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A Comparative GARCH Analysis of Macroeconomic Variables and Returns on Modelling the Kurtosis of FTSE 100 Implied Volatility Index

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Modelling the volatility (or kurtosis) of the implied volatility is an important aspect of financial markets when analysing market consensus and risk strategies. The purpose of this study is to evaluate the ability of symmetric and asymmetric GARCH systems to model the volatility of the FTSE 100 Implied Volatility Index (IV). We use GARCH, EGARCH, GJR-GARCH and GARCH-MIDAS to model variance. We also introduce FTSE 100 returns and several macroeconomic variables (UK industrial production, 3MLIBOR, GBP effective exchange rate and unemployment rate) to investigate whether they explain variance. Our results show that market returns is a major explanatory factor besides macroeconomic variables. Also, GARCH (1,1) outperforms other asymmetric models unless there is exceptionally high volatility such as the crisis of 2008 in which case EGARCH performs better. GJR-GARCH is outperformed by all other models. GARCH-MIDAS shows that both macroeconomic variables and market returns are useful when estimating IV. (JEL: C22, E32, E44, G12)

Keywords: FTSE 100 implied volatility index (IV); GARCH; EGARCH; GJR-GARCH; GARCH-MIDAS; FTSE 100 index returns; macroeconomic variables

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I. Introduction

Within financial markets, it is important to engage in volatility estimation and forecasting. This stems from the need to anticipate future fluctuations for risk management and investment purposes. Implied volatility captures to a certain degree the future realized volatility of market returns and market expectations (Canina and Figlewski, 1993). Implied volatility indices capture different types of index options, and therefore provide information about expected future returns. Modelling and explaining implied volatility indices holds great importance in the literature. Previous studies tend to explain implied volatility indices movements by different methodologies, mainly using its realized volatility, or by including exogenous variables such as market returns or macroeconomic factors. The relationship between implied volatility, market returns, and macroeconomic variables has been investigated from many different perspectives (See figure 1). However, what remains unexplored is the relationship between the volatility (or kurtosis) of implied volatility and exogenous variables such as macroeconomic factors and market returns. We wish to take the ‘volatility of implied volatility’ literature a step further and employ a variety of GARCH systems to model the impact of exogenous variables on the ‘volatility of the implied volatility’ index. Research in this area is virtually non-existent.¹ For the UK market which is the focal point of this study, there is not even an index that captures the ‘volatility of the implied volatility index’.² Identifying the factors that may (or may not) have an impact on ‘the volatility of implied volatility’, will help market participants decide if there is a consensus (and which factors affect the formation of consensus) on the future movements of the implied volatility index and the market itself.³ Most importantly, it will also help them design their risk strategies in order to hedge tail risk returns or capture the volatility risk premium.^{4,5}

1. Research in the area of volatility of volatility (captured by VVIX and created by CBOE, VVIX stands for volatility of the VIX) has concentrated on the effect of VVIX on tail risk hedging returns (see Yang-Ho Park, 2015) and on expected stock returns & variance risk premium (Wang et al., 2013). It is specific only to the US market.

2. The CBOE in the US has created an index which captures the volatility of the implied volatility index (VIX). This new index is called VVIX. See <http://www.cboe.com/products/vix-index-volatility/volatility-on-stock-indexes/the-cboe-vix-index/vvix-whitepaper>

3. Yang-Ho Park (2015) considers volatility of volatility as a proxy for uncertainty over volatility.

4. Yang-Ho Park (2015) finds that the volatility of volatility or VVIX has strong

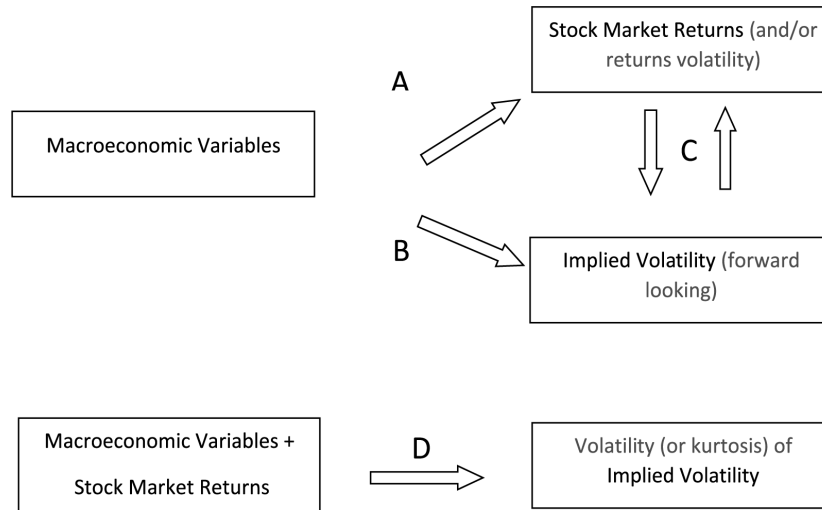


FIGURE 1.— Plot of research patterns between macroeconomic variables, stock market returns, volatility and implied volatility indices

Note: This figure shows research patterns between macroeconomic variables, stock market returns, volatility and implied volatility. The first pattern is the study of how macroeconomic variables affect stock market returns (and/or volatility), which is denoted by (A). The second pattern is about measuring the effect of macroeconomic announcements on implied volatility, denoted by (B). The third research pattern is about measuring the effect of stock market returns (and/or volatility) on implied volatility and vice versa, denoted by (C). The last pattern which is the focal point of this study, investigates the effect of both macroeconomic variables and stock market returns on the volatility (or kurtosis) of implied volatility.

Research pattern (A) in figure 1, holds the largest literature among other patterns, where a vast number of studies analyse the effect of macroeconomic variables (inflation, industrial production, GDP, exchange rate, interest rate, and unemployment rate) on stock market

predictability for tail risk hedging returns. Knowing which factors affect VVIX will help with hedging tail risk returns.

5. The CBOE explains in their VVIX Whitepaper (See <http://www.cboe.com/products/vix-index-volatility/volatility-on-stock-indexes/the-cboe-vix-index/vvix-whitepaper>) what strategies can be pursued to capture the volatility risk premium among other reasons regarding the usefulness of the VVIX. We elaborate further below. We wish to thank the reviewer for urging us to include reasons that market participants would be interested in modelling the volatility (kurtosis) of implied volatility.

returns. Changes in those variables affect the existence of available real investment opportunities, the firm's cash flows and the risk-adjusted discount rate (Flannery and Protopapadakis, 2002). Officer (1973), Campbell (1987), Breen et al. (1989), Engle and Rangel (2008), Engle et al. (2008), and Campbell and Diebold (2009), explained and related the fluctuation of stock market returns to several macroeconomic determinants.

The information content of macroeconomic variables also plays a major role in defining implied volatility movements (research pattern B). Ederington and Lee (1996), Heuson and Su (2003), Nofsinger and Prucyk (2003), Clements (2007), and Vähämaa (2009) indicated that macroeconomic announcements have an effect on implied volatility indices.

Now as far as the relationship between implied volatility and market returns is concerned, research pattern (C) in figure 1, shows that a two way relationship is present. The literature focuses on the effect of implied volatility on stock market returns. Empirical evidence indicates a negative and asymmetric relationship between market returns and implied volatility (Baillie and DeGennaro, 1990). Previous literature examined the role of implied volatility to capture the dynamics of market return volatility. For instance Day and Lewis (1992), Canina and Figlewski (1993) and Fleming (1998), find that implied volatility does not entirely capture the dynamics of market return volatility in the US. On the other hand, the reciprocal relationship, more specifically the role of market returns in estimating implied volatility, has not received much attention in the literature. There is a limited number of studies (see Whaley (2000) and Giot (2005)). They indicated a negative, significant relationship running from market returns indices such as the S&P100, S&P500 and NASDAQ 100 to implied volatility indices. Different research methodologies were used in analysing these relationships, but the most prominent model in understanding the behaviour of implied volatility is a GARCH model and its extended family.

In this paper, we are modelling the volatility of the log-returns FTSE 100 implied volatility index, 30 days option expiration. Studying the volatility of the implied volatility is tantamount to studying the kurtosis of the implied volatility. A leptokurtic implied volatility distribution means a high presence of outliers which shows lack of consensus and an unsettled market. It is important to model the kurtosis of the implied volatility index (IV) because market participants need information about the degree of consensus the market itself has on the future values of the implied volatility index (IV). In other words market participants need to

be clear about the strength of opinions formed regarding the future values of the implied volatility index (IV). Yang-Ho Park (2015) perceives volatility of volatility (VVIX) as a proxy for uncertainty over volatility and considers it a tail risk indicator in the US. A high volatility of the IV clearly indicates that there is no consensus about future movements and the stability of the IV itself. In addition, a high volatility of the IV could also indicate a looming crisis. Yang-Ho Park (2015) shows that in the US, an increase in the uncertainty measure (captured by VVIX or volatility of volatility) will raise current prices of tail risk hedging options and lower their subsequent returns over the next period. Our GARCH models will help identify which factors (returns, macroeconomic factors) could potentially play a role in predicting a looming crisis first captured by the volatility of the implied volatility index itself (IV).⁶ Also modelling the volatility of the implied volatility index (IV) will help market participants obtain a better understanding of the factors that determine the prices of implied volatility index options and futures as well as the IV itself.

According to the CBOE which have already developed a volatility index of the VIX called VVIX, trading strategies can be formed to help with risk management.⁷ This is achieved by forming a portfolio based on VVIX which essentially captures the price of a portfolio of VIX options. Selling this VVIX portfolio captures the volatility risk premium. If market participants believe that the VVIX is too high or too low at a particular point in time, they could buy or sell the underlying portfolio. Specifically buying a VVIX portfolio returns the difference between realized and expected volatility less the volatility risk premium. Conversely selling a VVIX portfolio returns the difference between expected and realized volatility plus the volatility risk. To the extent that volatility expectations are unbiased, consistently selling a VVIX portfolio captures the volatility risk premium. By modelling the volatility of the implied volatility irrespective of markets, we are identifying factors that could have an impact on the volatility of the implied volatility and in this way market participants could achieve

6. Even though there are no studies for the ‘volatility of the implied volatility index’ (IV) in the UK, the CBOE presents evidence that the VIX (implied volatility or fear index) and the VVIX (volatility of the VIX) are significantly correlated when the VIX (implied volatility or fear index) itself gets extreme values. This indicates why it is important to model the volatility of the IV in the UK.

7. See VVIX Whitechapter: <http://www.cboe.com/products/vix-index-volatility/volatility-on-stock-indexes/the-cboe-vvix-index/vvix-whitepaper>.

better control over their risk.

In order to model the volatility of the implied volatility index, we use several explanatory factors namely, its realized volatility, the FTSE 100 index log-returns (FTSE100R) and macroeconomic variables. This is research pattern (D) in figure 1. Using log-returns for both IV and FTSE 100 index yields better results because implied volatility indices and stock market returns are normally distributed (Bachelier, 2011). The macroeconomic variables that we use are: the UK industrial production (IP), the London 3 months interbank interest rate (LIBOR3M), GBP effective exchange rate (EEX), and unemployment rate (UR).⁸ We apply symmetric and asymmetric forms of GARCH models with different estimation methods. As a benchmark, we first analyse the conditional variance of the IV, its own volatility. Afterwards, we add FTSE100R and other macroeconomic variables individually with IV to study their effect on its variability. We try different combinations of these variables to produce the best results. We finally use GARCH-MIDAS (MIDAS: mixed data sampling) to capture the impact of FTSE100R and of other macroeconomic variables, sampled at monthly frequency, on the daily volatility of IV.⁹ GARCH-MIDAS is a univariate model which allows us to include only one variable at a time.

To the best of our knowledge, modelling the effect of macroeconomic variables and returns on the ‘volatility of the implied volatility index’ has not been investigated before.¹⁰ The IV reflects the future market fluctuations of FTSE100R, and enables investors to make better decisions in terms of investment and risk management. We believe these methods of evaluation, adding FTSE100R and other macroeconomic determinants as exogenous variables when analysing IV, could improve variance estimation and of out-of-sample estimations

8. We have excluded the UK inflation rate (CPI) and UK GDP. CPI is excluded because it is highly correlated with the UK unemployment rate and the three months London interbank rate (LIBOR3M). GDP is excluded because it is sampled quarterly.

9. GARCH-MIDAS conditional volatility consists of a short-term component specified by realized volatility of returns, and a long-term component that reflects macroeconomic fluctuations. In many cases, researchers tend to eliminate data from large datasets in order to match frequencies between high and low frequency variables. GARCH-MIDAS allows us to overcome the problem of non-aligned frequencies between high and low frequency variables and gives the estimated results more credibility.

10. Research in the area of ‘volatility of volatility’ is limited, specific to the US and has concentrated on the effect of VVIX on tail risk hedging returns (see Yang-Ho Park, 2015) and its effect on the equity premium (Wang et al, 2013).

of IV. Moreover, using the GARCH-MIDAS approach could either confirm the relationship between our chosen variables, or produce alternative results. The MIDAS approach could also improve our forecasting ability since it allows us to analyse all available data sampled at different frequencies. Macroeconomic variables theoretically are great candidates since they create the conditions where the financial assets are priced (Chen et al., 1986).

Our results show that FTSE100R and macroeconomic variables play a significant role in defining the volatility of IV. GARCH (1,1) outperformed other asymmetric models, EGARCH and GJR-GARCH. FTSE100 returns, IP, LIBOR 3M, EER, and UR helped in explaining IV volatility, and provided significant outputs using both symmetric and asymmetric GARCH models. The GARCH-MIDAS approach also confirmed the ability of macroeconomic variables in estimating IV's volatility.

The remainder of the paper is organized as follows: Section II presents the literature review. Data and the volatility models are explained in sections III and IV. Section V contains the empirical results and analysis followed by conclusion.

II. Literature Review

The literature review section aims at categorising empirical findings that explain the research patterns which are demonstrated in Figure 1. Section II.A sheds light on studies that adopted GARCH models in modelling stock market returns based on macroeconomic variables, research pattern A. Section II.B presents empirical work on how macroeconomic announcements affect implied volatility, research pattern B. Section II.C discusses a two-way relationship between implied volatility, stock market returns and returns volatility, research pattern C.

A. The use of macroeconomic variables in GARCH models to estimate market returns and returns volatility – Pattern A

With regards to conditional volatility, Flannery and Protopapadakis (2002) analysed the impact of several macroeconomic series on both returns and returns' conditional volatility over the 1980-1996 period using GARCH. Six risk factors showed a significant effect: consumer

and producer price indices, balance of trade, unemployment rate, housing starts and monetary aggregate.

Engle and Rangel (2008), observed macroeconomic effects on returns in about 50 countries using spline-GARCH, and found evidence that mainly GDP and interest rates caused market volatility. Similarly, Engle et al. (2013) used GARCH-MIDAS to investigate the link between returns and macroeconomic determinants. Their core finding is the accuracy of this model when adding long-term macroeconomic variables. These variables are tested in terms of pseudo out-of-sample predictions in long horizons, and were proven to outperform traditional statistical models. The long components refer to the macroeconomic variables (inflation and industrial production) that are sampled at longer periods, for example monthly and quarterly. The short component is represented by daily stock returns. The data set that was used in this new class model ranges from 1890 to 2010 and it is relevant to the US market.

Several studies also applied different forms of GARCH models to study the effect of macroeconomic factors on returns. Sariannidis et al. (2009) and Cho & Elshahat (2014) using different approaches of GARCH models, state that GDP, changes of oil prices, 10-year bond returns and exchange rate do influence US aggregate stock market volatility. Pelloni and Polasek (2003) using VAR-GARCH-M style showed that the unemployment rate has an effect on the US, UK and Germany stock markets. Mangani (2009) also claimed that the discount rate (Bank/repo rate) and gold prices affect returns in South Africa, while Oseni and Nwosa (2011) followed an EGARCH model in analysing Nigeria's stock market, and showed that GDP does affect returns.

To estimate the volatility of the US stock returns, Asgharian et al. (2013) used 'embedded principle components' into GARCH-MIDAS to combine several macroeconomic factors: interest rate, unemployment rate, term premium, inflation rate, exchange rate, default rate, industrial production and growth rate. GARCH-MIDAS with principal components outperforms other GARCH models and forecasting specifications. Girardin and Joyeux (2013) also used GARCH-MIDAS and succeeded to relate CPI to China's market volatility.

B. The effect of macroeconomic announcements on implied volatility – Pattern B

The impact of information releases on market uncertainty measured by

implied volatility has been investigated by many studies, suggesting that implied volatility can be predicted by macroeconomic announcements (Heuson and Su, 2003). Ederington and Lee (1996), investigated the impact of scheduled and unscheduled macroeconomic announcements on market uncertainty captured by implied volatility of option prices. They discovered that scheduled announcements lead to lower levels of implied standard deviation (ISD), and vice versa concerning unscheduled announcements.

Nofsinger and Prucyk (2003) examined the reaction of trading volume of S&P 500 option index (OEX) following scheduled economic news in 1993 and 1994. Out of many types of announcements, consumer confidence, new home sales, factory orders and construction spending directly affect option trading volume. Vähämaa (2009) used different methodologies and a large set of macroeconomic announcements and showed that there is an effect on S&P 500 option index (VIX) using data from 1999 to 2003. Clements (2007) examined the role of monetary policy announcements on the (VIX), and found that meetings of the Federal Open Market committee have a major effect on (VIX). Several studies also investigated the effect of announcements on implied volatility in other countries, and they also found a strong link. For example Äijö (2008) used FTSE-100 index options in the UK and Füss et al. (2011) measured the effect of macroeconomic announcements on the German implied volatility index (VDAX) and (VIX). Also, Shaikh and Padhi (2013) used the Indian (VIX) and Tanha et al. (2014) undertook research in Australian index options, yielding similar results.

C. Implied volatility (forward looking), stock market returns and conditional volatility: a two way relationship – Pattern C

A number of studies have examined the informational content of implied volatility in forecasting conditional volatility of market returns. Day and Lewis (1992) model the volatility of S&P100 index, using the implied content of index options, an exogenous variable using GARCH and EGARCH, in order to conduct symmetric and asymmetric analysis. Their results showed that the information content of the implied volatility and the conditional volatility from GARCH and EGARCH do not completely characterize the conditional stock market volatility, in terms of both in and out-of-sample estimation and forecast in the US. Canina & Figlewski (1993) and Fleming (1998), also found that implied volatility, represented by S&P100 index option, produce weak forecasts

of subsequent realized volatility. However, unlike previous studies, Christensen and Prabhala (1998) used monthly frequency and a longer volatility time series span of S&P 100 index and its corresponding index option and found strong evidence that implied volatility can predict future realized volatility.

In contrast, a few studies examine the empirical link between changes in stock returns and how this affects implied volatility indices. Whaley (2000) investigated the Chicago Board Options Market Exchange's Volatility Indices (VIX and VXN), where the VIX and the VXN, correspond to the S&P 500, and the NASDAQ 100 respectively. Whaley has documented a negative and significant relationship between market returns and the implied volatility indices. In other words, positive stock returns reduce implied volatility and vice versa. Giot (2005), analysed the relationship between the S&P 100 and NASDAQ 100 returns, and their implied volatility indices (VIX and VXN respectively). The VIX shows a significant, asymmetric relationship, and a stronger response to negative market shocks than positive market returns. However, there is a weaker and asymmetric response of VXN to market returns changes.

In the previous sections, we discussed research patterns in the area of market returns, conditional/implied volatilities and macroeconomic variables. Research in the area of 'volatility of volatility' is limited, specific to the US and has concentrated on the effect of VVIX on tail risk hedging returns (see Yang-Ho Park, 2015) and its effect on the equity premium (Wang et al., 2013).¹¹ Having identified relations between macroeconomic variables, implied/conditional volatilities and returns, now we are venturing in a new area, namely the effect of macroeconomic variables and market returns on the volatility (or kurtosis) of implied volatility which is worthy of exploration given the absence of literature for the UK market.¹²

11. We do not include this research pattern in figure 1 because research is quite limited and not of direct interest to our study even though it is useful for motivation purposes. Figure 1 is a graphical representation of relationships between macroeconomic variables, returns, conditional/implied volatilities, and volatility of volatility. The effect of 'volatility of volatility' on hedging and the equity premium is a different research area. Introducing a new separate research pattern in figure 1 and in the literature review would unnecessarily increase the size of the literature review without adding any benefits to the study itself.

12. To the best of our knowledge, there is no other study that investigates the effect of macroeconomic variables and returns on volatility of volatility for the UK or any other country.

III. Data and Methodology

The data in this study is drawn from two main sources. The log-returns of FTSE100 implied volatility index, 30 days expiration, (IV), observations are obtained from FTSE Russell, covering a period from 4/1/2000 to 31/12/2015. We used the following samples in the analysis: full sample (From 1/4/2000 to 31/12/2015), subsample 1 (From 4/1/2000 to 8/8/2007), and subsample 2 (From 9/8/2007 to 31/12/2015). The first subsample is the period from the start of the IV indices until the start of the financial crisis in 2007, where the sub-prime mortgage bubble was acknowledged for the first time and the consequences started to become obvious. The second subsample represents the period after the financial crisis to the end of 2015. Splitting the sample into before/after the financial crisis that started in August 2007, will allow us to examine whether the financial crisis had a detrimental effect on the ability of the financial models to predict volatility.

Regarding IV, there are several IV indices with different interpolated annualised implied volatility dates of the underlying FTSE100 index namely 30, 60, 90, 180 and 360 days. We chose the 30 days expiration index, since it has the highest volume of trades. We used daily and monthly data of IV in the analysis, due to the requirements of GARCH models in terms of frequencies. The IV index is calculated from out-of-the money options prices using the following formula:

$$\sigma_{IV}^2 = \frac{2}{T} \left(1 + \log \frac{F}{K_*} - \frac{F}{K_*} + e^{-rT} \int_0^{K_*} \frac{P(K)}{K^2} dK + e^{rT} \int_{K_*}^{\infty} \frac{C(K)}{K^2} dK \right), \quad (1)$$

Where σ_{IV}^2 , is the FTSE 100 implied volatility index (IV), and r is the free risk interest rate. K_* is the strike immediately below F , the forward price, and $P(K)$ and $C(K)$ are the put and call prices at strike K .

Monthly observations of the FTSE100 index log-returns (FTSE100R), and the first differences of the macroeconomic variables, namely: industrial production (IP), London interbank 3 months interest rate (LIBOR3M), effective exchange rate (EEXR), and unemployment rate (UR), are collected from Datastream for the same period.

A. Volatility models

Modelling time series is a big challenge due to statistical irregularities

such as non-stationarity and non-normal distribution. Classical linear regression models (CLRM) follow several assumptions, mainly the homoscedasticity assumption, in which the variance of the errors term is constant over time (Francq and Zakoian, 2011). CLRM also assume that volatility forecast is equal to current estimates, since the expected value of the error terms is the same at any given time when it's squared (Engle, 2001). These assumptions are unrealistic since volatility of financial assets changes overtime. Volatility can be exceptionally high or low in different periods (Alexander, 2008). This feature of financial series is called clustering. Clustering patterns show that the variance of the errors is not constant over the time, indicating heteroscedasticity in time series analysis. Engle (1982) introduced autoregressive conditional heteroscedasticity model (ARCH) model and its extension the generalized ARCH (GARCH) by Bollerslev (1986), to capture the volatility of heteroscedastic data.

In our comparative analysis, we will apply several specifications and forms of GARCH models to estimate the conditional variance of IV, based on both daily and monthly frequencies.

Symmetric GARCH models

GARCH Models

The classic GARCH (1,1) model uses its own lags to generate the conditional variance, and its specification is given below:

$$r_t = \mu + \varepsilon_t, \quad (2)$$

$$\sigma_{t|t-1}^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \beta_1 \sigma_{t|t-1}^2 \quad (3)$$

The mean equation (2) is specified and written as a function of a constant and an error term, where $\varepsilon_t = \sigma_t z_t$, and z_t is the standardized residual returns. In the conditional variance equations, the $\sigma_{t|t-1}^2$ represent the conditional variance, and ω is the constant GARCH term. The ARCH error term in equation (3), ε_{t-1}^2 captures volatility news from last period, and the GARCH term, $\sigma_{t|t-1}^2$ is the forecasted variance of the last period.

To add exogenous variables, regressors, Xs, in the variance equation, equation number (3) is extended to be:

$$\sigma_{t|t'}^2 = \omega + \sum_{j=1}^1 \beta_j \sigma_{t|t'-1}^2 + \sum_{i=1}^1 \alpha_j \varepsilon_{t-i}^2 + Z_t' \pi \quad (4)$$

The parameters constraints $\omega > 0$, $\alpha, \beta \geq 0$, and $\alpha + \beta \leq 1$, are proposed by Bollersleve (1986) to ensure that the conditional variance is positive and finite. However, many authors, mainly Nelson and Charles (1992) and Alexander (2008), have reported several violations of those constraints without indications of statistical or sampling errors. They state that it is a practitioner's choice to impose any of these parameters' constraints (p.136 Alexander, 2008).

Asymmetric GARCH models

Asymmetric volatility suggests that there are higher volatility levels in downswings of the market than in upswings. Symmetric forms of GARCH models cannot deal with asymmetries. It is important that conditional variance captures this asymmetry to explain the behaviour of market returns and its leverage effect. We will use two asymmetric GARCH Models, the exponential GARCH (EGARCH), and the threshold GARCH (GJR-GARCH) models.

EGARCH

The exponential GARCH model was developed by Nelson (1991) to detect the presence of shocks, while the log function imposes positive results of the conditional variance parameter. Since EGARCH attaches more importance to negative shocks than positives ones, it will provide a different interpretation of IV conditional volatility. IV showed exceptional spikes especially in 2002, and between 2007 and 2008. Therefore, dependence only on symmetric GARCH models can provide ambiguous results. The model specification is:

$$\begin{aligned} \log(\sigma_{t|t'}^2) = & \omega + \sum_{j=1}^q \beta_j \log(\sigma_{t-j}^2) + \sum_{i=1}^p \alpha_i \left(\left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}^2} \right| - E \left| \frac{\varepsilon_{t-i}}{\sigma_{t-i}^2} \right| \right) \\ & + \sum_{k=1}^r \gamma_k \left| \frac{\varepsilon_{t-k}}{\sigma_{t-k}^2} \right| \end{aligned} \quad (5)$$

The leverage effect in the model is exponential, implied by the log

function of the conditional variance, and therefore it's always positive. γ represents the asymmetric response parameter, and the impact is asymmetric when $\gamma_i \neq 0$. The positive effect, good news, has an impact of α_i , and the negative effect, bad news, has an impact of α_i and γ_k .

GJR-GARCH

Since we are using the log-returns data of IV, using the log function in estimating the conditional variance can affect the significance level of the estimated parameter. Hence, we are using different forms of asymmetric models. GJR-GARCH, or the threshold GARCH, was presented by Zakoian (1994) and Glosten et al. (1993). GJR-GARCH is a model that introduces a threshold effect into the volatility by specifying that conditional variance is a function of the positive and negative parts of the residuals (Francq and Zakoian, 2011). The GJR-GARCH conditional variance is estimated by the following formula:

$$\sigma_{t_{IV}}^2 = \omega + \sum_{j=1}^q \beta_j \sigma_{t-j}^2 + \sum_{i=1}^p \alpha_j \varepsilon_{t-i}^2 + \sum_{k=1}^r \gamma_k \varepsilon_{t-k}^2 I_{t-k} \quad (6)$$

Where I_t is a function, that is $I_t=1$ if $\varepsilon_t < 0$, and 0 otherwise.

The GARCH-MIDAS model

Engle et al. (2013) developed a new GARCH model with mixed data sampling GARCH-MIDAS, which decompose short- and long run components. The model was used to measure the effect of the low frequency, long term component specified by macroeconomic variables, on high frequency, short term component, the market returns. GARCH-MIDAS model is described by equations (7) to (11):

$$r_{i,t} = \mu + \sqrt{\tau_t g_{IV_{i,t}}} \varepsilon_{i,t}, \quad (7)$$

The $r_{i,t}$ is the daily returns i , and monthly t observations. The conditional variance is represented by the short-run component $g_{i,t}$, and the long-run component τ_t . The conditional variance of the short-term component, follows a daily GARCH (1,1) process, which is:

$$g_{IV_{i,t}} = (1 - \alpha - \beta) + \alpha \frac{(r_{i-1,t} - \mu)^2}{\tau_t} + \beta g_{IV_{i-1,t}} \quad (8)$$

While the conditional variance of the long-term component is determined by the realized volatility of the returns and macroeconomic variables, and implemented in the MIDAS equation:

$$\tau_t = m + \theta \sum_{k=1}^K \varphi_k(\omega) V_{t-k}, \quad (9)$$

The next equation represents the average of monthly realized volatility of an exogenous variable:

$$V_t = \frac{1}{N} \sum_{i=1}^N x_{i,t}, \quad (10)$$

The macroeconomic variables, x_i are fixed value for $i = 1, \dots, N$, and the long-term volatility is captured by beta polynomials for $V_{t-1}, V_{t-2}, \dots, V_{t-k}$:

$$\varphi_k(\omega) \propto \left(1 - \frac{k}{K}\right)^{c1-1} \left(\frac{k}{K}\right)^{c2-1}. \quad (11)$$

B. Model specifications

We specified four different types of equations in estimating the IV using daily and monthly data, based on their realized volatility, the FTSE100R_t, and the following macroeconomic variables: IP_t, LIBOR3M_t, EEX_t, and UR_t. Below we discuss our model specifications:

For our benchmark case, we use only IV, into univariate, symmetric and asymmetric GARCH models, (see equations (3), (5), and (6)). The reason we use different frequencies is because we would like to investigate if different frequencies of the same index produce different results.

The second stage involves introducing FTSE 100_t, using multivariate GARCH models with equal data frequencies, in estimating IV, described by equations (4), (5), and (6). This will allow us to determine whether FTSE 100_t can alone improve the estimation results as an exogenous determinant.

Thirdly, we add the first difference of our macroeconomic variables at time t to identify their effect on IV, along with FTSE 100_t, also using equations (4), (5), and (6). Our purpose is to find the optimal combination of these variables that will produce the best results.

Lastly, GARCH-MIDAS will be applied to determine if mixed data frequency, daily and monthly, will produce different results in terms of the significance of the estimation parameter. We will use IV, with FTSE 100t and other macroeconomic factors, once at a time as exogenous variable, using equations (7) to (11).

IV. Results and Analysis

A. Descriptive analysis

Correlation matrix analysis

Table 1 shows correlations between monthly observations of the independent variables, which are the FTSE100 index log-returns, and the macroeconomic variables.

Based on table 1, LIBOR3M is positively correlated (0.430) with EER and negatively correlated with UR (−0.346) at 1%. EER on the other hand, has a negative (−0.170) correlation with UR at 5%. Furthermore, IP is positively correlated with LIBOR30 (0.194) at 1%, and negatively correlated (0.123) with EER at 10%. In order to test for possible multicollinearity among the independent variables, we conducted variance inflation factor tests (VIF), (see John et al, 1996).¹³ Table 1 shows the VIF values between the independent variables, which indicate no multicollinearity. VIF results are less than 4, which is the cut off value that is recommended by several researchers (e.g, Regreson (2001), Pan and Jackson (2008)).

Lastly, FTSE 100_t has no significant correlation with any other exogenous, macroeconomic variables in the UK market. This result contradicts some of the previous empirical researches, for example Olawale et al. (2014). A possible explanation for this contradiction could be that we used the first difference of the macroeconomic factors and the log-returns of FTSE 100_t, while other studies used levels for all factors.

13. The (VIF), an indicator of multicollinearity, is calculated as: $VIF = 1 / (1 - R^2)$. It is the reciprocal of tolerance. R^2 is obtained by regressing each independent variable on the remaining independent variables using OLS. This is given by $X_1 = \alpha_2 X_2 + \alpha_3 X_3 + \dots + \alpha_k X_k + e$.

TABLE 1. Correlation matrix: January 2000 to December 2015

Variables	FTSE100R	IP	LIBOR 3M	EER	UR	VIF
FTSE100 - Returns	1.000					1.038
IP	0.106	1.000				1.061
LIBOR 3M	-0.076	0.194***	1.000			1.391
EER	-0.072	0.123*	0.430***	1.000		1.233
UR	-0.088	-0.134*	-0.346***	-0.170**	1.000	1.158

Note: The table below shows correlations between monthly observations of FTSE100 index log-returns (FTSE100R), and the first difference of macroeconomic variables. The macroeconomic variables are: industrial production measure (IP), London 3 months interbank rate (LIBOR3M), UK GBP sterling effective exchange rate (EER) and Unemployment rate (UR). It also displays the variance inflation factor (VIF), an indicator of multicollinearity. As a rule of thumb, and since none of the chosen independent variables exceed the value of 5, there is no evidence of multicollinearity. The (VIF) is calculated as: $VIF = 1/(1-R^2)$. It is the reciprocal of tolerance. R^2 is obtained by regressing each independent variable on the remaining independent variables using OLS. This is given by $X_1 = a_2X_2 + a_3X_3 + \dots + a_kX_k + e$.

Descriptive statistics

Table 2 shows descriptive statistics. Looking at the results of IV, daily and monthly frequency, they both have similar means but different maxima, minima standard deviations etc. This is because monthly data captures only the last day (value) of a month and does not consider any values in-between the ends of consecutive months. The reason that the mean and median of our macroeconomic variables are close to zero is because we present the first differences.

Figure 2 presents plots of all variables. Looking at IV log-returns and FTSE100R log-returns charts, we observe that significant spikes in FTSE100 returns coincide with high implied volatility. These spikes represent incidents where IV increased accompanied by a decrease in FTSE 100 returns between 2001 and 2002, and in 2008, mainly during the global recessions in 2002 and 2008. These recessions were attributed to the negative economic trends in the UK economy. IV also exhibits high volatility between 2010 and 2011 and in 2015 due to market expectations, but this does not coincide with high spikes in the FTSE00R chart. The industrial production plot shows the negative effect of the recession in 2002 and also the negative shock effect of 2007 that appears with a delay after approximately two years, in 2009. Moreover, in 2012 IP turned negative but due to spending cuts to reduce the government long-term budgetary deficit. LIBOR 3M, effective

TABLE 2. Descriptive statistics of the variables

Variables	Daily IV	Monthly IV	FTSE100R	IP	LIBOR 3M	EER	UR
Observations	4014	191	191	191	191	191	191
Mean	0.000	0.000	-0.001	-0.076	-0.003	-0.049	-0.008
Median	-0.004	-0.046	0.006	-0.100	0.000	0.006	0.000
Maximum	0.540	0.812	0.091	2.600	0.000	3.543	0.500
Minimum	-0.738	-0.677	-0.243	-5.300	-0.499	-5.816	-0.200
Std. Dev.	0.067	0.248	0.047	1.022	0.036	1.296	0.081
Skewness	0.356	0.472	-1.298	-0.935	-13.747	-0.768	1.670
Kurtosis	10.488	4.332	6.602	7.294	189.997	5.654	11.057
Jarque-Bera	9462.478	21.199	156.868	175.540	285790.900	75.242	608.523
Probability	0.000	0.000	0.000	0.000	0.000	0.000	0.000

Note: This table shows summary statistics for the log-returns of FTSE100 implied volatility index, 30 days expiration (IV) based on the level of daily and monthly frequency from 4/1/2000 to 31/12/2015, the FTSE 100 log-returns index (FTSE100R) industrial production measure (IP), London 3 months interbank rate (LIBOR3M), UK GBP sterling effective exchange rate (EER) and Unemployment rate (UR).

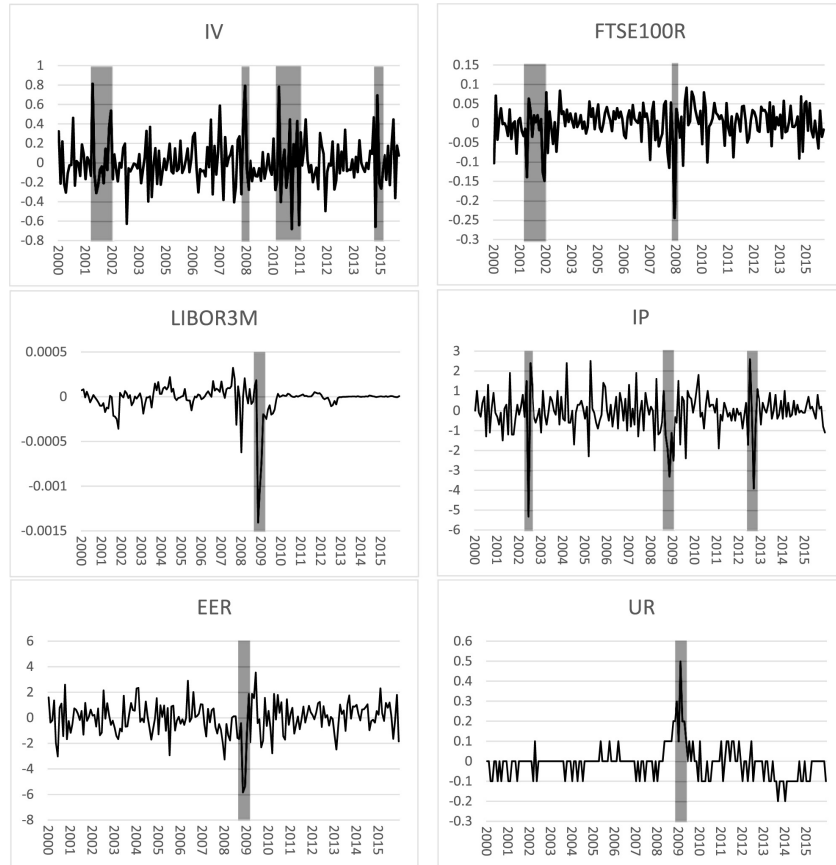


FIGURE 2.— Plot of log-returns of FTSE100 implied volatility index, FTSE100 log-returns, and the first difference of macroeconomic variables

Note: This figure shows monthly data of FTSE100 implied volatility index log-returns, with 30 days expiration (IV), FTSE 100 log-returns (FTSE100R), and the first difference of the macroeconomic variables from January 2000, to December 2015. The macroeconomic variables are the Industrial Production (IP), London 3 months Inter Bank Rate (LIBOR 3M), Effective Exchange Rate (EER), and Unemployment Rate (UR). Shaded areas in the charts show the most volatile periods for each variable.

exchange rate, and unemployment rate were affected mainly by the 2008 recession. LIBOR 3M was high at the beginning of 2007 since many of the financial institutions were in critical situation. This increased the

perceived risk of lending among banks that caused inadequate liquidity in the interbank market, which later introduced pressure on the economy. In 2009, LIBOR 3M decreased considerably since various central banks provided liquidity for financial institutions around the globe. Regarding the effective exchange rate, the largest decrease was in 2009. This fall can be attributed to the problems in equities and the banking sector in the UK. Similarly, unemployment rate shows a considerable spike in 2009 due to the effect of the financial crisis.

B. GARCH models parameters explanation and optimal choice

Tables 3, 4 and 5 present detailed, daily and monthly data, results of IV, when regressed on $FTSE\ 100_t$ returns and macroeconomic variables. GARCH parameters coefficients show their reaction to market shocks. These parameters according to Alexander (2008) are I) the mean of the returns (μ), II) the GARCH constant parameter (ω), which measures volatility's reaction, III) The first ARCH error parameter (α_1), which measures the reaction of conditional volatility to market shocks (the higher the value of α , the more sensitive volatility is to market events), IV) the leverage effect (γ_1), and V) the first GARCH parameter, the conditional variance (β_1), which measures the persistence of the conditional volatility regardless of the market volatility. When β is large (above 0.9), this means that volatility will persist for a long time following a market shock. The sum of α and β define the rate of convergence of the conditional volatility to the long term average volatility. When the sum of α and β is large, closer to 1.00, the term structure of the GARCH model is relatively flat, and conditional volatility takes longer to converge to average volatility.

The tables also present the parameters of the independent variables, the $FTSE\ 100_t$ log-returns and the macroeconomic variables. In order to decide which the best model is, we take into consideration the significance level of α and β , the Akaike Information Criterion (AIC), the Bayesian information criteria (BIC) and the log-likelihood function (LLF). In order to determine the best model, we use AIC and BIC and of course which independent variables fit best the sample data. In case they provide contradicting information, we choose the one with the higher LLF. The tables show five equations of IV, $FTSE\ 100_t$ log-returns, and the macroeconomic variables at time t for each of the GARCH models. The first two equations, in each table, show the estimation parameters for IV based only on its realized volatility

without adding independent variables. The last three equations, in each table, present the best fit models and the best combination of variables in the variance equation after having added our independent variables. Since there are 33 possible combinations when adding the independent variables, we included only the best three combinations respectively.

We used the classic order of ARCH term (q)=1, and the autoregressive order of GARCH term (p)=1, for GARCH, EGARCH and GJR-GARCH estimation. Even though the lag structure suggests an order of 3 for the GARCH term, it's not certain that it will always produce better results. Hansen and Lunde (2005) found that a GARCH (1,1) model provides better estimations and forecasts. We have also tested all possible lag structures, and the classic order for GARCH term (p)=1 produces the best results in our analysis.

C. GARCH models estimation results

Table 3 shows the estimation results of GARCH (1,1). Considering the full sample, and except for IV, with daily frequency, μ is significant in all equations. When μ is positive the higher the value of IV, the higher the variance of IV is. Similarly, ω is positive and significant in all equations meaning sensitive reactions to volatility, which also determines the change in the long-term volatility. The analysis of IV with monthly frequency, equation 2, based on its realized volatility, produces the highest ω value due to high market volatility. This happens because IV is monthly based and it doesn't take into account the values in between like daily data, which can reduce the effect of market shocks by the gradual change of the returns. However, equations 3, 4 and 5 in table 3, are also based on monthly data, but adding independent variables reduced the sensitivity of ω to market volatility, since there are now several determinant and explanatory factors. Moreover, α is also positive and significant in all cases confirming the existence of ARCH effects, the clustering patterns in the series. Since α is higher than 0.10 in all equations, except for subsample 1, this indicates a highly volatile and nervous market. Regarding subsample 1, α is lower than 0.10 in all equations, indicating a low volatility period. Also, GARCH persistence parameter β , is significant in most equations, also lower than 0.90, specifying that volatility doesn't take long time to converge to average volatility. The lower the β , the faster convergence is achieved to average volatility. The sum of α and β becomes lower when adding independent variables indicating that conditional volatility

TABLE 3. GARCH (1,1) estimation results with Normal distribution (Gaussian)

N	Samples	Variable in mean Equation	Variables in Variance Equation	Mean Equation (μ)
1		Daily IV	-	-0.001 (-0.576)
2		Monthly IV	-	18.149*** (35.504)
3	Full sample	Monthly IV	FTSE 100R, LIBOR3M, EER	-0.056*** (-3.459)
4		Monthly IV	FTSE 100R, LIBOR3M, EER, UR	-0.054*** (-3.947)
5		Monthly IV	FTSE 100R, EER	-0.056*** (-3.068)

(Continued)

TABLE 3. (Continued)

Variance Equation													
<i>N</i>	(<i>ω</i>)	(<i>α</i>)	(<i>β</i>)	FTSE 100R	IP	LIBOR 3M	EEX	UR	<i>α</i> + <i>β</i>	LLF	BIC	AIC	
1	0.000*** (9.970)	0.100*** (13.254)	0.827*** (63.262)	-	-	-	-	-	0.927	5368.200	-2.666	-2.673	
2	26.944*** (6.361)	0.752*** (6.828)	-0.071*** (-0.786)	-	-	-	-	-	0.681	-649.217	6.872	6.804	
3	0.020*** (4.149)	0.139** (1.964)	0.511*** (7.922)	-0.455*** (-4.882)	-	-29.694 (-1.745)	-0.008*** (-4.532)	-	0.650	16.083	0.024	-0.095	
4	0.023*** (-4.827)	0.154** (2.228)	0.459*** (5.518)	-0.443*** (-5.617)	-	-17.009 (-0.720)	-0.008*** (-2.621)	0.033*** (0.772)	0.613	16.982	0.042	-0.094	
5	0.022*** (3.758)	0.105* (1.725)	0.509*** (-6.554)	-0.338*** (-5.245)	-	-	-0.007*** (-6.053)	-	0.614	14.636	0.012	-0.090	
(Continued)													

(Continued)

TABLE 3. (Continued)

<i>N</i>	Samples	Variable in mean Equation	Variables in Variance Equation	Mean Equation (μ)
6		Daily IV	-	-0.001 (0.658)
7		Monthly IV	-	-0.007 (0.273)
8	Sub 1	Monthly IV	FTSE 100R, IP, UR	-0.057*** (2.705)
9		Monthly IV	FTSE 100R, LIBOR3M, UR	-0.057*** (2.634)
10		Monthly IV	FTSE 100R, EER, UR	-0.088*** (3.897)

(Continued)

TABLE 3. (Continued)

Variance Equation												
N	(ω)	(α)	(β)	FTSE 100R	IP	LIBOR 3M	EEX	UR	$\alpha+\beta$	LLF	BIC	AIC
6	0.000*** (4.636)	0.097*** (7.177)	0.817*** (29.188)	-	-	-	-	-	0.914	2850.493	-2.967	-2.979
7	0.001 (0.827)	-0.097*** (4.210)	1.069*** (3321.267)	-	-	-	-	-	0.972	17.912	-0.198	-0.309
8	0.022*** (3.122)	-0.075 (5.361)	0.574*** (5.740)	-0.467*** (3.228)	-0.006 (0.925)	-	-	-0.160 (1.224)	0.499	21.969	-0.138	-0.333
9	0.019*** (3.265)	-0.070*** (4.791)	0.635*** (9.345)	-0.516* (2.826)	-	-7.207 (0.244)	-	-0.132 (0.980)	0.564	23.009	-0.161	-0.356
10	0.023*** (3.060)	-0.077*** (3.713)	0.601*** (5.764)	-0.630*** (4.489)	-	-	-0.006 (1.242)	-0.171 (1.282)	0.524	23.162	-0.165	-0.359
(Continued)												

(Continued)

TABLE 3. (Continued)

<i>N</i>	Samples	Variable in mean Equation	Variables in Variance Equation	Mean Equation (μ)
11		Daily IV	-	0.000 (0.078)
12	Sub 2	Monthly IV	-	-0.008 (0.383)
13		Monthly IV	FTSE 100R, IP, LIBOR3M, EER	-0.011 (0.530)
14		Monthly IV	FTSE 100R, IP, LIBOR3M, UR	0.020 (0.875)

(Continued)

TABLE 3. (Continued)

Variance Equation												
N	(ω)	(α)	(β)	FTSE 100R	IP	LIBOR 3M	EEX	UR	$\alpha+\beta$	LLF	BIC	AIC
11	0.001*** (7.792)	0.118*** (10.228)	0.745*** (28.178)	-	-	-	-	-	0.862	2549.277	-2.410	-2.421
12	0.036** (2.018)	0.504** (2.241)	0.068 (0.278)	-	-	-	-	-	0.572	-6.207	0.306	0.202
13	0.033*** (3.711)	0.484** (2.256)	0.034*** (4.975)	-0.287* (1.923)	0.010 (1.563)	-35.291 (0.488)	0.002 (0.325)	-	0.519	4.831	0.270	0.063
14	0.035*** (2.745)	0.328** (2.111)	0.210*** (3.917)	-0.081 (0.782)	0.014*** (3.116)	69.180*** (3.153)	-	0.146** (2.006)	0.538	4.444	0.278	0.070
(Continued)												

(Continued)

TABLE 3. (Continued)

Note: This table presents the estimation results of GARCH (1,1) given in equations (2) and (4). This estimation is based on FTSE100 implied volatility index log-returns, 30 days expiration, using daily and monthly frequency of IV. In the variance equation, we have the FTSE 100 log-returns (FTSE100R_t), and our macroeconomic variables namely: Industrial Production (IP), London 3 months Inter Bank Rate (LIBOR3M_t), Effective Exchange Rate (EER_t), and Unemployment Rate (UR_t). The Full sample is from 4/1/2000 to 31/12/2015, subsample 1 is from 4/1/2000 to 8/8/2007, and subsample 2 is from 9/8/2007 to 31/12/2015. The parameters estimated are: the mean of the returns (μ), the first order of the GARCH constant parameters (ω), the first order of ARCH error term (α), and the first order of the GARCH term, (β). LLF is the value of the maximized likelihood function, BIC is the Bayesian information criterion, and AIC is the Akaike information criterion. IV index data is obtained from FTSE Russell, macroeconomic variables are obtained from Datastream. *** indicate significance at 1%, ** indicate significance at 5%, * indicate significance at 10%. The numbers in parentheses are t-statistics.

doesn't take longer time to return to the average level of volatility. Adding the independent variables helped in making conditional volatility more reactive to market shocks, and improved the significance of the estimation parameters. When evaluating models based on BIC, AIC, and LLF, equation 3 surpasses all other equations combining FTSE 100_{*t*}, LIBOR3M_{*t*}, and EEX_{*t*}. This means that adding market returns along with macroeconomic variables enhances the estimation process for the full sample. Results for subsample 1 and 2, are almost similar to the full sample in terms of parameters' significance and the convergence rate of conditional volatility. However, the combination of independent variables differ in the variance equations. For subsample 1, equation 8, the combination of FTSE100R_{*t*}, IP_{*t*}, and UR_{*t*} generate the best fit, while in equation 13, the group of FTSE100R_{*t*}, IP_{*t*}, LIBOR3M_{*t*}, and UR_{*t*} provide the best results for subsample 2. Besides, only two combinations of independent variables showed significant values of α and β , due to a highly volatile market since subsample 2 includes data from the beginning of financial crisis in 2007.

Some of the exogenous coefficients in the variance equations are negative, which could be due to sampling error and misspecification. With the introduction of (ARCH) model by Engel (1982) and (GARCH) by Bollerslev (1986), parameter constraints have been introduced to insure nonnegative conditional variance, more specifically: $\omega \geq 0$, $\beta_i \geq 0$ for all $i = 1$ to p , $\beta_j \geq 0$ for all $j = 1$ to q . Negative coefficients in GARCH models could be due to non-stationary data or residual serial correlation in the mean equation. However, Nelson and Charles (1992) and Alexander (2008), indicated that imposing constraints is a practitioner's choice, and these constraints are generally difficult to enforce, since several violations have been reported in the ARCH literature. Nelson and Charles (1992) claim that violations of Bollerslev's inequality constraints couldn't be due to statistical error or sampling problems. They have documented several violations of Bollerslev's constraints, specifically negative values of ARCH and GARCH terms α 's and β 's respectively, when estimating daily data of S&P 500, and for daily exchange rate of several currencies.¹⁴

14. Nelson and Charles (1992) encountered several violations of the GARCH parameters constraints in their study. They have reported several incidences of negative α_2 values, in ARCH terms, in their subsamples when estimating the volatility of the daily returns of S&P500. They have also reported negative α values for different orders of GARCH terms when estimating the conditional variance of three currencies against the US dollar, namely the British pound, the Japanese yen, and the Italian lira. Even though, they have not reported any negative β values in all cases of their empirical study, their decision based on the Akaike information criterion (AIC), includes selecting the best fit models with negative α values.

TABLE 4. EGARCH (1,1) estimation results with Normal distribution (Gaussian)

N	Samples	Variable in mean Equation	Variables in Variance Equation	Mean Equation (μ)
1		Daily IV	-	0.002** (2.28)
2		Monthly IV	-	17.621*** (37.74)
3	Full sample	Monthly IV	FTSE 100R, IP, UR	-0.039*** (3.22)
4		Monthly IV	FTSE 100R, IP, EER, UR	-0.033*** (2.59)
5		Monthly IV	FTSE 100R, IP, LIBOR3M, UR	-0.039*** (3.24)

(Continued)

TABLE 4. (Continued)

Variance Equation													
N	(ω)	(α)	(γ)	(β)	FTSE 100R	IP	LIBOR 3M	EEX	UR	$\alpha+\beta$	LLF	BIC	AIC
1	-0.209*** (10.99)	0.068*** (8.41)	0.125*** (18.70)	0.971*** (353.47)	-	-	-	-	-	1.039	5416.604	-2.689	-2.696
2	2.395*** (3.80)	0.757*** (3.25)	0.533*** (2.75)	0.209 (1.36)	-	-	-	-	-	0.967	-646.780	6.874	6.789
3	-5.570*** (21.16)	0.729*** (5.48)	0.104* (1.65)	-0.596*** (8.29)	-10.266*** (5.69)	0.208*** (3.34)	-	-	4.795*** (5.44)	1.325	29.456	-0.088	-0.225
4	-5.538*** (19.91)	0.735*** (5.30)	0.108 (1.55)	-0.583*** (7.77)	-10.071*** (5.42)	0.210*** (3.32)	-	0.043 (0.66)	4.980*** (5.50)	1.318	29.670	-0.063	-0.216
5	-5.559*** (20.00)	0.721*** (5.36)	0.104 (1.62)	-0.596*** (8.17)	-10.279*** (5.65)	0.206*** (3.15)	194.750 (0.22)	-	4.772*** (5.38)	1.317	0.887	-0.061	-0.214
(Continued)													

(Continued)

TABLE 4. (Continued)

<i>N</i>	Samples	Variable in mean Equation	Variables in Variance Equation	Mean Equation (μ)
6		Daily IV	-	0.001 (1.054)
7		Monthly IV	-	0.003 (0.123)
8	Sub1	Monthly IV	FTSE 100R, LIBOR3M	-0.033*** (2.804)
9		Monthly IV	FTSE 100R, IP, LIBOR3M, UR	-0.035*** (5.529)
10		Monthly IV	FTSE 100R, LIBOR3M, UR	-0.035*** (4.924)

(Continued)

TABLE 4. (Continued)

N	Variance Equation												
	(ω)	(α)	(γ)	(β)	FTSE 100R	IP	LIBOR 3M	EEX	UR	$\alpha+\beta$	LLF	BIC	AIC
6	-0.460*** (5.581)	0.118*** (6.053)	0.112*** (7.652)	0.937*** (74.006)	-	-	-	-	-	1.054	2864.750	-2.978	-2.993
7	-1.210*** (2.839)	-0.393** (2.309)	0.521*** (2.601)	0.521*** (3.132)	-	-	-	-	-	0.915	14.818	-0.079	-0.218
8	-0.263*** (13.966)	-0.846*** (722.260)	0.207** (1.966)	0.735*** (4.1E+103)	-10.642*** (4.340)	-	1440.465*** (3.408)	-	-	1.581	32.269	-0.367	-0.562
9	-0.400*** (13513.209)	-0.998*** (1.0E+103)	0.359*** (3.443)	0.683*** (5.7E+103)	-7.122 (2.566)	-0.265 (1.404)	1398.517*** (2.916)	-	-4.301*** (3.939)	1.681	34.035	-0.306	-0.556
10	-0.375*** (43.388)	-0.954*** (883288.889)	0.274*** (2.666)	0.692*** (5.8E+103)	-8.104*** (2.746)	-	1173.317*** (2.527)	-	-2.816*** (2.825)	1.645	32.708	-0.327	-0.549

(Continued)

TABLE 4. (Continued)

<i>N</i>	Samples	Variable in mean Equation	Variables in Variance Equation	Mean Equation (μ)
11		Daily IV	-	0.003** (2.002)
12		Monthly IV	-	0.007 (0.283)
13	Sub2	Monthly IV	FTSE 100R, LIBOR3M	-0.037 (1.476)
14		Monthly IV	FTSE 100R, LIBOR3M, EER	-0.041* (1.706)
15		Monthly IV	FTSE 100R, LIBOR3M, EER, UR	-0.034 (1.376)

(Continued)

TABLE 4. (Continued)

N	Variance Equation									
	(ω)	(α)	(γ)	(β)	FTSE 100R	IP	LIBOR 3M	EEX	UR	$\alpha+\beta$
11	-0.197*** (8.455)	0.029*** (3.671)	0.153*** (18.052)	0.966*** (249.181)	-	-	-	-	-	0.996
12	-1.820** (2.189)	0.636** (2.268)	0.221 (1.310)	0.527* (1.915)	-	-	-	-	-	1.163
13	-1.986** (2.414)	0.598** (2.121)	0.136 (0.589)	0.476* (1.760)	-8.802*** (2.933)	-	605.839 (0.871)	-	-	1.074
14	-1.745** (2.408)	0.497* (1.842)	0.098 (0.477)	0.528** (2.170)	-8.478*** (2.682)	-	1214.355 (1.197)	-0.113 (0.633)	-	1.026
15	-1.764** (2.572)	0.511** (1.982)	0.123 (0.600)	0.520** (2.255)	-7.890** (2.506)	-	1481.759 (1.403)	-0.107 (0.594)	1.137 (0.537)	1.031

(Continued)

TABLE 4. (Continued)

Note: This table present the estimation results of EGARCH (1,1) model given in equations (2) and (5). This estimation is based on FTSE100 implied volatility index log-returns, 30 days expiration, using daily and monthly frequency of IV. In the variance equation, we have the FTSE 100 log-returns (FTSE100R_t), and our macroeconomic variables namely: Industrial Production (IP), London 3 months Inter Bank Rate (LIBOR3M_t), Effective Exchange Rate (EER_t), and Unemployment Rate (UR_t). The Full sample is from 4/1/2000 to 31/12/2015, subsample 1 is from 4/1/2000 to 8/8/2007, and subsample 2 is form 9/8/2007 to 31/12/2015. The parameters estimated are: the mean of the returns (μ), the first order of the GARCH constant parameters (ω), the first order of ARCH error term (α), first order of the leverage effect (γ), and the first order of the GARCH term, (β). LLF is the value of the maximized likelihood function, BIC is the Bayesian information criterion, and AIC is the Akaike information criterion. IV index data is obtained from FTSE Russell, macroeconomic variables are obtained from Datastream. *** indicate significance at 1%, ** indicate significance at 5%, * indicate significance at 10%. The numbers in parentheses are t-statistics.

When adding exogenous variables in the variance equation, is tantamount to including a high order of GARCH terms in our estimation. Adding a covariate, as we do, improves volatility estimation and any negative coefficients could not be due to misspecification. In addition, we used the log returns of the volatility index, and the first difference of the exogenous variables and all of those variables are stationary as indicated by our stationarity tests. Also, no serial correlation is present in the residuals. Considering the absence of pathological effects (no misspecification, stationary data and no serial correlation in the residuals), we believe that our models do not ‘misbehave’ since negative values have been reported in the literature before.

Table 4 shows results of IV with market returns alongside macroeconomic factors using EGARCH (1,1). The parameter coefficients are mostly significant, and the information criteria, BIC and AIC are lower than the ones provided by GARCH (1,1) results in most cases. However, the rates of convergence of the conditional volatility to long term average level measured by the sum of α and β is increasing (above 1.00), therefore it provides unrealistic estimations for most models. This could be explained by the specification of EGARCH_t, which considers the log of the variance to ensure that positive variance values are produced. This could have caused non-stationarity in most equations. In other words, the EGARCH_t asymmetric feature which includes the leverage effect caused in a way a trending pattern in the results. In equations 2, 7 and 11, the convergence rate is below 1.00. The first two equations include IV in full sample and IV in subsample 1, but they are outperformed by other specifications using GARCH (1,1). The only meaningful equation using EGARCH_t (1,1), is equation 11 in subsample 2, which provided lower information criterion values. EGARCH_t does not capture adequately the qualities of the data set in this case because of the log variance which potentially introduces non-stationarity, unless of course there is exceptionally high volatility.

Table 5 shows GJR-GARCH (1,1) estimation results, following the same approach of GARCH and EGARCH. For the full sample, the results of IV, equations 1 and 2, show high rates of convergence of conditional volatility to long term average, but broke the parameter constraints since the sum of α and β exceeded 1.00, indicating unrealistic results. However, adding the exogenous variables resulted only in two equations with significant ARCH and GARCH effects in the full sample namely equations 3 and 4. The first combination is

TABLE 5. GJR-GARCH (1,1) estimation results with Normal distribution (Gaussian)

<i>N</i>	Samples	Variable in mean Equation	Variables in Variance Equation	Mean Equation (μ)
1		Daily IV	-	0.001 (1.333)
2		Monthly IV	-	17.753*** (33.530)
3	Full sample	Monthly IV	FTSE 100R, EER	-0.018 (0.988)
4		Monthly IV	FTSE 100R, IP, UR	-0.010 (0.358)
5		Daily IV	-	0.002 (1.132)
6		Monthly IV	-	0.011 (0.437)
7	Sub2	Monthly IV	FTSE 100R, LIBOR3M, UR	0.019 (0.750)
8		Monthly IV	FTSE 100R, LIBOR3M, EER, UR	0.019 (0.761)
9		Monthly IV	FTSE 100R, IP, EER	0.021 (0.510)

(Continued)

TABLE 5. (Continued)

N	(ω)	(α)	(γ)	(β)	Variance Equation					$\alpha+\beta$	LLF	BIC	AIC
					FTSE 100R	IP	LIBOR 3M	EEX	UR				
1	0.000*** (11.450)	0.123*** (13.336)	-0.118*** (11.657)	0.881*** (99.872)	-	-	-	-	-	1.003	5399.548	-2.680	-2.688
2	29.474*** (6.171)	0.948*** (5.590)	-0.915** (2.123)	-0.068 (0.760)	-	-	-	-	-	1.015	-645.458	6.860	6.776
3	0.020*** (3.506)	0.247** (2.015)	-0.269 (1.730)	0.494*** (4.704)	-0.333*** (4.791)	-	-	-0.003 (1.202)	-	0.741	14.321	0.043	-0.077
4	0.037* (1.861)	0.030*** (5.247)	-0.150*** (4.244)	0.575** (2.316)	-0.166 (1.389)	0.018*** (4.199)	-	-	0.051 (0.814)	0.604	0.374	0.216	0.080
5	0.000*** (8.399)	0.122*** (9.275)	-0.124*** (8.906)	0.866*** (60.860)	-	-	-	-	-	0.988	2565.364	-2.422	-2.435
6	0.037*** (2.609)	0.838* (1.808)	-0.654 (1.315)	0.065 (0.315)	-	-	-	-	-	0.903	-4.887	0.325	0.196
7	0.030*** (5.261)	0.353* (1.817)	-0.317 (1.332)	0.402** (2.530)	-0.234 (1.542)	-	100.294** (2.243)	-	0.130*** (2.787)	0.756	3.870	0.289	0.082
8	0.030** (2.237)	0.354*** (21.440)	-0.313 (2.388)	0.417** (2.018)	-0.233 (1.299)	-	102.405* (1.801)	0.000 (0.044)	0.129** (2.193)	0.771	3.744	0.337	0.104
9	0.049** (2.077)	0.027*** (2.786)	-0.186 (3.081)	0.571** (2.500)	-0.174 (1.333)	0.021*** (3.078)	-	0.002 (0.593)	-	0.598	-8.183	0.528	0.320

(Continued)

TABLE 5. (Continued)

Note: This table present the estimation results of GJR-GARCH (1,1) model given in equations (2) and (6). This estimation is based on FTSE100 implied volatility index log-returns, 30 days expiration, using daily and monthly frequency of IV. In the variance equation, we have the FTSE 100 log-returns (FTSE100R_t), and our macroeconomic variables namely: Industrial Production (IP), London 3 months Inter Bank Rate (LIBOR3M_t), Effective Exchange Rate (EER_t), and Unemployment Rate (UR_t). The Full sample is from 4/1/2000 to 31/12/2015, subsample 1 is from 4/1/2000 to 8/8/2007, and subsample 2 is form 9/8/2007 to 31/12/2015. The parameters estimated are: the mean of the returns (μ), the first order of the GARCH constant parameters (ω), the first order of ARCH error term (α), first order of the leverage effect (γ), and the first order of the GARCH term, (β). LLF is the value of the maximized likelihood function, BIC is the Bayesian information criterion, and AIC is the Akaike information criterion. IV index data is obtained from FTSE Russell, macroeconomic variables are obtained from Datastream. *** indicate significance at 1%, ** indicate significance at 5%, * indicate significance at 10%. The numbers in parentheses are t-statistics.

TABLE 6. Most fitted equations based on GARCH models estimation

Analysis results of only daily data of IV				
Samples	Model ranking	Model	Variables	
			Mean Equation	Variance Equation
Full sample	1	GARCH	Daily IV	-
Subsample (1)	1	GARCH	Daily IV	-
Subsample (2)	1	EGARCH	Daily IV	-
Analysis results of monthly data of IV with exogenous variables				
Samples	Model ranking	Model	Variables	
			Mean Equation	Variance Equation
Full sample	1	GARCH	Monthly IV	FTSE 100R, LIBOR3M, EER
Subsample (1)	1	GARCH	Monthly IV	FTSE 100R, IP, UR
Subsample (2)	1	GARCH	Monthly IV	FTSE 100R, IP, LIBOR3M, EER

Note: The table below presents the best fit equations for all samples based on the analysis of IV. Our explanatory factors are: FTSE 100 log-returns (FTSE100R_{*t*}), industrial production (IP_{*t*}), the London three months Inter Bank Interest Rate (LIBOR3M_{*t*}), GBP Effective Exchange Rate (EER_{*t*}), and Unemployment Rate (UR_{*t*}). The analysis is conducted using several GARCH models, GARCH (1,1), EGARCH (1,1), and GJR-GARCH (1,1). The tables present the best fit equations for all samples by taking into account the parameters of: (μ) the mean coefficients of the returns, (ω) the unconditional variance, (α) the ARCH term, (γ) the leverage effect, and (β) the GARCH term, the conditional variance. Models with significant parameters were ranked based on the lowest values of the Bayesian information criterion (BIC), and the Akaike information criterion (AIC).

presented by equation 3 which includes FTSE100R_{*t*} with EER_{*t*}, and the second is equation 4 which includes FTSE100R_{*t*} with IP_{*t*} and UR_{*t*}. We eliminated the GJR-GARCH estimation results for subsample 1 since we couldn't find any possible combination of variables that provide significant ARCH and GARCH effects. As for subsample 2, except for IV, with monthly data, analysis shown in equation 6, GJR-GARCH model provided significant parameters using only IV's daily realized volatility in equation 5, and also when adding exogenous variables, described by equations 7, 8 and 9. However, all of GJR-GARCH results and equations are outperformed by GARCH and EGARCH.

To summarize, Table 6 presents the best fit equations that model the conditional volatility of IV. We cannot compare the daily and the monthly results of IV, with or without independent variables because of the different data frequencies. When analysing IV based on its daily

TABLE 7. GARCH-MIDAS estimation results with maximum likelihood

<i>N</i>	Variables	Sample	Specification	(μ)	(α)	(β)	(θ)	(ω)	(<i>m</i>)
1			FW	0.000 (-0.388)	0.102*** (11.899)	0.820*** (45.985)	0.002 (0.003)	19.932 (0.999)	0.001*** (27.579)
2		Full Sample	RW	0.000 (-0.303)	0.105*** (11.754)	0.787*** (43.764)	0.174 (13.463)	1.002*** (6.396)	0.000*** (5.615)
3	Independent variable: IV	Subsample 1	FW	0.000 (-0.392)	0.098*** (5.997)	0.812*** (23.560)	0.004 (0.003)	4.890 (0.001)	0.000*** (8.036)
4	Dependent Variable: -		RW	0.000 (-0.375)	0.102*** (6.130)	0.801*** (23.088)	0.125*** (2.579)	1.001** (2.114)	0.000*** (3.992)
5		Subsample 2	FW	0.000 (-0.249)	0.135*** (9.691)	0.716*** (24.232)	0.003 (0.010)	30.649 (0.002)	0.000*** (27.395)
6			RW	0.000 (-0.237)	0.133*** (3.527)	0.711*** (23.402)	0.076*** (2.452)	2.273 (0.495)	0.000*** (15.997)
7			FW	0.000 (-0.547)	0.050 (0.241)	0.900*** (11.614)	0.079*** (2.749)	5.000*** (5.326)	0.000*** (3.030)
8		Full Sample	RW	0.000 (0.012)	0.050 (0.732)	0.900*** (7.966)	0.100 (1.581)	5.000*** (6.908)	0.000 (1.640)
9	Independent variable: IV	Subsample 1	FW	0.000 (0.057)	0.050 (0.349)	0.900*** (9.087)	0.39*** (1.999)	5.000*** (4.433)	0.000** (2.454)
10	Dependent Variable: FTSE		RW	0.000 (0.222)	0.050 (0.543)	0.900*** (5.848)	0.100 (1.406)	5.000*** (6.690)	0.000 (1.473)
11	100R		FW	0.000 (-0.017)	0.050 (0.818)	0.900*** (8.992)	0.100* (1.903)	5.000*** (31.322)	0.000* (1.953)
12		Subsample 2	RW	0.000 (-0.002)	0.050 (0.474)	0.900*** (5.285)	0.100 (0.758)	5.000*** (6.184)	0.000 (0.758)

(Continued)

TABLE 7. (Continued)

<i>N</i>	Variables	Sample	Specification	$\alpha + \beta$	LLF	BIC	AIC
1			FW	0.922	21311.700	-42573.700	-42611.500
2		Full Sample	RW	0.892	21325.600	-42601.400	-42639.200
3	Independent variable: IV	Subsample ₁	FW	0.910	9104.190	-18163.000	-18196.400
4	Dependent Variable: -		RW	0.903	9104.660	-18164.000	-18197.300
5		Subsample ₂	FW	0.851	9783.180	-19520.500	-19554.400
6			RW	0.844	9783.680	-19521.500	-19555.400
7			FW	0.951	15688.400	-31327.100	-31364.900
8		Full Sample	RW	0.950	15307.300	-30564.900	-30602.700
9	Independent variable: IV	Subsample ₁	FW	0.950	7212.980	-14380.600	-14414.000
10	Dependent Variable: FTSE		RW	0.951	6641.540	-13237.700	-13271.100
11	100R	Subsample ₂	FW	0.950	8023.550	-16001.200	-16035.100
12			RW	0.950	7628.190	-15210.500	-15244.400

(Continued)

TABLE 7. (Continued)

<i>N</i>	Variables	Sample	Specification	(μ)	(α)	(β)	(θ)	(ω)	(<i>m</i>)
13			FW	0.000 (-0.378)	0.102*** (12.750)	0.818*** (56.911)	0.000 (-0.956)	36.129 (0.275)	0.000*** (26.174)
14		Full Sample	RW	0.000 (-0.003)	0.050 (0.327)	0.900*** (4.131)	0.100 (0.904)	5.000*** (13.009)	0.002 (0.913)
15	Independent variable: IV	Subsample 1	FW	0.000 (-0.126)	0.076*** (6.916)	0.908*** (60.591)	0.000 (0.263)	49.039 (0.048)	0.000*** (6.129)
16	Dependent Variable: IP		RW	0.000 (-0.172)	0.252*** (11.103)	0.748*** (32.901)	0.131 (1.588)	2.571*** (2.606)	0.001* (1.675)
17		Subsample 2	FW	0.000 (-0.088)	0.131*** (9.246)	0.685*** (18.923)	0.000*** (7.867)	1.845*** (5.455)	0.000*** (24.336)
18			RW	0.000 0.000	0.050 (0.161)	0.900*** (3.952)	0.100 (0.180)	5.000*** (17.939)	0.000 (0.180)
19			FW	0.000 (-0.095)	0.050*** (10.326)	0.900*** (79.493)	0.100*** (9.300)	5.000*** (6.917)	0.000 (26.140)
20		Full Sample	RW	0.000 (-0.096)	0.050*** (10.514)	0.900*** (80.592)	0.100*** (9.726)	5.000*** (7.094)	0.000*** (26.738)
21	Independent variable: IV	Subsample 1	FW	0.000 (-0.126)	0.050*** (5.825)	0.900*** (41.977)	0.100*** (3.637)	5.000* (1.846)	0.000*** (15.781)
22	Dependent Variable: LIBOR 3M		RW	0.000 (-0.126)	0.050*** (5.807)	0.900*** (41.619)	0.100*** (3.593)	5.000* (1.810)	0.000*** (15.706)
23		Subsample 2	FW	0.000 (-0.042)	0.050*** (9.085)	0.900*** (66.468)	0.100*** (7.507)	5.000*** (2.246)	0.000*** (22.085)
24			RW	0.000 (-0.042)	0.050*** (9.088)	0.900*** (66.497)	0.100*** (7.052)	5.000*** (2.380)	0.000*** (22.013)

(Continued)

TABLE 7. (Continued)

<i>N</i>	Variables	Sample	Specification	$\alpha + \beta$	LLF	BIC	AIC
13			FW	0.920	21312.100	-42574.500	-42612.300
14		Full Sample	RW	0.950	9768.120	-19486.500	-19524.200
15	Independent variable: IV	Subsample ₁	FW	0.984	9098.450	-18151.600	-18184.900
16	Dependent Variable: IP		RW	0.999	9059.680	-18074.000	-18107.400
17			FW	0.816	9792.200	-19538.500	-19572.400
18		Subsample ₂	RW	0.950	5013.150	-9980.390	-10014.300
19			FW	0.950	21251.900	-42454.000	-42491.800
20		Full Sample	RW	0.950	21259.900	-42470.000	-42507.800
21	Independent variable: IV	Subsample ₁	FW	0.951	9092.560	-18139.800	-18173.100
22	Dependent Variable: LIBOR 3M		RW	0.950	9092.950	-18140.600	-18173.900
23			FW	0.950	9776.990	-19508.100	-19542.000
24		Subsample ₂	RW	0.951	9776.650	-19507.400	-19541.300

(Continued)

TABLE 7. (Continued)

<i>N</i>	Variables	Sample	Specification	(μ)	(α)	(β)	(θ)	(ω)	(<i>m</i>)
25		Full Sample	FW	0.000 (-0.008)	0.050 (0.684)	0.900*** (4.872)	0.100 (1.170)	5.000*** (48.095)	0.003 (1.171)
26			RW	0.000 (-0.263)	0.107*** (16.686)	0.893*** (139.170)	0.000 (0.179)	5.152 (0.123)	0.000** (2.414)
27	Independent variable: IV	Subsample 1	FW	0.000 (-0.012)	0.050 (0.410)	0.900*** (5.018)	0.100 (1.133)	5.000*** (7.066)	0.000 (1.134)
28	Dependent Variable: EEXR		RW	0.000 (-0.003)	0.050 (0.335)	0.900*** (4.219)	0.100 (0.921)	5.000*** (14.135)	0.001 (0.937)
29		Subsample 2	FW	0.000 (-0.118)	0.231*** (14.581)	0.769*** (48.518)	0.029 (0.757)	4.962*** (5.138)	0.000 (0.842)
30			RW	0.000 (0.000)	0.050 (0.102)	0.900*** (3.791)	0.100 (0.106)	5.000*** (8.448)	0.001 (0.011)
31		Full Sample	FW	0.000 (1.176)	0.050 (1.248)	0.901*** (12.120)	0.020*** (3.467)	5.000*** (9.233)	0.000*** (3.576)
32			RW	0.000 (-0.564)	0.124*** (23.609)	0.876*** (167.500)	0.122** (2.443)	1.056*** (78.401)	0.000*** (2.443)
33	Independent variable: IV	Subsample 1	FW	0.000 (-0.022)	0.050 (1.360)	0.900*** (10.849)	0.100** (2.287)	5.000*** (8.391)	0.000** (2.411)
34	Dependent Variable: UR		RW	0.000 (-0.009)	0.050 (0.501)	0.9000*** (5.677)	0.100 (1.298)	5.000*** (6.966)	0.000 (1.310)
35		Subsample 2	FW	0.000 1.176	0.050 (1.248)	0.900*** (12.120)	0.020*** (3.467)	5.000*** (9.233)	0.000*** (3.576)
36			RW	0.000 (-0.564)	0.124*** (23.609)	0.876*** (167.500)	0.123** (2.442)	1.056*** (78.401)	0.000** (2.443)

(Continued)

TABLE 7. (Continued)

<i>N</i>	Variables	Sample	Specification	$\alpha + \beta$	LLF	BIC	AIC
25			FW	0.950	8841.700	-17633.600	-17671.400
26		Full Sample	RW	0.950	21249.700	-42449.700	-42487.400
27	Independent variable: IV	Subsample ₁	FW	0.951	4479.230	-8913.130	-8946.460
28	Dependent Variable: EEXR		RW	0.950	4506.440	-8967.550	-9000.880
29		Subsample ₂	FW	0.951	9718.760	-19391.600	-19425.500
30			RW	0.951	4637.080	-9228.250	-9262.150
31			FW	0.951	17164.000	-34278.200	-34316.000
32		Full Sample	RW	0.999	21243.200	-42436.700	-42474.500
33	Independent variable: IV	Subsample ₁	FW	0.950	6932.990	-13820.600	-13854.000
34	Dependent Variable: UR		RW	0.950	6916.240	-13877.100	-13910.500
35		Subsample ₂	FW	0.950	3598.000	-34278.200	-34316.000
36			RW	0.999	21243.200	-42436.700	-42474.500

(Continued)

TABLE 7. (Continued)

Note: This table present the estimation results of GARCH-MIDAS model with 2 MIDAS lag years, given in equations (7), (8) and (9). This estimation is based FTSE100 implied volatility index log-returns, 30 days expiration, with daily monthly frequency of IV. In the variance equations are the FTSE 100 log-returns (FTSE100R_t), and our macroeconomic variables namely: Industrial Production (IP), London 3 months Inter Bank Rate (LIBOR3M), Effective Exchange Rate (EER), and Unemployment Rate (UR). Full sample is from 4/1/2000 to 31/12/2015, subsample 1 is from 4/1/2000 to 8/8/2007, and subsample 2 is from 9/8/2007 to 31/12/2015. The parameters are the mean coefficients of the mean of the returns (μ), the first order of the GARCH constant parameters (ω), first of GARCH error term (α), first order of GARCH term, the conditional volatility (β), and the moving average variance (m). LLF is the value of the maximized likelihood function, BIC is the Bayesian information criterion, and AIC is the Akaike information criterion. IV index data is obtained from FTSE Russell, macroeconomic variables obtained from Datastream. *** indicate significance at 1%, ** indicate significance at 5%, * indicate significance at 10%. The numbers in parentheses are t-statistic values.

realized volatility, GARCH (1,1) outperformed other models for the full sample and subsample 1. The symmetric GARCH (1,1) model was more accurate in a low volatility period. However, EGARCH was able to capture existent volatility in a more volatile set of data, which is the case of subsample 2, where the market was highly volatile especially between 2007 and 2008 due to the financial crisis.

When IV is regressed on its monthly realized volatility, market returns and macroeconomic variables, GARCH (1,1) models outperformed other models. This indicates that asymmetric models don't provide better estimations in such volatile environments, especially when adding exogenous variables. So overall GARCH (1,1) appears to be the best fit model unless there is exceptionally high volatility in which case EGARCH would perform better. In the next section, we take the analysis further by using GARCH-MIDAS, which enables us to analyse the effect of the chosen exogenous variables on IV using a mixed data approach.

D. GARCH-MIDAS estimation results

Table 7 displays GARCH-MIDAS output using six equations, the IV regressed on its realized monthly volatility alongside five independent variables namely $FTSE100R_t$, IP_t , $LIBOR3M_t$, EEX_t , and UR_t , introduced once at a time. We used 24 lags, which are two years of realized volatility (24 and 416 observations for the long, and the short components consecutively). The lags are averaged by the MIDAS equation in order to estimate the long run conditional variance. Apart from using a fixed window approach (FW), we also used a rolling window (RW) specification to see if it produces different results. A rolling analysis allows the model parameters to change overtime to capture any instability in the economic determinants over time.

According to table 7, it is clear that the mean of the returns, μ is insignificant in all equations, specifying that the mean does not explain returns volatility. However, as indicated before, we rely on the significance of the ARCH error term α , and GARCH conditional volatility β , parameters in model selection. In most cases, these parameters are significant showing the existence of conditional heteroscedasticity and autocorrelation. Based on the results, a rolling window approach provides the most significant outputs, lower information criterion values, and higher LLF. For the whole sample, equation 2, IV regressed on its monthly realized volatility using a rolling

window, produces the best fit model. It generates significant α and β terms, and has the lowest information criterion. The ARCH term α in equation 2, reaches the highest value of all the full sample equations in the GARCH-MIDAS analysis (0.105) at 1% significant level, indicating high sensitivity to market shocks. The conditional variance, on the other hand reaches its minimum value, showing the lowest convergence rate of the conditional volatility to average volatility. For subsample 1, when regressing IV on its monthly realized volatility using a rolling window, (equation 4), produces the best fit model. We obtain the lowest values for AIC and BIC but not the highest α or the lowest β . As for subsample 2, and due to the high volatility observed, regressing IV on UR_t , equation 36, using a rolling window provided the best fit model. Its ARCH term, α has the highest value showing high reaction to market volatility. It also has the lowest AIC and BIC values, but not the lowest β term, meaning that it doesn't have the highest convergence rate.

GARCH-MIDAS clearly pointed out the significance of the ARCH error term α , and conditional volatility effect β , in our results. It is clear that modelling the variance of the equation with AR (p), using MIDAS in analysing IV has considerable benefits in several cases. In other words, GARCH-MIDAS provides further support for the effect that exogenous factors have on IV. For the whole sample and for subsample 1, regressing IV on its realized volatility, equations 2 and 4, provide the best fit. However, for subsample 2, adding UR_t as an independent variable, equation 36, outperformed the results produced only by IV and its realized volatility. However, adding FTSE100R_t to the IV regression did not generate a significant α . Equations from 7 to 12, indicate that volatility is not sensitive to market shocks. As for macroeconomic determinants, adding LIBOR3M_t to IV (equations 19 to 24) and UR_t to IV (equations 31 to 36), provided significant α and β parameters in all samples using FW and RW. The other two variables when added as explanatory factors specifically IP in equations 13 to 18, and EEXR_t in equations 25 to 30, provided mostly significant results, but not for all samples when using FW and RW.

GARCH-MIDAS results support symmetric and asymmetric GARCH models since adding macroeconomic variables to market returns helps in the estimation of daily and monthly data of IV. Also, in terms of mixed frequencies, it sometimes provides better estimation than depending solely on its monthly and daily realized volatility. However, it's impossible to compare the GARCH-MIDAS approach with other GARCH symmetric and asymmetric models due to the

different data frequencies that have been used. The selection criteria AIC and BIC, which determines the best models, cannot be compared in this case, because mixed data frequency provides higher values of these criterions due to the higher number of observations used in the analysis.

V. Conclusion

In this study, we investigated the volatility and the conditional variance of the FTSE100 implied volatility index with 30 day expiration, IV, using daily and monthly data. We used several forms of GARCH models, the symmetric GARCH (1,1), and asymmetric GARCH models such as EGARCH (1,1) and GJR-GARCH (1,1). We also investigated the ability of the mixed data analysis approach namely GARCH-MIDAS to improve our modelling. We used several explanatory factors in the analysis, FTSE 100 index log-returns (FTSE100R) and macroeconomic determinants. The macroeconomic variables we used are the first difference of industrial production (IP), LIBOR three months rate (LIBOR3M), GBP effective exchange rate (EEX), and unemployment rate (UR). Our sample covers a 15 year period from January 4, 2000 to December 31, 2015. Besides analysing the whole sample, we also divided the sample into two subsamples, pre and post financial crisis. GARCH (1,1) outperformed other models for the full sample and for subsample 1 when daily IV is regressed on its realized volatility. However, due to the highly volatile period from the middle of 2007, which is included in subsample 2, EGARCH (1,1) was able to model the volatility of daily IV much better and outperformed all other models. Adding macroeconomic factors into the analysis namely FTSE100R, IP, LIBOR 3M, EER, and UR has improved the modelling process. Unlike other models, GJR-GARCH (1,1) did not produce any significant results with or without exogenous variables. However, GARCH (1,1) outperformed all other models with different specification lags starting from (1,1) and ending to (10,10) which is explained in table 7.

Using GARCH-MIDAS, showed the usefulness of the selected exogenous variables in modelling daily IV. Monthly realized volatility gave the best results for the full sample and subsample 1. For subsample 2, which is characterised by the highest average volatility, adding UR provided better estimation than realized volatility. Other independent variables also exhibit a clear effect on the estimation of daily IV, but not for both fixed window (FW), and rolling window (RW).

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