Rightarrow s < v^{\mathrm{D}}.
\end{aligned}
$$

Similarly, $\widehat{v} < s \Leftrightarrow v^{\mathrm{D}} < s$.

(ii) Let $s = v^{\mathrm{D}}$. The equilibrium condition is then $\frac{q\cdot v^{\mathrm{D}}+(1-q)((1-p)\mu+pG(\widehat{v})\mathbb{E}[v|v\leq\widehat{v}])}{q+(1-q)(1-p+pG(\widehat{v}))} = \widehat{v}$.
From Lemma 0, we have that $\frac{(1-p)\mu+pG(\widehat{v})\mathbb{E}[v|v\leq\widehat{v}]}{1-p+pG(\widehat{v})} = v^{\mathrm{D}}$. From the uniqueness of the threshold we get that $\frac{q\cdot v^{\mathrm{D}}+(1-q)((1-p)\mu+pG(\widehat{v})\mathbb{E}[v|v\leq\widehat{v}])}{q+(1-q)(1-p+pG(\widehat{v}))} = \widehat{v} \Leftrightarrow \widehat{v} = v^{\mathrm{D}}$.
Therefore, together with the result that $h_b(s \leq \widehat{v}, p, q) = h_a(\widehat{v} < s, p, q) \Leftrightarrow s = v^{\mathrm{D}}$ we get

that $v^{\mathrm{L}}(s)$ is continuous in s.

Using Proposition 2, we calculate $\frac{\partial}{\partial s}v^{\mathrm{L}}(s|s \leq v^{\mathrm{D}}) = \frac{q}{q+(1-q)(1-p+pG(\widehat{v}))}$ and $\frac{\partial}{\partial s}v^{\mathrm{L}}(s|s > v^{\mathrm{D}}) = \frac{q(1-p)}{q(1-p)+(1-q)(1-p+pG(\widehat{v}))}$. Thus, $\frac{\partial}{\partial s}v^{\mathrm{L}}(s|s > v^{\mathrm{D}}) < \frac{\partial}{\partial s}v^{\mathrm{L}}(s|s \leq v^{\mathrm{D}})$, $\forall\ p, q$.

Proof of Proposition 5: Suppose the investors conjecture that the disclosure threshold is $\widehat{v}$. If the manager discloses her information, she expects that the price is $\mathbb{E}_s[P(s, v)] = P(s = v) = v$. If the manager does not disclose she expects that the price is

$$\begin{aligned}\mathbb{E}_s[P(s,\varnothing) \mid v] &= q \cdot \mathbb{E}_s[P(s,\varnothing \mid s = v \leq \widehat{v})] \\ &\quad +(1-q)G(\widehat{v})\mathbb{E}_s[P(s,\varnothing \mid s \leq \widehat{v}) \mid s = x] \\ &\quad +(1-q)(1-G(\widehat{v}))\mathbb{E}_s[P(s,\varnothing \mid \widehat{v} < s) \mid s = x].\end{aligned}$$

Simplifying, we get

$$\begin{aligned}\mathbb{E}_s[P(s,\varnothing) \mid v] &= q \cdot \frac{q \cdot v + (1-q)((1-p)\mu + pG(\widehat{v})\mathbb{E}[v|v \leq \widehat{v}])}{q + (1-q)(1-p+pG(\widehat{v}))} \\ &\quad +(1-q)G(\widehat{v})\frac{q \cdot \mathbb{E}[x|x \leq \widehat{v}] + (1-q)((1-p)\mu + pG(\widehat{v})\mathbb{E}[v|v \leq \widehat{v}])}{q + (1-q)(1-p+pG(\widehat{v}))} \\ &\quad +(1-q)(1-G(\widehat{v}))\frac{q \cdot (1-p)\mathbb{E}[x|\widehat{v} < x] + (1-q)((1-p)\mu + pG(\widehat{v})\mathbb{E}[v|v \leq \widehat{v}])}{q(1-p) + (1-q)(1-p+pG(\widehat{v}))}\end{aligned}$$

In a threshold equilibrium, it has to hold that $T^{\mathrm{E}}(\widehat{v} = v^{\mathrm{E}}) = 0$ where $T^{\mathrm{E}}(\widehat{v}) \equiv \mathbb{E}_s[P(s,\varnothing) \mid v = \widehat{v}] - \mathbb{E}_s[P(s, v = \widehat{v})]$. Note that $T^{\mathrm{E}}(\widehat{v})$ is decreasing in $\widehat{v}$ because $\mathbb{E}_s[P(s, v = \widehat{v})] = \widehat{v}$ is increasing in $\widehat{v}$ at a rate of 1 and $\mathbb{E}_s[P(s,\varnothing) \mid v = \widehat{v}]$ is increasing in $\widehat{v}$ at a rate of less than 1. Now $\lim_{\widehat{v}\to 0} T^{\mathrm{E}}(\widehat{v}) > 0$ because $\lim_{\widehat{v}\to 0} \mathbb{E}_s[P(s,\varnothing \mid \widehat{v}) \mid v = \widehat{v}] = q\frac{(1-q)(1-p)\mu}{q+(1-q)(1-p)} + (1-q)\frac{(1-p)\mu}{q(1-p)+(1-q)(1-p)} > 0$, while $\lim_{\widehat{v}\to 0} \mathbb{E}_s[P(s, v = \widehat{v})] = 0$. In addition, $\lim_{\widehat{v}\to 1} T^{\mathrm{E}}(\widehat{v}) < 0$ because $\lim_{\widehat{v}\to 1} \mathbb{E}_s[P(s,\varnothing) \mid v = \widehat{v}] = q(q + (1-q)\mu) + (1-q)\mu < 1$ while $\lim_{\widehat{v}\to 1} \mathbb{E}_s[P(s, v = \widehat{v})] = 1$. Therefore, a unique threshold v^{E} exists such that the manager discloses if and only if v exceeds v^{E}.

To compare v^{E} and v^{D}, we use that v^{D} satisfies $T^{\mathrm{D}}(\widehat{v} = v^{\mathrm{D}}) = 0$ where $T^{\mathrm{D}}(\widehat{v}) \equiv P(v =$

$v^{\mathrm{D}}) - P(\varnothing) \Rightarrow v^{\mathrm{D}} = \frac{(1-p)\mu + pG(v^{\mathrm{D}})\mathbb{E}[v|v \le v^{\mathrm{D}}]}{1-p+pG(v^{\mathrm{D}})}$. Note that $\lim_{\widehat{v} \to v^{\mathrm{D}}} \mathbb{E}_s[P(s, v = \widehat{v})] = v^{\mathrm{D}}$ while

$$\begin{aligned}
\lim_{\widehat{v} \to v^{\mathrm{D}}} \mathbb{E}_s[P(s, \varnothing) \mid v] &= q \cdot \frac{q \cdot v^{\mathrm{D}} + (1-q)v^{\mathrm{D}}(1-p+pG(v^{\mathrm{D}}))}{q + (1-q)(1-p+pG(v^{\mathrm{D}}))} \\
&\quad +(1-q)G(v^{\mathrm{D}}) \frac{q \cdot \mathbb{E}[v|v \le v^{\mathrm{D}}] + (1-q)v^{\mathrm{D}}(1-p+pG(v^{\mathrm{D}}))}{q + (1-q)(1-p+pG(v^{\mathrm{D}}))} \\
&\quad +(1-q)(1-G(v^{\mathrm{D}})) \frac{q \cdot (1-p)\mathbb{E}[v|v^{\mathrm{D}} < v] + (1-q)v^{\mathrm{D}}(1-p+pG(v^{\mathrm{D}}))}{q(1-p) + (1-q)(1-p+pG(v^{\mathrm{D}}))} \\
&= q \cdot v^{\mathrm{D}} + (1-q)G(v^{\mathrm{D}})v^{\mathrm{D}} + (1-q)(1-G(v^{\mathrm{D}}))v^{\mathrm{D}} \\
&\quad +(1-q)G(v^{\mathrm{D}}) \frac{q}{q + (1-q)(1-p+pG(v^{\mathrm{D}}))} \cdot (\mathbb{E}[v|v \le v^{\mathrm{D}}] - v^{\mathrm{D}}) \\
&\quad +(1-q)(1-G(v^{\mathrm{D}})) \frac{q \cdot (1-p)}{q(1-p) + (1-q)(1-p+pG(v^{\mathrm{D}}))} \cdot (\mathbb{E}[v|v > v^{\mathrm{D}}] - v^{\mathrm{D}}) \\
&> v^{\mathrm{D}}.
\end{aligned}$$

Lastly, to compare v^{E} and μ, we use that $\lim_{\widehat{v} \to \mu} \mathbb{E}_s[P(s, v = \widehat{v})] = \mu$ while

$$\begin{aligned}
\lim_{\widehat{v} \to \mu} \mathbb{E}_s[P(s, \varnothing) \mid v] &= q \cdot \frac{q \cdot \mu + (1-q)((1-p)\mu + p\frac{1}{2}\mathbb{E}[v|v \le \mu)}{q + (1-q)(1-p/2)} \\
&\quad +(1-q)\frac{1}{2} \frac{q \cdot \mathbb{E}[x|x \le \mu] + (1-q)((1-p)\mu + p\frac{1}{2}\mathbb{E}[v|v \le \mu])}{q + (1-q)(1-p/2)} \\
&\quad +(1-q)\frac{1}{2} \frac{q \cdot (1-p)\mathbb{E}[x|\mu < x] + (1-q)((1-p)\mu + p\frac{1}{2}\mathbb{E}[v|v \le \mu])}{q(1-p) + (1-q)(1-p/2)} \\
&< \mu.
\end{aligned}$$

Proof of Proposition 6: The proof follows from the discussion in the text and is omitted.

Proof of Proposition 7: The proof follows from the discussion in the text and is omitted.

Proof of Corollary 2: The proof follows from the discussion in the text and is omitted.

Proof of Corollary 3: The comparison between $P_4^{\mathrm{L}}(s, v)$ and $P_3^{\mathrm{L}}(s)$ is straightforward. Here, we only compare $P_4^{\mathrm{L}}(s, \varnothing)$ with $P_3^{\mathrm{L}}(s)$. Recall that, $P_3^{\mathrm{L}}(s) = qs + (1-q)\mu$ and $P_4^{\mathrm{L}}(s, \varnothing) = \widetilde{v}^{\mathrm{L}}(s)$. Applying the Envelope Theorem, $\frac{d}{ds}\widetilde{v}^{\mathrm{L}}(s) = \frac{1-p}{1-p+pG(\widetilde{v}^{\mathrm{L}}(s))} \cdot q + \frac{pG(\widetilde{v}^{\mathrm{L}}(s))}{1-p+pG(v^{\mathrm{L}}(s))} \cdot \frac{q}{q+(1-q)G(\widetilde{v}^{\mathrm{L}}(s))} > q = \frac{d}{ds}P_3^{\mathrm{L}}(s)$.

Recall that s_o is the signal realization that satisfies $s_o = v^{\mathrm{L}}(s = s_o)$. We note that $P_3^{\mathrm{L}}(s = s_o) = q \cdot s_o + (1-q)\mu > s_o = v^{\mathrm{L}}(s = s_o) = P_4^{\mathrm{L}}(s = s_o, \varnothing)$, because $s_o < \mu$. Hence, we have $P_3^{\mathrm{L}}(s) > P_4^{\mathrm{L}}(s, \varnothing)$ for any $s \ge s_o$. It remains to show that this inequality holds for $s < s_o$. We note that

$$\begin{aligned}
\min P_4^{\mathrm{L}}(s,\varnothing) &= P_3^{\mathrm{L}}(s=0,\varnothing) \\
&= \frac{(1-p)(1-q)\mu + pG(\widetilde{v}^{\mathrm{L}}(s=0))\left(1-\frac{q}{q+(1-q)G(\widetilde{v}^{\mathrm{L}}(s=0))}\right)\mathbb{E}[v \mid v \leq \widetilde{v}^{\mathrm{L}}(s=0)]}{1-p+pG(\widetilde{v}^{\mathrm{L}}(s=0))} \\
&< \frac{(1-p)(1-q)\mu + pG(\widetilde{v}^{\mathrm{L}}(s=0))\,(1-q)\,\mathbb{E}[v \mid v \leq \widetilde{v}^{\mathrm{L}}(s=0)]}{1-p+pG(\widetilde{v}^{\mathrm{L}}(s=0))} \\
&< (1-q)\mu = P_3^{\mathrm{L}}(s=0) = \min P_3^{\mathrm{L}}(s).
\end{aligned}$$

Therefore, $P_4^{\mathrm{L}}(s,\varnothing) < P_3^{\mathrm{L}}(s)$ for any s.

Proof of Proposition 8: The equilibrium condition is $\Delta\Pi(\widehat{v}=\widetilde{v}^{\mathrm{E}}) = \Pi(v) - \Pi(\varnothing) = 0$. Using the notation in the proof of Proposition 5, we can simplify,

$$\Delta\Pi(\widehat{v}) = T^{\mathrm{D}}(\widehat{v}) + (\delta+\delta^2+\delta^3)T^{\mathrm{E}}(\widehat{v}).$$

Both $T^{\mathrm{D}}(\widehat{v})$ and $T^{\mathrm{E}}(\widehat{v})$ are decreasing in $\widehat{v}$. Thus, $\Delta\Pi(\widehat{v})$ is also decreasing. Furthermore,

$$\begin{aligned}
\lim_{\widehat{v}\to 1}\Delta\Pi(\widehat{v}) &= \underbrace{\lim_{\widehat{v}\to 1} T^{\mathrm{D}}(\widehat{v})}_{<0} + (\delta+\delta^2+\delta^3)\underbrace{\lim_{\widehat{v}\to 1} T^{\mathrm{E}}(\widehat{v})}_{<0} < 0; \\
\lim_{\widehat{v}\to 0}\Delta\Pi(\widehat{v}) &= \underbrace{\lim_{\widehat{v}\to 0} T^{\mathrm{D}}(\widehat{v})}_{>0} + (\delta+\delta^2+\delta^3)\underbrace{\lim_{\widehat{v}\to 0} T^{\mathrm{E}}(\widehat{v})}_{>0} > 0.
\end{aligned}$$

Therefore, there exists a unique threshold $\widetilde{v}^{\mathrm{E}} \in (0,1)$ such that the manager discloses if and only if $v > \widetilde{v}^{\mathrm{E}}$. To see that $\widetilde{v}^{\mathrm{E}} \in (v^{\mathrm{D}}, v^{\mathrm{E}})$ note that

$$\begin{aligned}
\lim_{\widehat{v}\to v^{\mathrm{D}}}\Delta\Pi(\widehat{v}) &= \underbrace{\lim_{\widehat{v}\to v^{\mathrm{D}}} T^{\mathrm{D}}(\widehat{v})}_{=0} + (\delta+\delta^2+\delta^3)\underbrace{\lim_{\widehat{v}\to v^{\mathrm{D}}} T^{\mathrm{E}}(\widehat{v})}_{>0} > 0; \\
\lim_{\widehat{v}\to v^{\mathrm{E}}}\Delta\Pi(\widehat{v}) &= \underbrace{\lim_{\widehat{v}\to v^{\mathrm{E}}} T^{\mathrm{D}}(\widehat{v})}_{<0} + (\delta+\delta^2+\delta^3)\underbrace{\lim_{\widehat{v}\to v^{\mathrm{E}}} T^{\mathrm{E}}(\widehat{v})}_{=0} < 0.
\end{aligned}$$

Lastly, using the Implicit Function Theorem,

$$\frac{\partial}{\partial\delta}v^{\mathrm{E}} \propto \frac{\partial}{\partial\delta}\Delta\Pi(\widetilde{v}^{\mathrm{E}}) = (1+2\delta+3\delta^2)\underbrace{T^{\mathrm{E}}(\widehat{v}=\widetilde{v}^{\mathrm{E}})}_{>0} > 0.$$

Proof of Corollary 4: Follows from the discussion in the text and is omitted.

Proof of Corollary 5: We know that $P_2^{\mathrm{E}}(\varnothing \mid \widetilde{v}^{\mathrm{E}}) - P_3^{\mathrm{E}}(s,\varnothing \mid \widetilde{v}^{\mathrm{E}}) = P(\varnothing \mid \widetilde{v}^{\mathrm{E}}) - P(s,\varnothing \mid$

$\widetilde{v}^{\mathrm{E}}$), where

$$\begin{aligned}
P(\varnothing \mid \widetilde{v}^{\mathrm{E}}) &= \frac{1-p}{1-p+p\cdot G(\widetilde{v}^{\mathrm{E}})}\cdot\mu + \frac{p\cdot G(\widetilde{v}^{\mathrm{E}})}{1-p+p\cdot G(\widetilde{v}^{\mathrm{E}})}\cdot\mathbb{E}[v \mid v\le \widetilde{v}^{\mathrm{E}}] \\
P(s,\varnothing \mid \widetilde{v}^{\mathrm{E}}, s\le \widetilde{v}^{\mathrm{E}}) &= \frac{1-p}{1-p+p\cdot G(\widetilde{v}^{\mathrm{E}})}[(1-q)\cdot\mu + q\cdot s] \\
&\quad + \frac{p\cdot G(\widetilde{v}^{\mathrm{E}})}{1-p+p\cdot G(\widetilde{v}^{\mathrm{E}})}[(1-q\cdot\pi(\widetilde{v}^{\mathrm{E}}))\cdot\mathbb{E}[v \mid v\le \widetilde{v}^{\mathrm{E}}] + q\cdot\pi(\widetilde{v}^{\mathrm{E}})\cdot s] \\
P(s,\varnothing \mid \widetilde{v}^{\mathrm{E}}, s> \widetilde{v}^{\mathrm{E}}) &= \frac{1-p}{1-p+p\cdot G(\widetilde{v}^{\mathrm{E}})}[(1-q)\cdot\mu + q\cdot s] \\
&\quad + \frac{p\cdot G(\widetilde{v}^{\mathrm{E}})}{1-p+p\cdot G(\widetilde{v}^{\mathrm{E}})}\cdot\mathbb{E}[v \mid v\le \widetilde{v}^{\mathrm{E}}].
\end{aligned}$$

Therefore:

$$P_2^{\mathrm{E}}(\varnothing \mid \widetilde{v}^{\mathrm{E}}, s>\widetilde{v}^{\mathrm{E}}) - P_3^{\mathrm{E}}(s,\varnothing \mid \widetilde{v}^{\mathrm{E}}, s>\widetilde{v}^{\mathrm{E}}) \;\propto\; \mu-(1-q)\cdot\mu - q\cdot s \propto \mu - s,$$

which is positive for $s<\mu$ and negative otherwise. Furthermore,

$$\begin{aligned}
w(s) &\equiv P_2^{\mathrm{E}}(\varnothing \mid \widetilde{v}^{\mathrm{E}}, s\le\widetilde{v}^{\mathrm{E}}) - P_3^{\mathrm{E}}(s,\varnothing \mid \widetilde{v}^{\mathrm{E}}, s\le\widetilde{v}^{\mathrm{E}}) \\
&= \frac{1-p}{1-p+p\cdot G(\widetilde{v}^{\mathrm{E}})}\cdot(\mu-s) + \frac{p\cdot G(\widetilde{v}^{\mathrm{E}})}{1-p+p\cdot G(\widetilde{v}^{\mathrm{E}})}\pi(\widetilde{v}^{\mathrm{E}})(\mathbb{E}[v \mid v\le \widetilde{v}^{\mathrm{E}}] - s)
\end{aligned}$$

This term is decreasing in s. It is immediate that $w(s=0)>0$. Furthermore,

$$w(s=\widetilde{v}^{\mathrm{E}}) = \frac{1-p}{1-p+p\cdot G(\widetilde{v}^{\mathrm{E}})}\cdot(\mu-\widetilde{v}^{\mathrm{E}}) + \frac{p\cdot G(\widetilde{v}^{\mathrm{E}})}{1-p+p\cdot G(\widetilde{v}^{\mathrm{E}})}\pi(\widetilde{v}^{\mathrm{E}})(\mathbb{E}[v \mid v\le \widetilde{v}^{\mathrm{E}}] - \widetilde{v}^{\mathrm{E}}).$$

From the proof of Proposition 8 recall that $\widetilde{v}^{\mathrm{E}}$ satisfies $\Delta\Pi(\widehat{v}=\widetilde{v}^{\mathrm{E}})=0$. Rearranging,

$$\begin{aligned}
\Delta\Pi(\widehat{v}=\widetilde{v}^{\mathrm{E}}) &= (1+\delta+\delta^2+\delta^3)(v-\mathbb{E}[P(s,\varnothing) \mid v=\widehat{v}, \widehat{v}=\widetilde{v}^{\mathrm{E}}]) \\
&\quad +\mathbb{E}[P(s,\varnothing) \mid v=\widehat{v}, \widehat{v}=\widetilde{v}^{\mathrm{E}}] - P(\varnothing \mid \widehat{v}=\widetilde{v}^{\mathrm{E}}).
\end{aligned}$$

Because $\widetilde{v}^{\mathrm{E}}\in(v^{\mathrm{D}}, v^{\mathrm{E}})$, it holds that $(1+\delta+\delta^2+\delta^3)(v-\mathbb{E}[P(s,\varnothing) \mid v=\widehat{v}, \widehat{v}=\widetilde{v}^{\mathrm{E}}])<0$. Therefore, it has to be that $\mathbb{E}[P(s,\varnothing) \mid v=\widehat{v}, \widehat{v}=\widetilde{v}^{\mathrm{E}}] - P(\varnothing \mid \widehat{v}=\widetilde{v}^{\mathrm{E}})>0$. Lastly, note that

$$w(s=\widetilde{v}^{\mathrm{E}}) = -(\mathbb{E}[P(s,\varnothing) \mid v=\widehat{v}, \widehat{v}=\widetilde{v}^{\mathrm{E}}] - P(\varnothing \mid \widehat{v}=\widetilde{v}^{\mathrm{E}}))<0.$$

Therefore, there exists $s^{\dagger\dagger}\in(0,\widetilde{v}^{\mathrm{E}})$ such that: $P_2^{\mathrm{E}}(\varnothing)\ge P_3^{\mathrm{E}}(s,\varnothing)$ if $s\in[0,s^{\dagger\dagger}]$ or $s\in[\widetilde{v}^{\mathrm{E}},\mu]$. However, $P_2^{\mathrm{E}}(\varnothing)\le P_3^{\mathrm{E}}(s,\varnothing)$ if $s\in[s^{\dagger\dagger},\widetilde{v}^{\mathrm{E}}]$ or $s\in[\mu,1]$.

References

[1] Acharya, V., DeMarzo, P., Kremer, I. 2011. Endogenous information flows and the clustering of announcements. *American Economic Review* 101, 2955-2979.

[2] Beyer, A., Cohen, D., Lys, T., Walthers, B. 2010. The financial reporting environment: review of the recent literature. *Journal of Accounting and Economics* 50, 296–343.

[3] Bertomeu, J., Cheynel, E., 2015. Asset measurement in imperfect credit markets. *Journal of Accounting Research* 53, 965-984.

[4] Bonham, J. 2021. News management, moral hazard, and the properties of earnings, prices, and compensation. Working paper.

[5] Cianciaruso, D., Marinovic, I., Smith, K. 2020. Information design in financial markets. Working paper.

[6] Dordzhieva, A., Laux, V., Zhang, R. 2020. Signaling private information via accounting system design. Working paper.

[7] Dye, R. 1983. Communication and post-decision information. *Journal of Accounting Research* 514-533.

[8] Dye, R., 1985. Disclosure of non-proprietary information. *Journal of Accounting Research* 23, 123-145.

[9] Dye, R., Sridhar, S., 1995. Industry-wide disclosure dynamics. *Journal of Accounting Research*, 33, 157-174.

[10] Dutta, S., Trueman, B., 2002. The interpretation of information and corporate disclosure strategies. *Review of Accounting Studies* 7, 75-96.

[11] Ebert, M., Schäfer, U., Schneider, G.T., 2019. Information leaks and voluntary disclosure. Working paper.

[12] Einhorn, E., 2018. Competing information sources. *The Accounting Review,* 93, 151-176.

[13] Frenkel, S., Guttman, I., Kremer, I., 2019. The effect of exogenous information on voluntary disclosure and market quality. *Journal of Financial Economics*, forthcoming

[14] Friedman, H., Hughes, J., Michaeli, B., 2020. Optimal reporting when additional information might arrive. *Journal of Accounting and Economics*, 69.

[15] Friedman, H., Hughes, J., Michaeli, B., 2021. A rationale for imperfect reporting standards. *Management Science*, forthcoming

[16] Gigler, F., Hemmer, T. 1998. On the frequency, quality, and informational role of mandatory financial reports. *Journal of Accounting Research* 36, 117-147.

[17] Gigler, F., Hemmer, T. 2001. Conservatism, optimal disclosure policy, and the timeliness of financial reports. *The Accounting Review* 76, 471-493.

[18] Göx, R., Michaeli, B. 2019. Optimal information design and incentive contracts with performance measure manipulation. Working paper.

[19] Göx, R., Wagenhofer, A. 2009. Optimal impairment rules. *Journal of Accounting and Economics*, 48, 2-16.

[20] Gregor, M., Michaeli, B. 2020. Board bias, information, and investment efficiency. Working paper.

[21] Guttman, I., Kremer, I., and Skrzypacz, A., 2014. Not only what but also when: A theory of dynamic voluntary disclosure. *American Economic Review* 104, 2400-2420.

[22] Grossman, S.J., 1981. The informational role of warranties and private disclosure about product quality. *Journal of Law and Economics* 24, 461-484.

[23] Jiang, X., Yang, M. 2017. Properties of optimal accounting rules in a signaling game. *Journal of Accounting and Economics*, 63, 499-512.

[24] Jung, W., Kwon, Y., 1988. Disclosure when the market is unsure of information endowment of managers. *Journal of Accounting Research* 26, 146-153.

[25] Kamenica, E., Gentzkow, M. 2011. Bayesian persuasion. *American Economic Review* 101, 2590-2615.

[26] Menon, R., 2020. Voluntary disclosures when there is an option to delay disclosure. *Contemporary Accounting Research* 37, 829-856.

[27] Michaeli, B. 2017. Divide and inform: Rationing information to facilitate persuasion. *The Accounting Review* 92(5), 167-199.

[28] Milgrom, P., 1981. Good news and bad news: Representation theorems and applications. *The Bell Journal of Economics* 12, 380-391.

[29] Sletten, E., 2012. The effect of stock price on voluntary disclosure. *Review of Accounting Studies* 17.

[30] Suijs, J., 2007. Voluntary disclosure of information when managers are uncertain of investor response. *Journal of Accounting and Economics* 43, 391-410.