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**Article**

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# The Risk-Asymmetry Index as a new Measure of Risk

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The aim of this paper is to propose a simple and unique measure of risk that subsumes the conflicting information contained in volatility and skewness indices and overcomes the limitations of these indices in accurately measuring future fear or greed in the market. To this end, the concept of upside and downside corridor implied volatility, which accounts for the asymmetry in the risk-neutral distribution, is exploited. The risk-asymmetry index is intended to capture the investors' pricing asymmetry towards upside gains and downside losses. The results show that the proposed risk-asymmetry index can play a crucial role in predicting future returns, at various forecast horizons, since it subsumes the information embedded in both the volatility and skewness indices. Furthermore, the risk-asymmetry index is the only index that, at very high values, possesses the ability to clearly highlight a risky situation for the aggregate stock market. (JEL: G13, C02)

**Keywords:** risk-asymmetry; corridor implied volatility; risk-neutral moments; risk measures; return predictability

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## I. Introduction

The Chicago Board Options Exchange skewness index (*CBOE SKEW*) is designed to measure the perceived tail risk, i.e., the probability that investors attach to extreme negative returns. As such, the *CBOE* skewness index is intended to supplement the information contained in the *CBOE* volatility index (*CBOE VIX*), which measures the overall risk in the 30-day S&P500 log-returns. Despite its critical role in describing the return distribution, the *CBOE SKEW* index has not acquired the same outstanding reputation as the *CBOE VIX* index. This may at least be partially due to the positive relationship between changes in the *CBOE SKEW* index and those of the market returns that associate a positive change in the *CBOE SKEW* index to an increase in market returns (see e.g., Chang, Christoffersen, and Jacobs, 2013). Two points are notable in this connection. First, while the volatility index (*CBOE VIX*) spikes during periods of market downturn, the skewness measure (*CBOE SKEW*) is known to spike in both calm and turmoil periods (Faff and Liu, 2017). Second, the US risk-neutral distribution of returns is more negatively skewed during bullish market periods than during bearish market periods, indicating higher levels of risk during the former than the latter market periods (Faff and Liu, 2017).[1] These two points raise questions about the usefulness of the *CBOE SKEW* index as an indicator of the US market fear (see e.g. Chang, Christoffersen, and Jacobs, 2013; Faff and Liu, 2017; Elyasiani, Gambarelli and Muzzioli, 2016), namely a barometer that spikes during periods of high volatility and market downturn. This calls for alternative asymmetry measures that are better suited to describe market fear.[2] Moreover, from the investors' point of view, it is difficult to combine the information

1. The risk-neutral distribution of the returns of an asset is the expected distribution, under the risk-neutral measure, of the future distribution of the stock returns. It can be obtained from option prices listed on the underlying asset.

2. An index is said to act as a measure of market fear (greed), when high (low) values in the index can be regarded as an early warning indicator of future negative (positive) market returns.

contained in the *CBOE VIX* and *CBOE SKEW* indices to serve as a single indicator of market conditions. Specifically, high levels of *CBOE VIX* may be associated with both high and low levels of the *CBOE SKEW* index and this may give rise to confusion rather than confidence on the part of the investors.

The aim of this paper is to offer investors a simple measure of risk that subsumes the information embedded in the *CBOE VIX* volatility index while disentangling good volatility (volatility due to positive returns) from bad volatility (volatility due to negative returns) in order to overcome the limitations of both the *CBOE VIX* and the *CBOE SKEW* indices in measuring fear or greed in the market place. To this end, the concept of upside and downside corridor implied volatility measures is exploited in order to construct a new indicator of risk. This framework accounts for the asymmetry in the risk-neutral distribution, i.e., the fact that investors like positive spikes while they dislike negative spikes in returns. This is achieved by focusing on positive and negative returns above and below the forward price. Upside and downside implied volatilities are combined in an asymmetry index, called the risk-asymmetry index (*RAX*), that serves as our proposed new measure of risk. The *RAX* index is meant to capture the investors' pricing asymmetry towards upside gains and downside losses. The numerator is standardized by total volatility in order to leave the investors' pricing asymmetry free from the level of volatility. In this way, the *RAX* index is not influenced by bullish or bearish market periods.

The properties of the *RAX* index are examined in the Italian stock market over the period 2005-2014. This sample constitutes an ideal setting for investigating tail-risk measures since the Italian market suffered two major declines during this period (the subprime crisis in 2007-2008, and the European debt crisis in 2011-2012). It should be noted that the Italian market is taken here just for explanatory purposes: the *RAX* index and the other option-implied measures used in this study can be computed for any market. Even if the results are deemed to be country-specific, analogous results can be expected for the southern European economies of similar size and industrial structure, given that most of these markets were impacted in a similar way by the two recent crises mentioned earlier.

The paper proceeds as follows. First, both a skewness index and a volatility index for the Italian market are computed in line with the *CBOE* methodology (*CBOE* (2009; 2011)) for explanatory purposes. These indices are called the Italian skewness index (*ITSKEW*), and the

Italian volatility index (*ITVIX*) and are used as a benchmark for measuring market skewness and market volatility. Second, upside and downside corridor implied volatilities ($CIV_{UP}$ and $CIV_{DW}$) are introduced and combined in order to construct the risk-asymmetry index (*RAX*), designed to disentangle the contribution of positive and negative shocks to the risk-neutral distribution of returns. Third, the relationship between future market returns, the proposed risk-asymmetry index (*RAX*), the Italian skewness index (*ITSKEW*), the Italian volatility index (*ITVIX*), and the two corridor upside and downside indices ($CIV_{UP}$ and $CIV_{DW}$) is investigated within univariate and multiple regression models in order to establish whether high index values are associated with positive or negative future returns. Fourth, in order to provide investors with a sound indicator of future fear or greed, the paper examines whether extremely high or low levels of the index may be related to positive or negative future returns.

In order to provide investors with a time-frame for the relation between the risk-asymmetry index and future returns, their relation is investigated in the very short (1, 7 days), short (30 days) and medium-term (60, 90 days) windows. This allows us to address two interesting and controversial issues in the literature: the relationship between skewness and future returns, and the forecast horizon of the returns predictability. In this connection, while Bali and Murray (2013) and Conrad, Dittmar, and Ghysels (2013) find a negative relation between risk-neutral skewness and future stock returns (i.e. stocks with a left skewed risk-neutral distribution earn higher future returns to compensate for their higher left-tail risk), many authors find the opposite relationship. In particular, Xing, Zhang and Zhao (2010), Cremers and Weinbaum (2010), Yan (2011), Faff and Liu (2017) and Stilger, Kostakis and Poon (2017), find a positive relation between future stock returns and risk-neutral skewness or other proxies for skewness. As for the forecast horizon of returns predictability, some authors (e.g., Pan and Poteshman, 2006) argue that publicly observable option signals are able to predict stock returns only for the next one or two trading days, while others (e.g. Xing, Zhang and Zhao, 2010) find that the predictability horizon extends to longer periods of up to six months.

The following results are obtained. First, the Italian volatility and skewness indices (*ITVIX*, *ITSKEW*, respectively) are found to move together but in opposite directions and, as a result, combining their information on future fear could be problematic. On the other hand, unlike the *ITSKEW* index, the risk-asymmetry index *RAX* moves in the same direction as the *ITVIX*, in the sense that *RAX* strengthens when the *ITVIX* rises. Second, the *RAX* index provides useful information about future returns for the entire sample period, while the volatility indices (*ITVIX*, and upside and downside corridor volatilities $CIV_{UP}$ and $CIV_{DW}$) provide useful information about future returns only in the high volatility periods and only for the medium-term forecast horizons (60 and 90 days).

Third, the risk-asymmetry index *RAX* subsumes all the information in *ITSKEW* and *ITVIX* in predicting future market returns: once the *RAX* index is included in the model, the other two indices have no additional explanatory power on future returns. This suggests that the risk-asymmetry index (*RAX*) is the only index investors need to rely on for the purpose of predicting future returns, without having to complement the information in the implied volatility index with that of the skewness index that might be contradictory to it.

Fourth, the *RAX* index gives a clear and unambiguous signal to investors as extremely low (high) values of the risk-asymmetry index signal a buy (sell) opportunity. More specifically, when the *RAX* index falls below the threshold value of 101.15 (which represents the 10$^{th}$ percentile for the *RAX* index values), the average *FTSE MIB* index return over the next 30 days takes a value of 2.81% (34.16% on an annual basis), pointing to a strong short-term bullish market. Conversely, in cases in which the *RAX* index spikes above 103.70, the average *FTSE MIB* index return over the next three months will take the value of –11.17% (–45.29% on an annual basis), pointing to a strong medium-term bearish market. As a result, the risk-asymmetry index can be interpreted as a short-term greed index and a medium-term fear index.

To sum up, the advantages of the *RAX* index are threefold. First, it is a measure of bad volatility in the sense that it spikes when volatility of the left part of the distribution (i.e. volatility of negative returns) increases as a percentage of the total. Second, it correctly signals high risk when the risk-neutral distribution is riskier (more volatile and skewed to the left). Third, the *RAX* index is a better indicator of early-warnings than both the *ITSKEW* and the *ITVIX* indices in the sense that the *ITSKEW* signals negative future market returns only for extremely low values and the *ITVIX* index provides different indications about future returns for high and very high volatility levels, leaving investors without a clear warning.

This work extends that of previous researchers who have analyzed the relation between risk-neutral moments and future returns. Most closely related to this paper is Rubbaniy et al. (2014), whose methodology is applied to the volatility index and extended in order to examine the predictive power of the third moment and the risk-asymmetry index in predicting future returns. To the best of our knowledge, this is the first paper proposing a risk index, based on model-free moments, accounting for both the second and the third moments of the risk-neutral distribution.

The structure of the paper is as follows. In Section II, the risk-asymmetry index (*RAX*) is introduced. In Section III, the data and the methodology used to compute the Italian skewness index, risk-asymmetry index, volatility index and upside and downside corridor implied volatility indices are described. In Section IV, the properties of the risk-asymmetry index and the other indices are discussed. In Section V, the relation between the indices and future market returns in both high and low volatility periods is investigated. In Section VI the relation between extreme levels of the risk-asymmetry index and future aggregate market returns is assessed and the results are compared with the volatility indices and the skewness index. The final section concludes.

## II. The risk-asymmetry index (*RAX*)

In this section the risk-asymmetry index (*RAX*) is introduced. This measure is based on upside and downside corridor implied volatilities. Corridor implied variance (*CIV*), introduced by Carr and Madan (1998) and Andersen and Bondarenko (2007) is obtained from model-free implied variance due to Britten-Jones and Neuberger (2000) by truncating the integration domain between two barriers.

$$\hat{E}[CIV(0,T)] = \hat{E}\left[\frac{1}{T}\int_0^T \sigma^2(t,\ldots) I_t(B_1 B_2)\,dt\right] \tag{1}$$

The indicator function $I_t(\ldots)$ implies that variance is accumulated only if the underlying asset lies between the two barriers ($B_1$ and $B_2$).

Demeterfi et al. (1999) and Britten-Jones and Neuberger (2000) show that it is possible to compute the expected value of corridor implied variance (*CIV*) and, consequently, the corridor implied volatility measure as its square root, under the risk-neutral probability measure. This objective is achieved by using a portfolio of options with strikes ranging from $B_1$ to $B_2$, as described by:

$$\hat{E}[CIV(0,T)] = \frac{2e^{rT}}{T} \int_{B_1}^{B_2} \frac{M(K,T)}{K^2} dK \tag{2}$$

In this specification, $M(K, T)$ is the minimum between a call or put option price with strike price $K$ and maturity $T$, $r$ is the risk-free rate, and $B_1$ and $B_2$ are the barrier levels within which the variance is accumulated. In particular, if $B_1$ and $B_2$ are set equal to zero and infinity, respectively, the corridor implied variance coincides with model-free implied variance. The square root of model-free implied variance is model-free implied volatility (*MFIV*). Downside corridor implied variance is obtained by setting $B_1$ equal to zero and $B_2$ equal to the forward price, $F_t$. On the other hand, upside corridor implied variance is computed by setting $B_1$ equal to the forward price, $F_t$, and $B_2$ equal to infinity (∞). Note that the sum of upside and downside corridor implied variances coincides with model-free implied variance. Downside corridor implied volatility $(\sigma_{DW})$ is the square root of downside corridor implied variance and is described by equation (3); upside corridor implied volatility $(\sigma_{UP})$ is the square root of upside corridor implied variance and is described by equation (4):

$$\sigma_{DW}(0,T) = \sqrt{\frac{2e^{rT}}{T} \int_{0}^{F_t} \frac{M(K,T)}{K^2} dK} \tag{3}$$

$$\sigma_{UP}(0,T) = \sqrt{\frac{2e^{rT}}{T} \int_{F_t}^{\infty} \frac{M(K,T)}{K^2} dK} \tag{4}$$

In these models, $F_t = K^* e^{rT} * difference$, where $K^*$ is the reference strike price (i.e. the strike at which the difference in absolute value between the at-the money call and put prices is the smallest).[3]

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3. In this application, corridor implied volatility is computed as a discrete version of equations (3) and (4) with integration domain equal to $[K_{min}, F]$ and $[F, K_{max}]$, $K_{min}$ and $K_{max}$ correspond to the minimum and maximum strike price ensuring an insignificant truncation

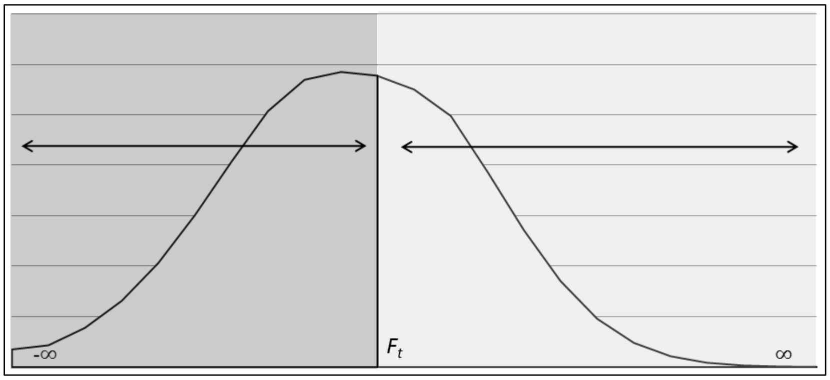


FIGURE 1.— Graphical presentation for upside and downside corridor volatility measures

**Note:** The figure proposes a stylized representation of the market return distribution in order to enhance the interpretation of upside and downside corridor volatility measures. Downside (upside) corridor volatility measures the variability of returns below (above) the threshold given by the forward rate.

Figure 1 exhibits a graphical representation of the upside and downside corridor volatility measures. Downside (upside) corridor volatility measures the expected variability in the left (right) part of the distribution. Downside (upside) corridor volatility is accumulated when the underlying asset lies between zero and $F_t$ ($F_t$ and $+\infty$). Downside corridor volatility is a proxy for bad volatility, since it measures the variability of negative returns. On the other hand, upside corridor volatility is a proxy for good variance, since it measures the variability of positive returns.

The risk-asymmetry measure $RAX(0, T)$ is derived as the difference between the volatility of the right part of the distribution (upside corridor implied volatility, $\sigma_{UP}$ ) and the volatility of the left part of the distribution (downside corridor implied volatility, $\sigma_{DW}$ ) as a ratio of total volatility ( $\sigma_{TOT}$ ):

$$RAX(0,T) = \frac{\sigma_{UP}(0,T) - \sigma_{DW}(0,T)}{\sigma_{TOT}(0,T)} \tag{5}$$

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error (for more details see Muzzioli, 2015).

The reason for this choice is as follows. First, as explained below, the numerator of the *RAX* index can be considered to measure the investors' asymmetry in the preferences for the upside gain and downside loss. Second, the standardization by total volatility is meant to isolate the effect of asymmetry from the volatility level. To elaborate, a corridor variance swap is a variant of a variance swap which takes into account daily stock variations only when the underlying asset lies in a specific corridor. At maturity, the long side pays a fixed rate and receives a floating rate (the realized variance which is accumulated only if the underlying asset lies in a pre-specified range). A notional dollar amount is multiplied by the difference between the two rates. The payoff at maturity of a long position in an upside (downside) corridor variance swap is:

$$N\left(\sigma_{RUC}^2 - \sigma_{UP}^2\right)$$

$$N\left(\sigma_{RDC}^2 - \sigma_{DW}^2\right)$$

where $N$ is the notional amount of the contract, $\sigma_{UP}^2$ and $\sigma_{DW}^2$ are the strike prices of the variance swap contracts, $\sigma_{RUC}^2$ and $\sigma_{RDC}^2$ are the realized measures for upside and downside corridor variance and can be obtained as:

$$\sigma_{RUC}^2 = \frac{1}{T}\sum_{i=1}^{n}\left(\ln\left(\frac{S_i}{S_{i-1}}\right)\right)^2 1_{S_i > F_t}$$

$$\sigma_{RDC}^2 = \frac{1}{T}\sum_{i=1}^{n}\left(\ln\left(\frac{S_i}{S_{i-1}}\right)\right)^2 1_{S_i < F_t}$$

If a long position in a downside corridor variance swap contact is combined with a short position in an upside corridor swap contract with the same notional ($N$), the payoff of the portfolio at maturity is equal to:

$$N\left(\sigma_{RDC}^2 - \sigma_{DW}^2\right) - N\left(\sigma_{RUC}^2 - \sigma_{UP}^2\right) = N\left(\sigma_{RDC}^2 - \sigma_{DW}^2 - \sigma_{RUC}^2 + \sigma_{UP}^2\right)$$

If a one-dollar notional ($N$=1) is assumed, and, on average, the upside realized volatility measure is not significantly different from the downside realized volatility measure (see e.g., empirical evidence in

Muzzioli, 2013a),[4] the payoff can be simplified as:

$$\left(\sigma_{UP}^2 - \sigma_{DW}^2\right).$$

Therefore, an insight into the meaning of the *RAX* index, with the appropriate simplifications, since it uses volatilities and not variances,[5] is as follows. The numerator of the *RAX* index $(\sigma_{UP} - \sigma_{DW})$ represents how much an investor is willing to pay to hedge against peaks of good volatility, minus the amount the same investor is willing to pay to hedge against peaks of bad volatility. The *RAX* index, therefore, measures the investors' pricing asymmetry towards upside gains and downside losses. The numerator is standardized by total volatility in order to leave the investors' pricing asymmetry free from the level of volatility. In this way the *RAX* should behave in a proper manner during both high and low volatility periods.

In order to have a constant 30-day measure for the risk-asymmetry index, the $RAX_{30}$ index is obtained by using a linear interpolation of the near and next term options:

$$RAX_{30} = wRAX_{near} + (1-w)RAX_{next} \tag{6}$$

where $w = (T_{next} - 30)/(T_{next} - T_{near})$, and $T_{near}(T_{next})$ is the time to expiration of the near (next) term options, $RAX_{near}(RAX_{next})$ is the annualized asymmetry index measure referring to the *near* (*next*) term options, respectively.

In order to have the same interpretation as the *SKEW* index and for ease of comparison, the final daily value for the *RAX* index is computed as:

$$RAX = 100 - 10 \times RAX_{30} \tag{7}$$

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4. Even if the underlying source of risk is the same (the underlying spot index) for upside and downside realized volatility and for upside and downside implied volatilities, the levels of the two (realized versus implied) are different since investors' expectations affect upside and downside implied volatilities, with upside and downside risk premia.

5. A replicating portfolio exists only in the case of a corridor variance swap, not in the case of a corridor volatility swap. However, square root of variance is used instead of variance in order to have a direct comparison with the other volatility measures. Using variance instead of volatility in the definition of the *RAX* index would not have changed the results (available upon request).

Formula (7) is adopted in order to enhance the interpretation of the risk-asymmetry index: since the $RAX_{30}$ attains typically negative values, the value of the *RAX* is positive and higher than 100. The higher the $RAX_{30}$ in absolute terms, the higher the *RAX* index.

## III. Data and methodology

The data-set consists of closing prices on *FTSE MIB*-index options (MIBO) and closing prices of the *FTSE MIB*-index recorded from 3 January 2005 to 28 November 2014.[6] The choice of the sample period is motivated by the fact that the Italian market suffered two major declines in this period due to the subprime crisis of 2007-2008 and the European debt crisis of 2011-2012, making it an ideal candidate for investigating the predictive power of tail risk and risk-asymmetry measures such as the *SKEW* index and the *RAX* in predicting future positive or negative returns.

The *FTSE MIB* index value is adjusted for dividends in order to use it as the adjusted underlying asset in the option pricing formulas:

$$\hat{S}_t = S_t e^{-\delta_t \Delta t} \tag{8}$$

where $S_t$ is the *FTSE MIB* index value at time $t$, $\delta_t$ is the dividend yield at time $t$, and $\Delta t$ is the time to maturity of the option. Euribor rates with maturities of one week, and one, two, and three months are used as a proxy for the risk-free rate (the appropriate yield to maturity is computed by linear interpolation). The data-set for the MIBO is kindly provided by Borsa Italiana S.p.A. The time series of the *FTSE MIB* index, the dividend yield and the Euribor rates are obtained from Datastream.

Several filters are applied to the option data-set in order to eliminate arbitrage opportunities and other irregularities in the prices. First, consistent with the computational methods of other indices such as the *CBOE SKEW*, options with time-to-maturity of less than eight days that may suffer from pricing anomalies that might occur close to expiration are eliminated. Second, following Ait-Sahalia and Lo (1998), only at-the-money and out-of-the-money options are retained (put options

6. Financial Times Stock Exchange Milano Indice di Borsa: a capital-weighted index consisting of 40 major stocks listed on the Italian market.

with moneyness lower than 1.03 and call options with moneyness higher than 0.97). In the money options are excluded. Last, option prices violating the standard no-arbitrage constraints and positive prices for butterfly spreads are eliminated (Carr and Madan, 2005). Following Muzzioli (2013a), the volatility-strike knots are interpolated by using cubic splines and extrapolated outside the existing domain of strike prices by using a constant extrapolation scheme. In this way, an insignificant truncation and discretization error is ensured (for more details see Muzzioli (2013a; 2013b)).

In order to obtain 30-day constant maturity indices, each day the following indices are computed by linear interpolation between near-term and next-term maturity options: (i) the Italian skewness index (*ITSKEW*) using the formula of the *CBOE SKEW* index, adapted to the Italian market, (ii) the Italian volatility index *ITVIX*, as , $ITVIX = MFIV$ *100 and (iii) the upside and downside corridor implied volatility indices ( $CIV_{UP} = \sigma_{UP} * 100$ and $CIV_{DW} = \sigma_{DW} * 100$ ).[7] Further details of the computation of the *ITSKEW* index are provided in appendix A. Physical skewness is obtained from daily *FTSE MIB* (Milano Indice Borsa) log-returns by using a rolling window of 30 calendar days that is then annualized. In this way the physical measure refers to the same time-period covered by the risk-neutral counterparts. Following the methodology adopted by the *CBOE*, the *ITSKEW* is computed as in equations (6 and 7). The physical skewness index ($SKEW_{PH}$) is also computed as in equation (7) for ease of comparison.

## IV. Descriptive analysis of the *RAX* index, the *ITSKEW* and the implied volatility indices

Table 1 provides the descriptive statistics for the *FTSE-MIB* index returns (*R*), the index of physical skewness ($SKEW_{PH}$), and the levels and the first differences of the Italian volatility index (*ITVIX*), the upside and downside corridor implied volatility indices ($CIV_{UP}$ and $CIV_{DW}$), the Italian skewness index (*ITSKEW*), and the risk-asymmetry index (*RAX*). The distribution of physical returns is found to be far from

7. The Italian Volatility Index (*IVI*) which is the Italian version of the *VIX* index is currently quoted in the Italian market. The *IVI* is computed by means of quoted option prices and does not use cubic splines interpolation and extrapolation. As a proxy for the volatility index, the *ITVIX* index is used instead of the *IVI* for consistency with the computational methodology of upside and downside corridor implied volatilities.

**TABLE 1. Descriptive statistics for returns and the risk measures**

| | $SKEW_{PH}$ | $ITSKEW$ | $RAX$ | $ITVIX$ | $CIV_{UP}$ | $CIV_{DW}$ | $R$ | $\Delta ITSKEW$ | $\Delta RAX$ | $\Delta ITVIX$ | $\Delta CIV_{UP}$ | $\Delta CIV_{DW}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Mean | 100.06 | 103.44 | 101.72 | 24.90 | 15.08 | 19.35 | –0.00 | –0.00 | –0.00 | 0.01 | 0.00 | 0.00 |
| Median | 100.06 | 103.17 | 101.66 | 23.15 | 14.11 | 17.93 | 0.00 | 0.02 | 0.01 | –0.05 | –0.03 | –0.04 |
| Maximum | 101.32 | 110.98 | 105.46 | 80.36 | 50.25 | 61.18 | 0.11 | 5.00 | 1.75 | 27.51 | 16.59 | 21.19 |
| Minimum | 98.74 | 100.08 | 100.11 | 8.76 | 4.78 | 6.95 | –0.09 | –8.37 | –2.89 | –14.96 | –9.16 | –11.20 |
| Std. Dev. | 0.34 | 1.43 | 0.56 | 10.77 | 6.32 | 8.40 | 0.02 | 0.95 | 0.30 | 1.77 | 1.12 | 1.41 |
| Skewness | –0.04 | 1.30 | 1.47 | 1.23 | 1.06 | 1.31 | –0.05 | –0.61 | –0.64 | 1.72 | 1.31 | 1.69 |
| Kurtosis | 3.85 | 5.55 | 8.55 | 5.08 | 4.50 | 5.38 | 7.72 | 10.71 | 11.73 | 34.39 | 30.82 | 31.26 |
| Jarque-Bera | 76.00 | 1365.00 | 4054.00 | 1070.00 | 693.00 | 1284.00 | 2291.00 | 6259.00 | 8006.00 | 102487.00 | 80307.00 | 83278.00 |
| p-value | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |

**Note:** The table shows the descriptive statistics of the physical skewness index $SKEW_{PH}$, the Italian skewness index *ITSKEW*, the risk-asymmetry index *RAX*, the Italian implied volatility index *ITVIX* and the corridor upside and downside implied volatility indices $CIV_{UP}$ and $CIV_{DW}$, along with *FTSE MIB* returns. $SKEW_{PH}$ is the index of subsequently realized skewness in the next 30 days, *ITSKEW* is the skewness index computed using the *CBOE* method, adapted to the Italian market, *RAX* is the risk-asymmetry index defined as: $RAX = 100 - 10 * (CIV_{UP} - CIV_{DW})/MFIV$, *MFIV* is the model-free implied volatility, *R* is the *FTSE MIB* daily return (continuously compounded). The p-value refers to the Jarque-Bera test for normality (the null hypothesis is that both skewness and kurtosis are zero).

normal, displaying a slight negative skewness and a pronounced excess kurtosis. According to the test statistics reported in the last row, the hypothesis of normality is strongly rejected also for the implied volatility index, indicating the presence of extreme movements in volatility, i.e., fat tails on both sides of the distribution.

When the Italian volatility index (*ITVIX*) is split into its two components (upside and downside corridor implied volatilities), it is observed that each component is far from the normal distribution, with downside corridor implied volatility being on average higher than upside corridor volatility. This indicates that extreme movements are more often present on the left part (downside) of the risk-neutral distribution, suggesting that peaks of volatility are more often associated with increases in the volatility of the left-hand side of the distribution (bad news).[8]

The skewness index and the risk-asymmetry index are found to be on average higher than the threshold level of 100 (103.44, and 101.72, for *ITSKEW*, and *RAX*, respectively) pointing to a highly negatively skewed risk-neutral distribution. On the other hand, the physical skewness index ($SKEW_{PH}$) is close to 100 (100.06), pointing to an almost symmetrical distribution of physical returns. This suggests the presence of a positive skewness risk premium (i.e., the difference between physical and risk-neutral skewness) which is mainly attributable to the negative asymmetry of the risk-neutral distribution, i.e., to investor expectations that overestimate the probability of negative returns. This result is consistent with the evidence from the US market provided by Foresi and Wu (2005), and Kozhan, Neuberger and Schneider (2013). The excess kurtosis in all the risk-neutral asymmetry measures indicates the presence of extreme movements not only in implied volatility, but also in higher moments.

The fact that the risk-asymmetry measure (*RAX*) is on average higher than 100 suggests that downside corridor implied volatility is on average higher in percentage terms than the upside corridor implied

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8. Since returns are not normal, non-normality is expected also for implied volatility measures. Moreover, the fact that upside and downside corridor implied volatilities have different means highlights the intrinsic asymmetry in implied volatility. Even though normality is a desirable property from the statistical point of view, we chose not to express the variables in logarithmic terms in order to achieve normality in the subsequent regression analysis. This choice is motivated by the need to ascertain the relation between future returns and levels of the indices in Section V (examined in greater depth by also looking at extreme levels in Section VI).

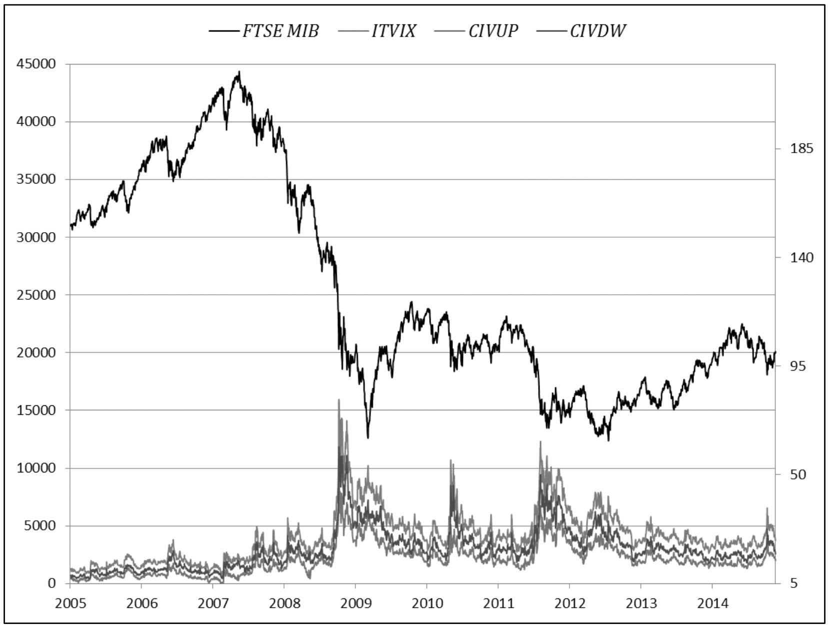


FIGURE 2.— Comparison between the *FTSE MIB* index and implied volatility indices

**Note:** The figure shows the closing values of the Italian market index *FTSE MIB*, the Italian volatility index (*ITVIX*) in blue, the upside corridor implied volatility index ($CIV_{UP}$) in green, and the downside corridor implied volatility index ($CIV_{DW}$) in red. *FTSE MIB* index refers to the left axis, while implied volatility indices (*ITVIX*, $CIV_{UP}$ and $CIV_{DW}$) refer to the right axis.

volatility. In fact, downside corridor implied volatility attains an average of 0.19, whereas upside corridor implied volatility an average of 0.15. The evidence of the difference between the volatility of the left and right part of the distribution is supported by a t-test, where errors are corrected by Newey West (t-stat = –81.70, p-value = 0.00). The difference between upside and downside volatility amounts to four basis points and highlights the fact that the expected variability of returns is higher when returns are negative. This means that investors are more concerned about losses than they are confident in positive returns. The *RAX* index measures the investors' asymmetry in how much they value potential upside gains and downside losses.

Figure 2 depicts the *FTSE MIB* index along with the Italian volatility index (*ITVIX*) and the upside and downside corridor implied volatility

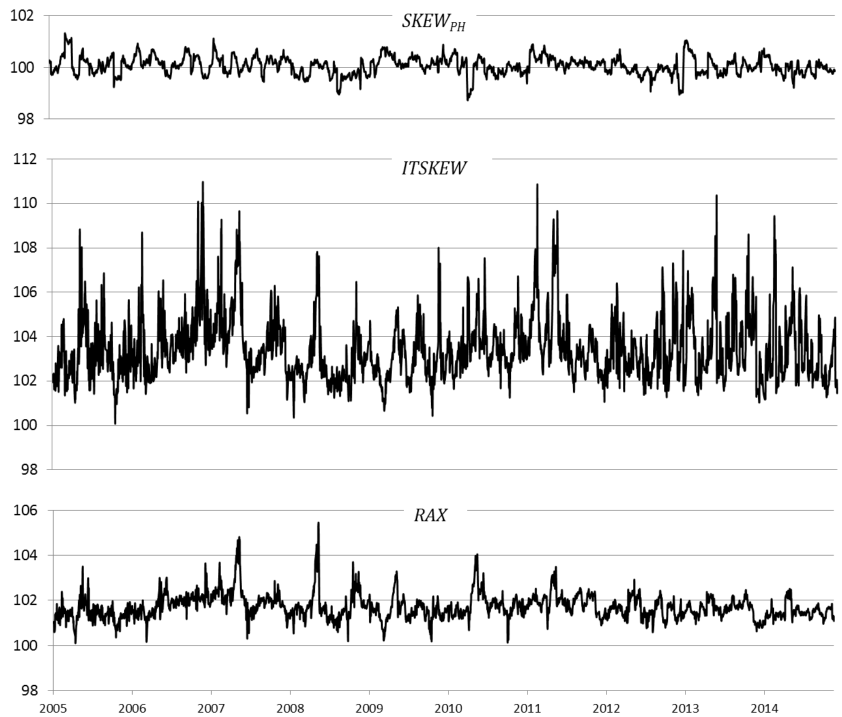


FIGURE 3.— Graphical comparison of physical skewness, the Italian skewness index *ITSKEW* and the risk-asymmetry index *RAX*

**Note:** $SKEW_{PH}$ is the index of subsequently realized skewness in the next 30 days, *ITSKEW* is the Italian skewness index, computed using the CBOE methodology adapted to the Italian market and *RAX* is the risk-asymmetry index defined as: $RAX = 100 - 10 * (CIV_{UP} - CIV_{DW}) / MFIV$.

indices ($CIV_{UP}$ and $CIV_{DW}$). Downside movements of the *FTSE MIB* index are associated with spikes of the three implied volatility indices: downside corridor implied volatility is on average higher than upside volatility, and during turbulent periods, the difference between downside and upside corridor measures is exacerbated. This is also clear from figure 3 showing plots of the Italian skewness index *ITSKEW* and the risk-asymmetry index *RAX*, along with physical skewness. It is apparent that the risk-asymmetry index (*RAX*), being normalized by model-free implied volatility, spikes when downside corridor implied volatility increases as a percentage of the total (or upside corridor implied volatility decreases as a percentage of the total). This represents an initial advantage of the risk-asymmetry index, that is able to account not only for volatility, but also for the asymmetric behavior of upside and downside volatility parts.

In table 2 the correlation coefficients between returns, physical skewness and the levels and the first differences of the *RAX* index, the *ITSKEW* index, and the model-free implied volatility indices (*ITVIX*, $CIV_{UP}$ and $CIV_{DW}$) are reported. Both the *RAX* and the *ITSKEW* index display a positive significant correlation with the realized skewness index ($SKEW_{PH}$), and, as a result, they can be used as predictors of future realized skewness. Notably, while the *ITSKEW* index displays a negative and significant correlation with the Italian volatility index (–0.16), the *RAX* index is positively and significantly correlated (0.099) with the Italian volatility index. The correlation between the Italian volatility index (*ITVIX*) and upside and downside corridor implied volatilities is close to the unit value, showing a high degree of association between them. The low positive correlation of both the levels and the daily changes of the *RAX* index with implied volatility constitutes a second advantage of the *RAX* index: it goes in the same direction as implied volatility, but it contains additional information beyond the information in volatility. As a result, according to the *RAX* index, the risk-neutral distribution of the *FTSE-MIB* index returns is more negatively skewed indicating a higher level of risk, when the implied volatility index is high. For the *ITSKEW* index the relation is exactly the opposite: it indicates a higher level of risk, when the implied volatility index is low. This can be considered as a third advantage of the *RAX*, since the risk-asymmetry measure is expected to positively spike in periods of turmoil (when *ITVIX* is high).

To sum up, the advantages of the *RAX* index are threefold. First, it increases when bad volatility (the volatility of the left part of the distribution) increases as a percentage of the total. Second, it is only marginally positively correlated with the Italian volatility index both in term of levels and daily changes, suggesting that the measure of asymmetry is not affected by the volatility level. Third, it correctly indicates that the risk-neutral distribution of the *FTSE-MIB* index returns is riskier when the implied volatility index is higher.

**TABLE 2. Correlation table**

| | $SKEW_{PH}$ | $ITSKEW$ | $RAX$ | $ITVIX$ | $CIV_{UP}$ | $CIV_{DW}$ | $R$ | $\Delta ITSKEW$ | $\Delta RAX$ | $\Delta ITVIX$ | $\Delta CIV_{UP}$ | $\Delta CIV_{DW}$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $SKEW_{PH}$ | 1.000 | | | | | | | | | | | |
| $ITSKEW$ | 0.154*** | 1.000 | | | | | | | | | | |
| $RAX$ | 0.073*** | 0.674*** | 1.000 | | | | | | | | | |
| $ITVIX$ | –0.104*** | –0.162*** | 0.099*** | 1.000 | | | | | | | | |
| $CIV_{UP}$ | –0.111*** | –0.253*** | –0.036* | 0.988*** | 1.000 | | | | | | | |
| $CIV_{DW}$ | –0.103*** | –0.128*** | 0.154*** | 0.998*** | 0.977*** | 1.000 | | | | | | |
| $R$ | 0.041** | 0.090*** | 0.034* | –0.074*** | –0.081*** | –0.072*** | 1.000 | | | | | |
| $\Delta ITSKEW$ | 0.024 | 0.331*** | 0.152*** | –0.000 | –0.019 | 0.006 | 0.230*** | 1.000 | | | | |
| $\Delta RAX$ | 0.006 | 0.153*** | 0.268*** | 0.007 | –0.026 | 0.021 | 0.170*** | 0.515*** | 1.000 | | | |
| $\Delta ITVIX$ | –0.019 | –0.035* | 0.006 | 0.080*** | 0.078*** | 0.080*** | –0.552*** | –0.051** | 0.059*** | 1.000 | | |
| $\Delta CIV_{UP}$ | –0.022 | –0.095*** | –0.096*** | 0.070*** | 0.086*** | 0.064*** | –0.620*** | –0.248*** | –0.321*** | 0.894*** | 1.000 | |
| $\Delta CIV_{DW}$ | –0.018 | –0.012 | 0.050** | 0.079*** | 0.070*** | 0.082*** | –0.499*** | 0.0228 | 0.222*** | 0.983*** | 0.800*** | 1.000 |

**Note:** The table shows the correlation coefficients between the measures used in the study. For the definition of the measures, see table 1. Significance at the 1% level is denoted by ***, at the 5% level by **, and at the 10% level by *.

## V. The risk-asymmetry index as an indicator of future index returns

This section examines the forecasting power of the risk-asymmetry index (*RAX*) both in univariate (subsection *A*) and encompassing (subsection *B*) regressions.

### *A. Univariate regressions*

Many authors (e.g., Pan and Poteshman, 2006; Cremers and Weinbaum, 2010) have pointed out that options have a predictive power on future market returns, since informed investors trade first in the option market and the information is reflected in stock prices in the spot market only subsequently. Specifically, Lin and Lu (2015) suggest that the return predictability is addressed by "analysts tipping", i.e., option traders receiving tips from analysis such as upcoming recommendation changes or earnings forecast revisions. To elaborate, when the news is positive (negative), informed option traders buy call (put) options carrying an upward pressure on call (put) prices and the volatility of the right (left) part of the distribution increases. As a result, the *RAX* index is expected to be low (high) when option traders receive and convey in the option market good (bad) news for the investment opportunity set.

Following this rationale, the information content embedded in option prices is expected to be reflected in a positive relation between future stock returns and option implied asymmetry or other proxies for asymmetry (Xing, Zhang and Zhao, 2010; Cremers and Weinbaum, 2010; Yan, 2011; Faff and Liu, 2017; Stilger, Kostakis and Poon, 2017). A theoretical explanation for the empirical results cited above is also provided in Sasaki (2016), indicating a significantly positive relation between the jump risk and future aggregate index market return, in a general equilibrium setting.

The univariate regressions analyzed are:

$$R_{t,t+n} = \alpha + \beta_1 index_t + \varepsilon_t \tag{9}$$

where $index_t$ is alternatively proxied by levels of *ITSKEW*, *RAX*, *ITVIX*, $CIV_{UP}$ and $CIV_{DW}$ and $R_{t,t+n}$ is the market aggregate log-return return computed between day *t* and day *t*+*n* ($R_{t,t+n}$= ln($FTSEMIB_{t+n}$/$FTSEMIB_t$), where $FTSEMIB_{t+n}$ and $FTSEMIB_t$ are the values of the *FTSE MIB* index

**TABLE 3. Univariate model output: $R_{t,t+n} = \alpha + \beta_1$ *index*$_t + \varepsilon_t$ equation (9)**

A. Entire sample

| | *ITSKEW* | *RAX* | *ITVIX* | $CIV_{UP}$ | $CIV_{DW}$ |
|---|---|---|---|---|---|
| $R_{t,t+1}$ | –0.0007*** | –0.0016*** | 0.0000 | 0.0001 | 0.0000 |
| | (–2.92) [0.29] | (–2.67) [0.26] | (0.60) [0.00] | (1.00) [0.04] | (0.44) [0.00] |
| $R_{t,t+7}$ | –0.0018*** | –0.0054*** | 0.0000 | 0.0000 | –0.0000 |
| | (–2.58) [0.69] | (–2.85) [0.93] | (0.16) [0.00] | (0.54) [0.04] | (–0.00) [0.00] |
| $R_{t,t+30}$ | –0.0058*** | –0.0204*** | 0.0000 | 0.0004 | –0.0001 |
| | (–2.70) [1.14] | (–3.69) [2.63] | (0.06) [0.00] | (0.48) [0.07] | (–0.14) [0.00] |
| $R_{t,t+60}$ | –0.0090** | –0.0321*** | 0.0003 | 0.0010 | 0.0002 |
| | (–2.55) [1.61] | (–3.47) [3.15] | (0.52) [0.06] | (0.93) [0.38] | (0.31) [0.00] |
| $R_{t,t+90}$ | –0.0114** | –0.0373*** | 0.0004 | 0.0013 | 0.0004 |
| | (–2.52) [1.88] | (–3.28) [3.04] | (0.60) [0.12] | (1.01) [0.46] | (0.41) [0.03] |
| B. Low volatility | | | | | |
| $R_{t,t+1}$ | –0.0003 | –0.0004 | 0.0001 | 0.0001 | 0.0001 |
| | (–1.42) [0.07] | (–0.87) [0.00] | (0.96) [0.00] | (1.08) [0.01] | (0.90) [0.00] |
| $R_{t,t+7}$ | –0.0010* | –0.0023 | 0.0001 | 0.0003 | 0.0002 |
| | (–1.78) [0.48] | (–1.60) [0.29] | (0.70) [0.06] | (0.87) [0.15] | (0.62) [0.03] |
| $R_{t,t+30}$ | –0.0046*** | –0.0100*** | 0.0002 | 0.0006 | 0.0002 |
| | (–2.92) [2.44] | (–2.61) [1.34] | (0.37) [0.00] | (0.67) [0.14] | (0.28) [0.00] |
| $R_{t,t+60}$ | –0.0057** | –0.0167*** | –0.0002 | 0.0003 | –0.0004 |
| | (–2.49) [2.14] | (–3.31) [2.19] | (–0.21) [0.00] | (0.28) [0.00] | (–0.39) [0.00] |
| $R_{t,t+90}$ | –0.0073** | –0.0286*** | 0.0002 | 0.0012 | –0.0000 |
| | (–2.53) [2.41] | (–3.83) [4.44] | (0.27) [0.00] | (0.92) [0.23] | (–0.04) [0.00] |
| C. High volatility | | | | | |
| $R_{t,t+1}$ | –0.0012*** | –0.0025** | 0.0001 | 0.0002** | 0.0001 |
| | (–3.00) [0.58] | (–2.56) [0.44] | (1.48) [0.17] | (2.00) [0.41] | (1.27) [0.10] |
| $R_{t,t+7}$ | –0.0033** | –0.0078** | 0.0002 | 0.0006 | 0.0003 |
| | (–2.52) [1.23] | (–2.51) [1.31] | (1.14) [0.38] | (1.60) [0.92] | (0.95) [0.23] |
| $R_{t,t+30}$ | –0.0088** | –0.0280*** | 0.0010 | 0.0024** | 0.0010 |
| | (–2.30) [1.72] | (–3.06) [3.46] | (1.62) [1.31] | (2.08) [2.62] | (1.41) [0.88] |
| $R_{t,t+60}$ | –0.0160** | –0.0430*** | 0.0025*** | 0.0055*** | 0.0029** |
| | (–2.51) [2.73] | (–2.90) [3.86] | (2.63) [4.29] | (3.00) [6.72] | (2.47) [3.37] |
| $R_{t,t+90}$ | –0.0211*** | –0.0421** | 0.0037*** | 0.0077*** | 0.0044*** |
| | (–2.63) [3.52] | (–2.26) [2.69] | (2.85) [6.96] | (3.31) [9.83] | (2.69) [5.85] |

**Note:** The table shows the estimated output of the following regression: $R_{t,t+n} = \alpha + \beta_1$ *index*$_t$ + $\varepsilon_t$ where *index*$_t$ is proxied by the Italian skewness index ($ITSKEW_t$), the risk-asymmetry index ($RAX_t$), the Italian implied volatility index ($ITVIX_t$), and corridor upside and downside implied volatility indices ($CIV_{UPt}$ and $CIV_{DWt}$); t-stats and R-squared (in percentage terms) are shown in parentheses and brackets respectively.

at time $t+n$ and $t$, respectively). The time horizon $n$ is chosen to be equal to 1, 7, 30, 60 and 90 calendar days, in order to consider both very short (1, 7 days), short (30 days) and medium-term (60, 90 days) returns.

Equation (9) is intended to establish whether the index values are associated with positive or negative future returns, thus highlighting the possibility of profits or losses in the market over a very short-, short- or medium-term forecast horizon.

The results for estimation of the model described by equation (9) are reported in table 3, Panel A. All the regressions were run by using the Ordinary Least Squares (OLS), with the Newey and West (1987, 1994) heteroscedasticity and autocorrelation consistent (HAC) covariance matrix. According to the figures in table 3 (Panel A), none of the implied volatility indices (*ITVIX*, $CIV_{UP}$ and $CIV_{DW}$) has a significant explanatory power for future stock returns in any considered forecast horizon. As a result, neither the Italian volatility index (*ITVIX*), nor upside corridor implied volatility index ($CIV_{UP}$), nor downside corridor implied volatility index ($CIV_{DW}$) can be used to forecast returns over the next 90 days.[9] On the contrary, the *RAX* index shows a significant (at the 1% level) negative relationship with future aggregate returns for all the forecast horizons considered. The *ITSKEW* index displays a highly significant relation with the aggregate returns in the very short- and short-term horizons (1-, 7- and 30- days), but the relationship is only marginally significant for medium-term horizons (60- and 90- days). These results suggest that there is an inverse relationship between the *RAX* and the *ITSKEW* index on the one hand and future market return on the other. In other words, high (low) values of the *RAX* and of the *ITSKEW* index are reflected in low (high) future market returns in the very short-, short- and medium–term forecast horizons.

The surprising results for the information content of model-free implied volatility indices (*ITVIX*, $CIV_{UP}$ and $CIV_{DW}$) drive the investigation further, since these indices are expected to be informative at least during turmoil periods. As a result, the study examines whether

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9. Elyasiani, Gambarelli and Muzzioli (2017) investigate the forecasting power of different volatility measures related to different portions of the risk-neutral distribution (corridor upside, downside, *RSV* and *SIX*). By examining their forecasting power on the next 30-day returns, they find that the latter measures have explanatory power only during high volatility periods. Moreover, by analyzing two crises, the European debt crisis and the subprime debt crisis, they find that the volatility measures have greater explanatory power during the European sovereign debt crisis (internal crisis) than the subprime crisis (external crisis).

the volatility level (*ITVIX*, $CIV_{UP}$ and $CIV_{DW}$) can affect the predictability of returns. Two main volatility regimes are evident during the sample period: one regime is characterized by low volatility and positive market returns (2005-2007 and 2013-2014) while the second regime is characterized by high volatility and a decline of about 70% in the stock market (2008-2012). Figure 2, showing the contrast between *FTSE MIB* index and the model-free implied volatility indices (*ITVIX*, $CIV_{UP}$ and $CIV_{DW}$), helps to enucleate the two sub-periods. In order to contrast the predictive power of skewness and volatility indices under calm and intense market volatility conditions, the models described by regression (9) are estimated for both sub-periods.

Overall, both the *RAX* and the Italian skewness index (*ITSKEW*), outperform volatility indices in forecasting future market returns, the implication being that forecasting market performance should be based on moments beyond the second moment. The results for the calm period, reported in table 3, Panel B, confirm the predictive power of the *RAX* and the *ITSKEW* indices only for short- and medium-term forecast horizons (30-, 60- and 90- days), while the results for the volatility indices do not exhibit any significant relationship with future market returns. This result could be attributed to the symmetric treatment of positive and negative changes in returns, linked to implied volatility. In the same spirit, the poor forecasting performance of upside and downside corridor volatility measures could be linked to the strong correlation of the two with implied volatility. A sharp increase in upside or downside corridor volatility is reflected in the same way in an increase in total implied volatility. On the other hand, the *RAX* is the only index that increases (decreases) when bad (good) volatility increases as a percentage of the total.

The results for the high volatility period, reported in table 3, Panel C, show that both the skewness index (*ITSKEW*) and the *RAX* index embed useful information for predicting future market returns in all considered forecast horizons. In this case, also the model-free implied volatility indices (*ITVIX*, $CIV_{UP}$ and $CIV_{DW}$) display a significant relation with future returns, but only over medium-term horizons. Among the volatility indices, the corridor upside volatility index ($CIV_{UP}$) achieves a better forecasting performance, since it embeds useful information to predict both very short- and medium aggregate market returns. The positive sign of the relationship between model-free implied volatility indices (*ITVIX*, $CIV_{UP}$ and $CIV_{DW}$) and future returns (*R*) is consistent with the findings in Rubbaniy et al. (2014), who document a significant positive medium-term relation between volatility indices and future stock returns in both the German and the US market. This is in line with the capital asset pricing model: when volatility is high, investors require a higher return in order to be compensated for the higher risk. In general, the R-squared statistics increase in the high volatility period compared to the low volatility period and with the forecast horizon. Similar results for volatility have been found in Rubbaniy et al., 2014.

*B. Encompassing regressions*

In order to establish whether the *RAX* index subsumes all the information contained in the Italian skewness index (*ITSKEW*) and the Italian volatility index (*ITVIX*) in predicting future market returns, the three indices are included in the same model, as described by equation (10). Within this model, we test whether the coefficient of the *ITSKEW* index ($\beta_1$) and the coefficient of the *ITVIX* index ($\beta_3$) are jointly zero ($\beta_1 = \beta_3 = 0$).

$$R_{t,t+n} = \alpha + \beta_1 ITSKEW_t + \beta_2 RAX_t + \beta_3 ITVIX_t + \varepsilon_t \qquad (10)$$

The results for the full sample period are reported in table 4, Panel A. All the regressions were run by using the OLS with the Newey and West (1987, 1994) heteroscedasticity and autocorrelation consistent (HAC) covariance matrix. Multicollinearity is checked by means of the variance inflation factor (VIF). The results (available upon request) show that the estimated model is not affected by significant multicollinearity.

The Wald test for the joint hypothesis of ($\beta_1 = \beta_3 = 0$) is reported in the last column of table 4. From this table, it is evident that only the *RAX* index shows a significant relation with future market returns. The coefficient of *RAX* is negative and statistically significant at the 1% level for both the short- and the medium-term forecast horizons (30-, 60- and 90-days). In line with equation (10), high values in the *RAX* index are associated with low future market returns (*R*). According to the results in table 4, both *ITSKEW* and *ITVIX* indices appear to be ineffective in terms of forecasting the market index (*FTSE MIB*) returns in the future. This is confirmed by a joint Wald test $\beta_1 = \beta_3 = 0$. This finding suggests that the risk-asymmetry index (*RAX*) subsumes all the information of *ITSKEW* and *ITVIX* for any considered forecast horizon. In other words, once *RAX* is included in the model, the other two indices

**TABLE 4. Encompassing model output: $R_{t,t+n} = \alpha + \beta_1\ ITSKEW_t + \beta_2\ RAX_t + \beta_3\ ITVIX_t + \varepsilon_t$ equation (10)**

| | *ITSKEW* | *RAX* | *ITVIX* | Adj. $R^2$ (%) | $\chi^2$ |
|---|---|---|---|---|---|
| A. Entire sample | | | | | |
| $R_{t,t+1}$ | –0.0004 | –0.0010 | 0.0000 | | |
| | (–1.32) | (–1.23) | (0.53) | 0.28% | 2.23 (0.33) |
| $R_{t,t+7}$ | –0.0006 | –0.0044$^{*}$ | 0.0000 | | |
| | (–0.87) | (–1.81) | (0.22) | 0.93% | 0.82 (0.66) |
| $R_{t,t+30}$ | –0.0004 | –0.0200$^{***}$ | 0.0001 | | |
| | (–0.17) | (–2.70) | (0.29) | 2.60% | 0.13 (0.94) |
| $R_{t,t+60}$ | 0.0005 | –0.0340$^{***}$ | 0.0005 | | |
| | (0.14) | (–2.88) | (0.81) | 3.34% | 0.66 (0.72) |
| $R_{t,t+90}$ | –0.0012 | –0.0364$^{***}$ | 0.0006 | | |
| | (–0.26) | (–2.85) | (0.83) | 3.30% | 0.87 (0.65) |
| B. Low volatility | | | | | |
| $R_{t,t+1}$ | –0.0003 | 0.0003 | 0.0000 | | |
| | (–1.08) | (0.35) | (0.86) | 0.00% | 1.94 (0.38) |
| $R_{t,t+7}$ | –0.0008 | –0.0006 | 0.0001 | | |
| | (–1.05) | (–0.30) | (0.58) | 0.44% | 1.39 (0.50) |
| $R_{t,t+30}$ | –0.0042$^{*}$ | –0.0016 | 0.0001 | | |
| | (–1.87) | (–0.29) | (0.18) | 2.32% | 3.63 (0.16) |
| $R_{t,t+60}$ | –0.0034 | –0.0103 | –0.0003 | | |
| | (–1.05) | (–1.40) | (–0.44) | 2.51% | 1.15 (0.56) |
| $R_{t,t+90}$ | –0.0011 | –0.0265$^{***}$ | –0.0000 | | |
| | (–0.34) | (–2.88) | (–0.06) | 4.31% | 0.11 (0.94) |
| C. High volatility | | | | | |
| $R_{t,t+1}$ | –0.0004 | –0.0022 | 0.0001 | | |
| | (–0.79) | (–1.47) | (1.56) | 0.75% | 4.38 (0.11) |
| $R_{t,t+7}$ | –0.0003 | –0.0084$^{*}$ | 0.0003 | | |
| | (–0.23) | (–1.94) | (1.42) | 2.02% | 2.41 (0.30) |
| $R_{t,t+30}$ | 0.0065 | –0.0438$^{***}$ | 0.0016$^{**}$ | | |
| | (1.48) | (–3.35) | (2.49) | 6.25% | 6.64 (0.04) |
| $R_{t,t+60}$ | 0.0100 | –0.0712$^{***}$ | 0.0035$^{***}$ | | |
| | (1.48) | (–3.94) | (3.46) | 10.60% | 12.02 (0.00) |
| $R_{t,t+90}$ | 0.0043 | –0.0648$^{***}$ | 0.0045$^{***}$ | | |
| | (0.49) | (–3.09) | (3.38) | 11.86% | 11.90 (0.00) |

**Note:** The table shows the estimated output of the following regression: $R_{t,t+n} = \alpha + \beta_1\ ITSKEW_t + \beta_2\ RAX_t + \beta_3\ ITVIX_t + \varepsilon_t$ ; t-stats are shown in parentheses. The $\chi^2$ shows the statistic of a $\chi^2$ test for the joint null hypotesis $\beta_1$=$\beta_3$=0 (p-values in parentheses).

fail to contribute a significant incremental explanatory power to future returns. This is clearly important for investors, who can rely on a single measure of risk, namely the *RAX* index, without having to consider the other two indices: the *ITSKEW* and the *ITVIX*.

The results for the low and high volatility periods are shown in Panels B and C of table 4, respectively. According to these results, during the low volatility period, only the *RAX* index has a significant explanatory power at the 1% level and only for the 90-day forecast horizon, as confirmed by the Wald test. During the high volatility period (2008-2012), both the *RAX* and the *ITVIX* indices are found to be significant, pointing to the usefulness of complementing the information in the *RAX* index with that provided in *ITVIX* (as confirmed by the Wald test that rejects the null hypothesis ($\beta_1 = \beta_3 = 0$). However, the signs on the betas show that the information in the *RAX* index and *ITVIX* move in the opposite direction: whereas high values of the *RAX* index are associated with low future market returns, high values in volatility are associated with high future returns.

To sum up, the *RAX* index outperforms both the Italian volatility index *ITVIX* and the *ITSKEW* index in forecasting future market returns. This result is important for investors, who can exploit the information of the *RAX* index in order to make profitable trades. Moreover, when the *RAX* index is high, they can promptly hedge their portfolios in order to avoid large upcoming losses. This information could have dramatically improved portfolio selection procedures during the recent financial crisis.

## VI. The information content of extreme values of the *RAX* index

In the financial literature it is recognized that high levels of volatility in the market are associated with investor fear and future downturns in the stock prices. This is explained by the strong negative relation between parallel changes in volatility indices and stock market returns (see e.g. Whaley, 2000; Giot, 2005). However, this relation is not necessarily true if future market returns are considered. Giot (2005) suggests that high, or very high, implied volatility levels may indicate an oversold market and, as a result, possible positive future returns for long positions in the underlying market. This hypothesis is investigated in Rubbaniy et al. (2014), who estimate the relation between different levels of implied volatility indices (index values higher than the 90%, 95% and 99% percentiles or lower than the 1%, 5% and 10% percentiles) and the corresponding future index returns. They find that very high levels of volatility are related to positive future market returns, in line with the argument put forward in Giot (2005). In keeping with this observation, the aim here is to investigate the relation between future returns and levels of the *RAX* index on the extreme quantiles. The analysis is conducted first by means of quantile regression in subsection *A*, and second by investigating extreme values of the *RAX* index in subsection *B*.

A. *The relation between returns and the RAX index at extreme quantiles*

An interesting approach to investigate the relation between future aggregate market returns and the proposed risk-asymmetry index is represented by quantile regression. Quantile regression is a type of regression analysis that aims at estimating the conditional median or other quantiles of the dependent variable, given certain values of the independent variable. As a result, it is particularly suitable for identifying different relationships in different parts of the distribution of the dependent variable. In this sense, quantile regression provides an alternative to ordinary least squares (OLS) regression, which typically assumes the relation between independent and dependent variables to be the same at all quantiles, resulting in estimates of the conditional mean of the dependent variable. However, as pointed out in Lê Cook and Manning (2013), it is important to note that quantile regression is not a regression estimated on a specific quantile, or subsample of data. In fact, estimating a $x$-th quantile regression fits a regression line through the scattered data so that the $x$ percentage of the observations are below the regression line, and $1-x$ percentage are above. As a result, quantile regression exploits all the observations and assigns different weights depending on positive and negative residuals to the distance between the values predicted by the regression line and the observed values.

In order to further investigate the relation between future market returns at the 1, 7, 30, 60 and 90-day forecast horizon and the proposed risk-asymmetry index, quantile regression is applied to our data by exploiting the "quantreg" package for *R*. The results are shown in figure 4, where a graphical representation of the quantile coefficients along with confidence intervals is proposed. Quantile regression is estimated

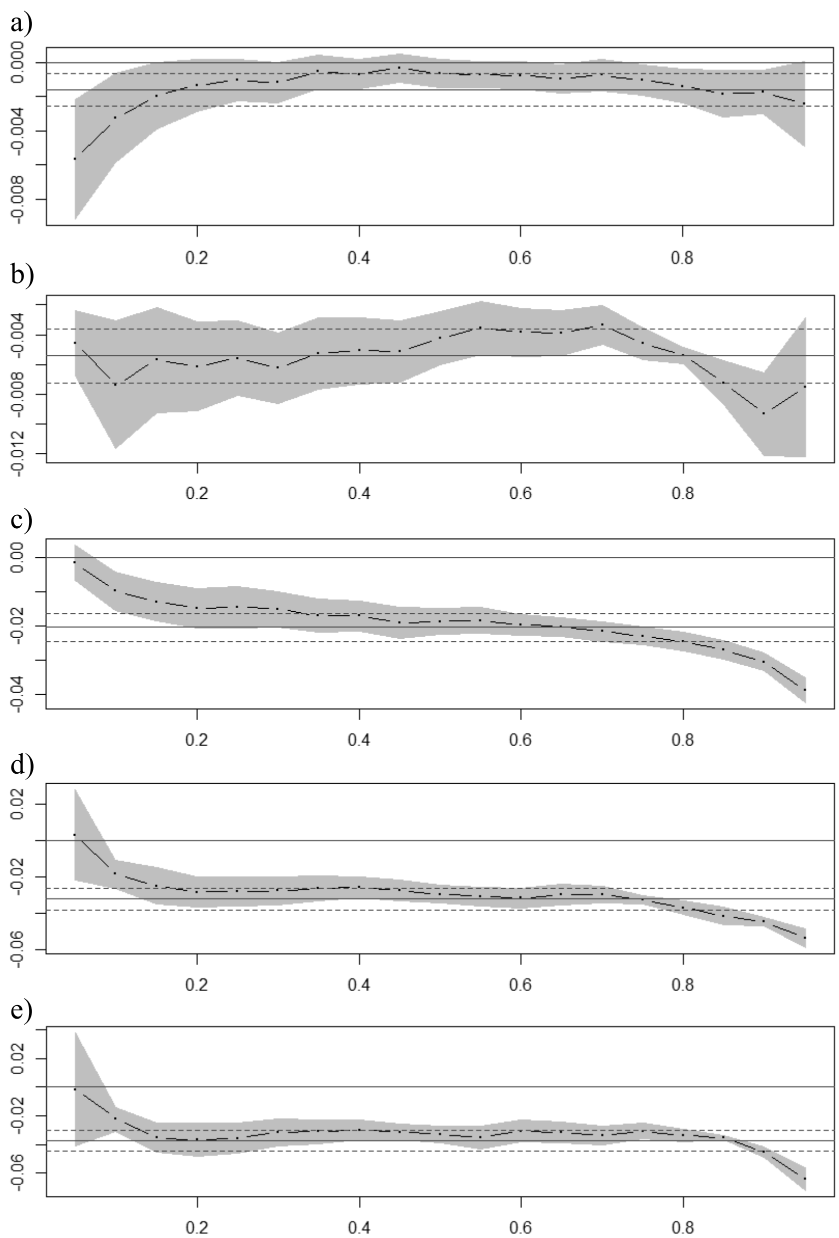


FIGURE 4.— Future market returns and the *RAX* index: quantile regression output

**Note:** The figure displays the output of quantile regression between the *RAX* index and future market returns for different forecast horizons: a) 1 day, b) 7-day, c) 30-day, d) 60-day, e) 90-day, (percentiles on the x-axis and beta coefficients on the y axis) on percentiles of the dependent variable which are spaced at 5% intervals. Each black dot is the slope coefficient estimated for the quantile indicated on the *x* axis, and the grey area represents its confidence interval. The red lines along with the corresponding confidence interval represent the least squares regression.

The least square slope is suitable for describing the relationship between the dependent and the response variable for most of the quantiles, since the slope coefficients obtained through quantile regression are inside the OLS confidence interval. The only estimates that lie outside of the OLS confidence interval are the ones for very low and very high quantiles, suggesting that the relation between the predictor and the dependent variable changes for very low and very high values of the latter. This will be investigated further in the next sub-section.

*B. The information content of extreme values of the RAX index*

The relation between future returns and extreme levels of the *RAX* index is investigated here in order to determine whether extremely low or extremely high values of the index are associated with positive or negative returns. In order to make a comparison, and determine which index is the most reliable one, the Italian skewness index (*ITSKEW*) and model-free implied volatility indices (*ITVIX*, $CIV_{UP}$ and $CIV_{DW}$) are included in the analysis. Only extreme index values are considered, namely, extremely low and very low values (index values lower than the 1% and the 5% percentiles, respectively) and extremely high and very high values (index values higher than the 99% and 95% percentiles, respectively). Accordingly, four subsamples are constructed for estimating the model described by equation (9).[10] In this way it is examined whether extreme upside or downside index levels can be considered as an early signal of future negative or positive returns.

The results for extremely low and very low index values (values lower than their 1%, and 5%, percentile) are shown in table 5, Panels A and B, respectively. All the models were estimated using the OLS with the Newey and West (1987, 1994) heteroscedasticity and autocorrelation consistent (HAC) covariance matrix. The results for extremely high and very high index values (values higher than their 95%

10. Results for the 10% and the 90% percentiles are not reported for reasons of space but are available on request and are similar to the 5% and 95% percentiles, respectively.

**TABLE 5. Regression output for extremely low values of skewness and volatility indices and future market returns: $R_{t,t+n} = \alpha + \beta_1\ index_t + \varepsilon_t$ equation (9)**

A. Regression output for indices values lower than their 1% percentile

| | *ITSKEW* | *RAX* | *ITVIX* | $CIV_{UP}$ | $CIV_{DW}$ |
|---|---|---|---|---|---|
| $R_{t,t+1}$ | 0.0145 | 0.0353** | 0.0081 | 0.0026 | –0.0080 |
| | (1.03) | (2.36) | (1.47) | (0.73) | (–1.25) |
| $R_{t,t+7}$ | 0.0371* | 0.0709** | 0.0323*** | 0.0094 | 0.0003 |
| | (1.84) | (2.23) | (4.63) | (1.08) | (0.02) |
| $R_{t,t+30}$ | 0.0732 | 0.2628** | 0.0467** | 0.0434*** | 0.0182 |
| | (1.12) | (2.48) | (2.57) | (2.94) | (0.95) |
| $R_{t,t+60}$ | 0.1311 | 0.2328 | 0.0117 | 0.0112 | 0.0311** |
| | (1.26) | (1.39) | (1.01) | (1.31) | (2.32) |
| $R_{t,t+90}$ | 0.1000 | 0.2536 | –0.0337** | –0.0228 | –0.0273 |
| | (0.95) | (1.71) | (–2.25) | (–1.67) | (–1.32) |

B. Regression output for indices values lower than their 5% percentile

| | *ITSKEW* | *RAX* | *ITVIX* | $CIV_{UP}$ | $CIV_{DW}$ |
|---|---|---|---|---|---|
| $R_{t,t+1}$ | –0.0093 | 0.0052 | 0.0001 | 0.0011 | 0.0015 |
| | (–1.61) | (1.03) | (0.14) | (0.91) | (1.24) |
| $R_{t,t+7}$ | –0.0210 | –0.0115 | 0.0034 | 0.0065** | 0.0044** |
| | (–1.39) | (–0.89) | (1.54) | (2.06) | (2.25) |
| $R_{t,t+30}$ | –0.0738** | –0.0150 | 0.0220*** | 0.0244*** | 0.0235*** |
| | (–2.08) | (–0.29) | (5.89) | (5.06) | (5.69) |
| $R_{t,t+60}$ | –0.1196* | –0.0039 | 0.0095*** | –0.0041 | 0.0176*** |
| | (–1.82) | (–0.07) | (3.13) | (–0.89) | (4.40) |
| $R_{t,t+90}$ | –0.1375** | –0.0124 | 0.0284*** | 0.0198** | 0.0330*** |
| | (–2.25) | (–0.19) | (5.24) | (2.09) | (5.90) |

**Note:** The table shows the estimated output of the univariate model: $R_{t,t+n} = \alpha + \beta_1\ index_t + \varepsilon_t$ where $index_t$ is proxied by the Italian skewness index ($ITSKEW_t$), the risk-asymmetry index ($RAX_t$), the Italian implied volatility index ($ITVIX_t$), and corridor upside and downside implied volatility indices ($CIV_{UPt}$ and $CIV_{DWt}$) on their 1% and 5% percentiles; t-stats are shown in parentheses.

and 99% percentiles) are shown in table 6, Panels A and B, respectively. As all the indices are constructed in order to have a constant forecast horizon of 30 days, they are expected to have the highest information content on the short-term forecast horizon.

Starting from the analysis of extremely low values (table 5), a desirable property of a fear index is that when the index is extremely low, investors may feel safe and expect positive future returns. This is exactly what it is found for the *RAX* index: when the level of the index is extremely low (lower than its 1% percentile), a positive and marginally

**TABLE 6. Regression output for extremely high values of skewness and volatility indices and future market returns: $R_{t,t+n} = \alpha + \beta_1\ index_t + \varepsilon_t$ equation (9)**

A. Regression output for indices values higher than their 95% percentile

| | *ITSKEW* | *RAX* | *ITVIX* | $CIV_{UP}$ | $CIV_{DW}$ |
|---|---|---|---|---|---|
| $R_{t,t+1}$ | –0.0010 | –0.0005 | 0.0006 | 0.0015 | 0.0008 |
| | (–1.40) | (–0.31) | (1.35) | (1.47) | (1.65) |
| $R_{t,t+7}$ | –0.0024* | –0.0091* | 0.0003 | 0.0021 | 0.0005 |
| | (–1.93) | (–1.89) | (0.54) | (1.26) | (0.53) |
| $R_{t,t+30}$ | –0.0018 | –0.0183*** | –0.0029*** | –0.0030 | –0.0028** |
| | (–0.56) | (–2.78) | (–2.75) | (–1.38) | (–2.27) |
| $R_{t,t+60}$ | 0.0005 | –0.0343*** | –0.0042*** | –0.0071*** | –0.0030*** |
| | (0.12) | (–3.25) | (–3.74) | (–3.00) | (–3.04) |
| $R_{t,t+90}$ | –0.0061 | –0.0342*** | –0.0065*** | –0.0102*** | –0.0059*** |
| | (–0.65) | (–2.70) | (–3.76) | (–3.15) | (–3.37) |

B. Regression output for indices values higher than their 99% percentile

| | *ITSKEW* | *RAX* | *ITVIX* | $CIV_{UP}$ | $CIV_{DW}$ |
|---|---|---|---|---|---|
| $R_{t,t+1}$ | –0.0027 | 0.0062 | 0.0036** | 0.0067*** | 0.0037 |
| | (–0.90) | (1.17) | (2.32) | (4.46) | (1.66) |
| $R_{t,t+7}$ | –0.0063 | 0.0007 | 0.0026 | 0.0043 | 0.0020 |
| | (–1.69) | (0.06) | (1.43) | (1.09) | (0.66) |
| $R_{t,t+30}$ | –0.0054 | –0.0483*** | 0.0049*** | 0.0063* | 0.0054* |
| | (–0.48) | (–4.43) | (3.63) | (1.74) | (1.86) |
| $R_{t,t+60}$ | 0.0122 | –0.0770*** | 0.0009 | –0.0015 | 0.0010 |
| | (0.94) | (–3.37) | (0.52) | (–0.40) | (0.40) |
| $R_{t,t+90}$ | 0.0186 | –0.0472* | 0.0029 | 0.0011 | 0.0025 |
| | (1.11) | (–1.98) | (1.32) | (0.28) | (0.83) |

**Note:** The table shows the estimated output of the univariate model: $R_{t,t+n} = \alpha + \beta_1\ index_t + \varepsilon_t$ where $index_t$ is proxied by the Italian skewness index ($ITSKEW_t$), the risk-asymmetry index ($RAX_t$), the Italian implied volatility index ($ITVIX_t$), and corridor upside and downside implied volatility indices ($CIV_{UPt}$ and $CIV_{DWt}$) on their 95% and 99% percentiles; t-stats are shown in parentheses.

significant relationship between the index and future aggregate market returns is found only for the very short and short forecast-horizons (1-, 7- and 30- days). As a result, extremely low values of the *RAX* index can be interpreted as indicators of short-term market greed. The same relation is found for *ITVIX*, $CIV_{UP}$ and $CIV_{DW}$, for the 7-day and the 30-day forecast horizons. For medium-term forecast horizons, very low implied volatility index (*ITVIX*) values have a significant relationship with future aggregate market returns, which, however, is not constant in sign, making it difficult for investors to interpret the signal.

For extremely high and very high levels of the *RAX* index (table 6), a negative and significant relation between the *RAX* index and future aggregate market returns, mainly for the short-term and the medium-term forecast horizons, is found. This suggests that extremely high and very high values in the *RAX* index are a clear signal of low future market returns in the short- and medium-term forecast horizons. In this framework the *RAX* index acts as a measure of medium-term market fear, since high values in the skewness index can be regarded as an early warning indicator of future market returns. No evidence is found for the *ITSKEW* index as a warning indicator of future negative returns.

For the Italian volatility index (*ITVIX*) a different signal is found depending on the level of volatility: very high values of the index are associated with negative future returns, but extremely high values are associated with positive future returns. This result may be interpreted as follows: if volatility becomes extremely high, then the market has already discounted all the fear, and positive returns can be expected. From $CIV_{UP}$ and $CIV_{DW}$ almost the same conflicting information is obtained as that obtained from *ITVIX*. However, given that from the investors' point of view it is impossible to assess whether the volatility level is very high (in this case "sell"), or extremely high (in this case "buy"), the mainstream information obtained from the *RAX* (if high then "sell", if low then "buy") is the simplest and the most valuable for investors. Moreover, given that it is preferable to correctly measure fear rather than greed, it can be stated that the *RAX* index is the only index able to indicate (when reaching very high values) a possible risky situation for the aggregate stock market. In figure 5, a comparison between the "buy" (green) and "sell" (red) signals given by the *RAX* index and the *FTSE MIB* returns is shown. In many points the *RAX* index correctly signals future market returns.

In order to assess the economic significance of the "buy" and "sell" signals provided by the *RAX* index, following Giot (2005), a trading strategy that takes a long position in the underlying asset (the *FTSE MIB* index) when the *RAX* index level is lower (higher) than its 1%, 5% and 10% (90%, 95%, 99%) percentiles is proposed. The profitability of the strategy both for very short-, short- and medium-term holding periods is assessed. In table 7, Panel A, the strategy that takes a long position on the *FTSE MIB* index when the *RAX* index level is very low earns on average a positive return for all the considered holding periods. For example, when the *RAX* index is lower than 101.15 (which correspond

**TABLE 7. Average returns for a long position on the *FTSE MIB* index set when the *RAX* index is lower (higher) than its 1%, 5% and 10% (90%, 95% and 99%) percentiles.**

| | A. *RAX* index lower than | | | B. *RAX* index higher than | | |
|---|---|---|---|---|---|---|
| | 100.63 (1%) | 101.00 (5%) | 101.15 (10%) | 102.32 (90%) | 102.55 (95%) | 103.70 (99%) |
| | 0.0042 | 0.0027** | 0.0018* | –0.0017 | –0.0024 | –0.0042 |
| $R_{t,t+1}$ | (0.0033) | (0.0012) | (0.0010) | (0.0012) | (0.0017) | (0.0027) |
| | [1.26] | [2.19] | [1.74] | [–1.41] | [–1.38] | [–1.54] |
| | 0.0171** | 0.0074** | 0.0047** | –0.0060** | –0.0075* | –0.0120** |
| $R_{t,t+7}$ | (0.0078) | (0.0029) | (0.0022) | (0.0023) | (0.0038) | (0.0044) |
| | [2.18] | [2.54] | [2.16] | [–2.59] | [–1.97] | [–2.76] |
| | 0.0489* | 0.0340*** | 0.0281*** | –0.0172*** | –0.0278*** | –0.0408*** |
| $R_{t,t+30}$ | (0.0249) | (0.0077) | (0.0046) | (0.0039) | (0.0049) | (0.0089) |
| | [1.96] | [4.43] | [6.10] | [–4.35] | [–5.71] | [–4.61] |
| | 0.0610* | 0.0517*** | 0.0389*** | –0.0300*** | –0.0529*** | –0.0930*** |
| $R_{t,t+60}$ | (0.0344) | (0.0111) | (0.0078) | (0.0057) | (0.0076) | (0.0156) |
| | [1.77] | [4.66] | [4.99] | [–5.23] | [–6.95] | [–5.97] |
| | 0.0752** | 0.0582*** | 0.0364*** | –0.0452*** | –0.0696*** | –0.1117*** |
| $R_{t,t+90}$ | (0.0341) | (0.0134) | (0.0101) | (0.0076) | (0.0113) | (0.0155) |
| | [2.21] | [4.35] | [3.60] | [–5.96] | [–6.17] | [–7.23] |

**Note:** The table shows the average returns for a strategy that take a long position when the *RAX* index is lower (higher) than its 1%, 5% and 10% (90%, 95% and 99%) percentiles. Standard errors are shown in parentheses; t-stats are shown below in square brackets.

to the 10% percentile), the average return of the strategy over a 30-day period is equal to 2.81% (34.16% on an annual basis), with the standard error (corrected by using Newey-West) equal to 0.005. On the other hand, the average return of the strategy that takes a long position in the underlying asset when the *RAX* index is very high is negative for all the considered holding periods (Panel B, table 7). The average return of the strategy increases (in absolute value) when the strategy is set for high values of the *RAX* index and for long holding periods. In particular, the 90-day average return for a buy position when the *RAX* index is higher than 103.70 (higher than its 99% percentile) is equal to –11.17% (–45.29% on an annual basis), with a standard error of 0.015. This result is important for investors who can promptly adopt trading strategies and use them to hedge their portfolios in order to avoid substantial losses.

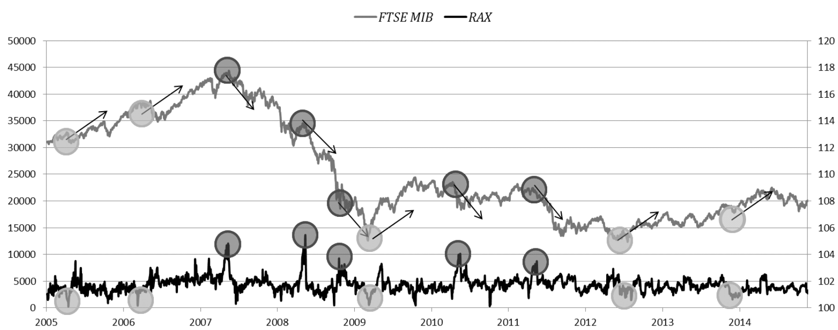


FIGURE 5.— Graphical comparison between the *FTSE MIB* index and the *RAX* index

**Note:** Green dots show the "buy" signals, and red dots show the "sell" signals as indicated by the *RAX* index.

## VII. Conclusions

Given the importance of disentangling positive and negative shocks to volatility, which are perceived by investors, respectively, as good or bad news, the information in upside and downside corridor implied volatilities is exploited in order to measure the asymmetry of the return distribution. Upside and downside corridor implied volatilities are aggregated into the risk-asymmetry index (*RAX*), which measures the difference between upside and downside corridor implied volatilities standardized by total volatility. The *RAX* index is meant to measure the investors' pricing asymmetry towards upside gains and downside losses. The numerator is standardized by total volatility in order to leave the investors' pricing asymmetry free from the level of volatility. In this way, the *RAX* index is not influenced by bullish or bearish market periods.

The two risk measures that capture the second and the third moments, namely the *CBOE VIX* and the *CBOE SKEW*, can provide conflicting information. To elaborate, it is difficult to interpret a high *CBOE VIX* index, which is meant to measure fear, together with a low *CBOE SKEW* index, which is intended to measure additional tail risk. The *RAX* index is intended to substitute the two risk measures, as the only index that investors need to rely on for determining portfolio strategies.

The forecasting power of the *RAX* index in predicting future returns is compared with those of the skewness index, the volatility index, and the corridor upside and downside implied volatility indices ($CIV_{UP}$ and $CIV_{DW}$) in the Italian market. It is worth recalling that the Italian market is taken just for explanatory purposes: the *RAX* and the other option implied measures used in this study can be computed for any market under investigation.

The following results were obtained. First, the risk-asymmetry index presents several advantages: it is positively correlated with realized skewness, as a result, it can be considered a market-based forecast of the latter and it is positively but weakly associated with the Italian volatility index (*ITVIX*). Therefore, according to the *RAX* index, the risk-neutral distribution of the *FTSE-MIB* index returns is riskier when model-free implied volatility is high, and as a result the *RAX* index is expected to positively spike in turmoil periods. Second, the *RAX* index subsumes all the information in the Italian volatility index as well as the information in the Italian skewness index in forecasting future market returns at any forecast horizon (1, 7, 30, 60 and 90 days). This result is important for investors who need to rely on just one simple indicator in order to plan profitable trades.

Third, the *RAX* index can be considered as a greed index in the short term (up to 30 days) and a fear index in the medium term (from 30 to 90 days), since extremely low values of the *RAX* index can be interpreted as indicators of future positive returns over the next 30 days and extremely high values of the *RAX* index indicate future negative returns over the next 30 to 90 days. Last, unlike the *ITVIX* index which cannot easily be used by investors since it indicates future negative returns if it is very high, but future positive returns if it is extremely high, the *RAX* index is the only index able to indicate (when reaching very high values) a clearly risky situation for the aggregate stock market. In particular, when the *RAX* index is higher than its 99% percentile (higher than 103.70) the average market return over the next three months is equal to –11.17% (–45.29% on an annual basis), pointing to a very risky condition for the aggregate stock market. This result is important for investors who can hedge their portfolios in order to avoid losses.

*Accepted by*: *Prof. G. Koutmos*, *Guest Editor*, *July* 2018
*Prof. P. Theodossiou*, *Editor-in-Chief*, *July* 2018

## Appendix A. The Italian skewness index (*ITSKEW*)

Currently, the only skewness index quoted is the *CBOE SKEW* index for the S&P500 market. This index is defined as:

$$SKEW = 100 - 10 \times SK \tag{A1}$$

where *SK* is the 30-day measure of risk-neutral skewness. Risk-neutral skewness measures the asymmetry in the returns distribution obtained from option prices. It is equal to zero for a normal distribution, indicating symmetry in the returns. If skewness is negative it means that the mass of the distribution is concentrated in the left tail or, more specifically, the left tail is longer or thicker, or both, compared to the right tail. Symmetrically, if skewness is positive, it means that the right tail is longer or thicker, or both, with respect to the left tail (the mass of the distribution is concentrated in the right tail). In order to have a 30-day measure of risk-neutral skewness, *SK* is computed by linear interpolation between two values of risk-neutral skewness: the first refers to near-term options, with maturity possibly less than 30 days, the second refers to next-term options with time to maturity possibly greater than 30 days. As the risk-neutral skewness attains typically negative values for equity indices, the *ITSKEW* index attains in general a value above 100; the higher the value of the *ITSKEW*, the higher the riskiness of the returns distribution. The risk-neutral skewness (*SK*) is computed by means of the Bakshi, Kapadia and Madan (2003) formula.

According to Bakshi, Kapadia and Madan (2003), model-free skewness is obtained from the following equation as:

$$\begin{aligned} SK(t,\tau) &\equiv \frac{E_t^q\left\{\left(R(t,\tau) - E_t^q[R(t,\tau)]\right)^3\right\}}{\left\{E_t^q\left(R(t,\tau) - E_t^q[R(t,\tau)]\right)^2\right\}^{3/2}} \\ &= \frac{e^{r\tau}W(t,\tau) - 3e^{r\tau}\mu(t,\tau)V(t,\tau) + 2\mu(t,\tau)^3}{\left[e^{r\tau}V(t,\tau) - \mu(t,\tau)^2\right]^{3/2}} \end{aligned} \tag{A2}$$

where $\mu(t,\tau)$, $V(t,\tau)$, $W(t,\tau)$ and $X(t,\tau)$ are the prices of the contracts, at time $t$ with maturity $\tau$, based on first, second, third and fourth moment of the distribution, respectively. Their values are computed as:

$$\mu(t,\tau) \equiv E^q \ln[S(t+\tau)/S(t)]$$

$$= e^{r\tau} - 1 - \frac{e^{r\tau}}{2}V(t,\tau) - \frac{e^{r\tau}}{6}W(t,\tau) - \frac{e^{r\tau}}{24}X(t,\tau) \tag{A3}$$

$$V(t,\tau) = \int_{S(t)}^{\infty} \frac{2(1-\ln[K/S(t)])}{K^2} C(t,\tau;K)\,dK$$

$$+ \int_{0}^{S(t)} \frac{2(1+\ln[S(t)/K])}{K^2} P(t,\tau;K)\,dK \tag{A4}$$

$$W(t,\tau) = \int_{S(t)}^{\infty} \frac{6\ln[K/S(t)] - 3(\ln[K/S(t)])^2}{K^2} C(t,\tau;K)\,dK +$$

$$- \int_{0}^{S(t)} \frac{6\ln[S(t)/K] + 3(\ln[S(t)/K])^2}{K^2} P(t,\tau;K)\,dK \tag{A5}$$

$$X(t,\tau) = \int_{S(t)}^{\infty} \frac{12(\ln[K/S(t)])^2 - 4(\ln[K/S(t)])^3}{K^2} C(t,\tau;K)\,dK +$$

$$+ \int_{0}^{S(t)} \frac{12(\ln[S(t)/K])^2 + 4(\ln[S(t)/K])^3}{K^2} P(t,\tau;K)\,dK \tag{A6}$$

where $C(t,\tau;K)$ and $P(t,\tau;K)$ are the prices of a call and a put option at time $t$ with maturity $\tau$ and strike $K$, respectively, $S(t)$, is the underlying asset price at time $t$.

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