Simionescu, Mihaela

Article

Forecasting economic growth using chaos theory

Article history: Received 22 April 2016; last revision 30 June 2016; accepted 12 September 2016

FORECASTING ECONOMIC GROWTH USING CHAOS THEORY

Mihaela Simionescu
Institute for Economic Forecasting of the Romanian Academy

Marek Śmiechowski
Statistical Office in Rzeszow

Abstract

This paper presents an original contribution of the application of Lorenz attractor in economic theory by modeling the chaos in economic systems and forecasts
We apply the chaos theory provisions and employ the provisions of Chaos Dynamics which represents a new trend in economic science and is based on endogenous approach*.*
Our results indicated a tendency of increase in world economic growth on the horizon 2015-2019, when specific values are given to the parameter in Lorenz system. This approach might be considered a new forecasting method in economic science.

Keywords: Lorenz system, economic forecasts, economic growth, chaos

JEL Classification: F41, F47

Introduction

The existence of chaos in economic series has been deeply analyzed in the last years. There are several concepts from chaos theory that converge with the interests in economic research, some of them being related to complex properties of dynamics and limited ability to forecast. There are many applications of chaos in economics, ranging from explaining business cycles to forecasting movements in stock markets. Many types of financial and economic time series were analyzed using techniques for identifying chaos. Even if the presence of chaotic movements in time series was weak or inexistent, the unexplained non-linear structure was detected in these series. However, the analyze of time series using chaos techniques is still an important research topic in macroeconomics and finance in order to explain some particular evolutions in time series in the context of small or big data.
In this paper, we provide an example of applying chaos theory in Economics with results that are closed to traditional forecasting methods using in making economic predictions. After the presentation of theoretical background regarding chaos theory and, especially, Lorenz attractor, an application is proposed on data regarding the predictions and the actual values of world economic growth. The last part concludes.

Theoretical background

Economic behavior is characterized by complexity, a property that needs to be studied by economists. There are many interesting examples when economists are trying to apply the tools of mathematics and chaos theory to model strange occurrences in market theory (see e.g. Filatov and Makolskaya, 2015) or even some unusual or often bizarre events (see Strielkowski et al., 2013). The new domain of research in economics is known as Chaotic Dynamics.

One would probably agree that economics developed in the context of explaining this complexity in an endogenous approach. The function of strange attractors is used to locate the chaos. Bifurcation diagrams are followed or the figures' profile using fractal geometry is studied. The chaos is considered as raw material associated to natural elements without shape being eager to take a form. The geometrics of chaos are represented by bending and stretching. Actually, a mechanism bifurcated in distinct actions produces the movement's irregularity. There are separate trajectories, spatial phase being stretched and this will doubles back into itself. The first phase of the process is explained by Lyapunov's exponents by evaluating the exponential separation of the adjacent trajectories. For a discrete system one can analyze the local instability in the Lyaponov approach:

$$x_{n+1} = f(x_n) \qquad (1)$$

This implies showing how the adjacent points divide the function's iterated application:

$$|f^n(x_0 + \epsilon) - f^n(x_0)| = \epsilon^{n\lambda(x_0)} \qquad (2)$$

$$\lambda(x_0) = \lim_{n\to\infty} \lim_{\epsilon\to 0} \frac{1}{n} \log \left| \frac{f^n(x_0+\epsilon) - f^n(x_0)}{\epsilon} \right| \qquad (3)$$

$$\lambda(x_0) = \lim_{n\to\infty} \frac{1}{n} \log \left| \frac{df^n(x_0)}{dx_0} \right| \qquad (4)$$

In a system of dimension *n*, a number of *n* Lyapunov exponents are considered. Each exponent refers to the mean of contraction or expansion rate of the phase space in any direction from the n ones under the dynamical system action. The distinction between attractors with complex and simple dynamics is made through Lyapunov exponents. These are known as traditional attractors until the Lorenz approach from 1963 (quasi-periodic tours, limit cycles and fixed points). Simple dynamics are considered inside tours, limit cycles and fixed points, because two orbits that begin to move arbitrary one near to the other stay adjacent. Therefore, long run predictability is ensured. A complex dynamical system can be observed inside the chaotic or strange attractor. Even if the bounded in the attractor is not locally stable, with aperiodic and recurrent cycles, with sensitive correlation to first conditions, it is difficult to provide forecasts on very short-run. The Lyapunov exponent is employed to identify the complex or the simple dynamics.

The sine qua non condition for chaos existence is the positive Lyapunov exponent (there is no local stability). Lorenz attractor is among the many types of attractors from literature, having the form of butterfly wings. It solves an important problem from meteorology consisting in atmospheric convection that supposes the evolution of a fluid layer that is heated from below. Other important attractors are Rössler and Hénon attractor with a more elegant structure.

In the chaos domain, Lorenz (1963) studied the main equations for fluid mechanics by describing characteristics like temperature, density and velocity as function of position or time. The problem formulated by Rayleigh-Benard showed that at different temperatures the fluid in a container has different surfaces. The difference between temperatures is increased in order to get transitions between steady flow, steady state and a potential chaotic flow. In economics, this demarche might be equivalent to the increase in shocks intensity in order to catch the economic phenomenon in various states.

The Lorenz equations are built around three differential equations. The Lorenz equations that describe the fluid behavior are given by:

$$\frac{dx}{dt} = \sigma(y - x) \quad (5)$$

$$\frac{dy}{dt} = x(\rho - z) - y \quad (6)$$

$$\frac{dz}{dt} = xy - \beta\beta z \quad (7)$$

Where:

x,y,z refers to temperature, density and velocity;

σ , ρ , β measures the differences in temperature across the studied fluid and other specific parameters of fluid;

σ- Prandtl number (computed as momentum diffusivity over thermal diffusivity);

ρ- Rayleigh number (depends on fluid effective force, measures the differences in temperature at top and bottom of the fluid);

β- physical proportion

In physics, the values that are taken as follows:

$$\sigma = 10$$
$$\rho = 28$$
$$\beta = \frac{8}{3}$$

If these parameters are given, Lyapunov exponents might be calculated. For negative Lyapunov exponents we have a stable system, while the presence of chaos can be reflected by the positive Lyapunov exponents.

Yu (1996) showed that a strategy based on a variable structure control can stabilize the Lorenz chaos. Switching between Lorenz systems with slightly different values for parameters can determine the sliding region. If the system trajectory convergence to equilibrium is asymptotical, chaos stabilization is made.

Tucker (2002) proved that Lorenz attractor is actually a strange attractor by combining the formal theory with the empirical validations.

Lorenz attractor is a type of strange attractors where one does not have information where on the attractor the system might be. Two close points on the attractor at a certain time will be randomly

far apart at next moments. Only a restriction is imposed in the sense that the system state will remain on the attractor. Another characteristic of this strange attractor is related to the fact that attractors never close on themselves. In other words, the system evolution never repeats. The chaotic behavior is represented by the movement on the strange attractors.
Chaos theory was also applied in economics by Benhabib and Day (1982), Day(1997), Goodwin (1990), Lorenz (1993), Medio and Gallo (1995) and Jablanović (2012a) and many other researchers. Jablanovic (2012b) built a simple model for chaotic general economic equilibrium That is able to generate the stable equilibrium, cycles, or chaos. An example for general economic equilibrium is given by the case of monopolies.
Lorenz system was not applied yet in Economics, but an application will be to propose these parameters for actual values and predictions of world economic growth.

Empirical model and application

The empirical application refers to the world economic growth in terms of forecasts and actual values. We consider two types of forecasts: naïve predictions (the value in the previous period remains constant in the current period) and OECD's forecast. In the system, Y represents the actual value of world GDP growth, X is the naïve forecasts and Z is the OECD's expectation for 2014. We have to use certain values for the parameters in the model. We computed those parameters by using the historical values for X, Y and Z. The results made us to consider the following parameters in the Lorenz system:

$$\sigma = 0.2$$
$$\rho = 12$$
$$\beta = 2.9$$

For the initial state, the following values were given to the variables: X=3.174, Y=346 and Z=3.6. All the computations were made in R using deSolve software package.

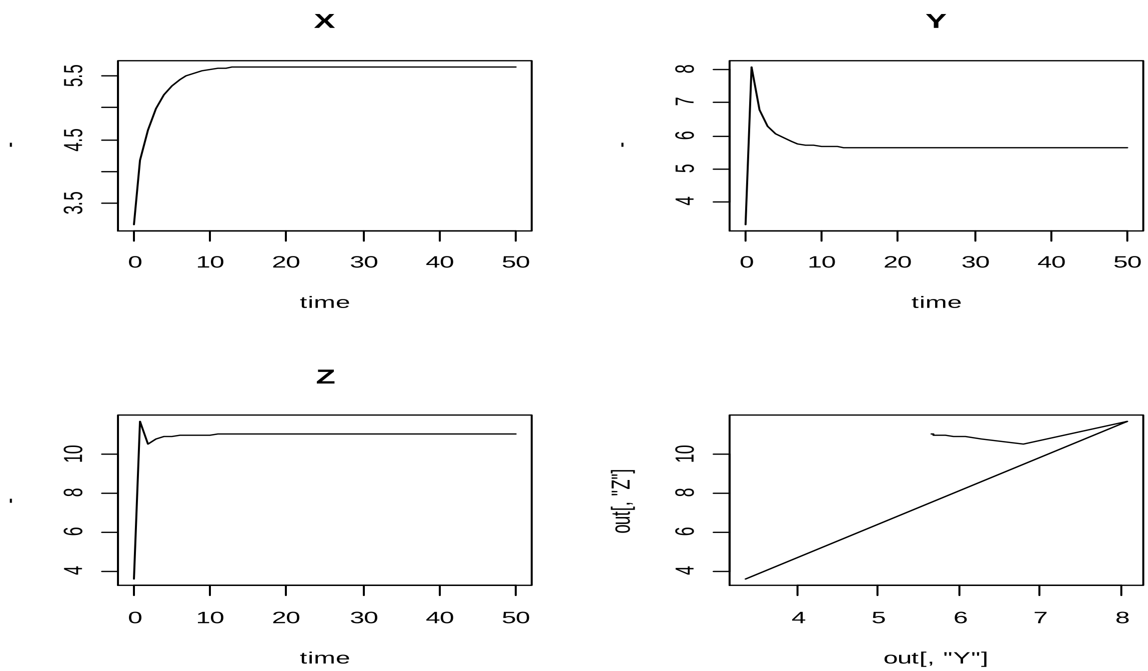
Table 1: Predictions of world economic growth (2015-2019)

Year	X	Y	Z
2015	4.167094	8.084633	11.64923
2016	4.643052	6.796598	10.51710
2017	4.981014	6.273177	10.77234
2018	5.197761	6.069903	10.86892
2019	5.341991	5.930440	10.91201

Source: Own results

The results obtained in course of our estimations are presented in Table 1. Looking at these results, one can deduct that for the next 5 years, X might take values between 4.16 and 5.34, a tendency of increase in global economic growth from a year to another being anticipated. The Lorenz model is presented in Figure 1 that follows.

Figure 1: Lorenz model representation



Source: Own results

Thence, it appears that the Lorenz curve provided acceptable forecasts for the next values of the world economic growth. Even if the butterfly wings were not identified, the predictions based on Lorenz attractor are in accordance with the expectations. The Lorenz system might be a suitable forecasting method in Economics.

Conclusions and implications

This paper provided the results of an original research that demonstrated an example for applying Lorenz system in economic research. As previous studies confirmed, the chaos theory does not provide in Economics the same results as in Physics, but important features of the economic phenomenon are identified. In economic forecasting, the Lorenz attractor might be applied, the main contribution being the proposal of new forecasts that were related to forecasters' expectations.

References

- Benhabib, J., Day, R.H. (1982), Characterization of erratic dynamics in the overlapping generation model, *Journal of Economic Dynamic*, Vol. 4, pp. 37-55.
- Day, R.H. (1997), *Complex Economic Dynamics Volume I: An Introduction to Dynamical Systems and Market Mechanism in Discrete Dynamics in Nature and Society*, MIT Press.
- Goodwin, R.M. (1990), *Chaotic Economic Dynamics*, Clarendon Press, Oxford.

- Filatov, A., & Makolskaya, Y. (2015). The equilibrium and socially effective number of firms at oligopoly markets: theory and empirics. *Czech Journal of Social Sciences Business and Economics*, Vol. 4, Issue 4, pp. 17-31.
- Jablanovic, V.D. (2012b), The chaotic long-run monopolistic competitor's output growth model, *African Journal of Business Management*, Vol. 48, No 8, pp. 11758-11761.
- Jablanović, V. (2012a), *Budget Deficit and Chaotic Economic Growth Models*, Aracne editrice Srl.
- Lorenz, E. N. (1963), Deterministic nonperiodic flow, *Journal of the atmospheric sciences*, Vol. 20, No. 2, pp.130-141.
- Lorenz, H.W. (1993), *Nonlinear Dynamical Economics and Chaotic Motion*, Springer-Verlag.
- Medio, A., Gallo, G. (1995), *Chaotic Dynamics. Theory and Applications to Economics*, Cambridge University Press.
- Strielkowski, W., Lisin, E., & Welkins, E. (2013), Mathematical models of interactions between species: peaceful co-existence of vampires and humans based on the models derived from fiction literature and films, *Applied Mathematical Sciences*, Vol. 7, No. 10, pp. 453-470
- Tucker, W. (2002), A rigorous ODE solver and Smale's 14th problem, *Foundations of Computational Mathematics*, Vol. 2, No. 1, pp. 53-117.
- Yu, X. (1996), Controlling Lorenz chaos, *International Journal of Systems Science*, Vol. 27, No. 4, pp. 355-359.

About the authors:

Mihaela Simionescu (mihaela_mb1(at)yahoo.com) is a Senior Researcher at the Institute for Economic Forecasting of the Romanian Academy, Bucharest, 050711, Casa Academiei, Calea 13 Septembrie nr.13, sector 5, Romania.

Marek Śmiechowski (mareksmiechowski(at)poczta.onet.pl) is a Ph.D. student and a Researcher at the Statistical Office in Rzeszow, ul. Jana III Sobieskiego 10, 35-959 Rzeszow, Poland.