Pandiangan, Naomi; Sukono; Hasbullah, Endang Soeryana

**Article**

# Quadratic investment portfolio based on value-at-risk with risk-free assets : for stocks of the mining and energy sector

International Journal of Energy Economics and Policy

**Provided in Cooperation with:**
International Journal of Energy Economics and Policy (IJEEP)

This Version is available at:
http://hdl.handle.net/11159/7766

# Quadratic Investment Portfolio Based on Value-at-risk with Risk-Free Assets: For Stocks of the Mining and Energy Sector

**Naomi Pandiangan[1], Sukono[2]\*, Endang Soeryana Hasbullah[2]**

[1]Master's Program in Mathematics, Universitas Padjadjaran, Bandung, Indonesia, [2]Department of Mathematics, Universitas Padjadjaran, Bandung 40132, Indonesia . *Email: sukono@unpad.ac.id

**ABSTRACT**

The mining and energy sector is still the driving force for economic development and community empowerment, especially around mining and energy activities. Therefore, increased investment in the mining and energy sectors needs to be increased and balanced with stricter safety and environmental policies. This paper aims to formulate a quadratic investment portfolio optimization model, and apply it to several stocks in the mining and energy sectors. In this paper, it is assumed that risk is measured using Value-at-Risk (VaR), so that the optimization modeling is carried out using the quadratic investment portfolio approach to the Mean-VaR model with risk-free assets. Furthermore, the model is used to determine the efficient portfolio surface based on several values of risk aversion levels. Based on the results of the analysis, it is found that an efficient portfolio surface has a minimum portfolio return value with an average of 0.766522 and a VaR risk of 0.038687. In addition, the results of the analysis can be concluded that the greater the level of risk aversion, the smaller the VaR value, which is followed by the smaller the portfolio average value.

**Keywords:** Mining and Energy Sector, Risk Free Assets, Investment, Value-at-Risk, Portfolio Optimization
**JEL Classifications:** A12, C61, G11, Q48.

## 1. INTRODUCTION

The mining and energy sectors continue to play an important role in national economic growth. The mining and energy sectors can also be the driving force of economic development and community empowerment, especially around mining and energy activity areas (Devi and Prayogo, 2013; Priyarsono et al., 2012). Positive information related to activities must be conveyed to the community so that activities are not always synonymous with natural damage (Deller and Schreiber, 2012). The current difficult situation has encouraged all stakeholders to innovate and formulate new strategies in mining and energy management that are in line with the economic development steps undertaken by the government (Devi and Prayogo, 2013).

One of the steps for the development of the Indonesian economy is by increasing investment in the mining and energy sectors.

Increased investment is carried out through the down streaming of mining and energy commodities. In the future, the more companies that carry out downstream mining and energy commodities, the more mining and energy products can be enjoyed by the wider community (Elinu et al., 2010). The government has provided maximum incentives for companies that carry out mining and energy down streaming. It is hoped that both fiscal and non-fiscal incentives will attract investors to be able to build downstream infrastructure in Indonesia (Patricia et al., 2016; Kitula, 2005).

This increased investment in the mining and energy sectors must be balanced with the development of stricter safety and environmental policies. For example, the obligation to carry out reclamation and post-mining activities with a 100% success rate, showcasing innovation, performance and company achievements, including economic development programs and community empowerment around the mining area. This program is to show the public that the

current economic conditions, the mining and energy industries still have an important role in efforts to restore the national economy (Adiatma et al., 2019; Er et al., 2018).

Investments in mining and energy exploration in Indonesia need to be a priority and continue to be spurred on to maintain the level of reserves as raw material for future industrial development, including down streaming. If the current economic condition does not change, it is feared that the level of mining and energy reserves in Indonesia will quickly run out. In fact, the discovery of new reserves is needed to support the long-term mining and energy industries. The failure of these activities can be caused by several factors, ranging from commodity prices to unsupportive regulations. Regulations that do not help investors on the exploration side are for example auction prices that are too high. Companies that want exploration have to pay a high price and it is not clear whether they can get it or not (Taušová et al., 2017; Lawal et al., 2020).

The main key to increasing investment, particularly exploration, is in policy. Policies for the mining and energy sectors should not be too rigid because of the different characteristics of each commodity. For example, gold, which is relatively stable, cannot be equated with a highly volatile nickel policy. Investment has indeed increased, but not significantly. Several activities in the mining and energy sectors are relatively stagnant. Investments in mining and new energy are almost non-existent (Gurrib et al., 2020). It has been a long time since we discovered world class mining and energy. Investment for mining exploration in Indonesia is still in the range of 1.5% of world exploration costs. This portion is very small compared to Indonesia's role as one of the main exporters of mining minerals in the world. Meanwhile, investment in the mining and energy sectors is dominated by smelters. Indonesia must be able to attract investment, especially in the exploration sector. All stakeholders must always direct so that there are more attractive regulations to investors (Warburton, 2017).

Exploration activities carry a very high risk with a success rate of below 10%, depending on the location. In addition, the time required can reach 10 years before the company can increase its activities to the production operation stage. A number of companies have explored and failed because their reserves are not economical. In fact, the funds that have been spent can reach tens of millions of U.S. dollars. Mining and energy are long-term industries that need investors who are willing to have a long-term commitment. It is up to investors whether they are State-Owned Enterprises (BUMN) or private. As long as the rules are attractive, they will definitely want to enter (Adiatma et al., 2019).

To catch up with potential investors, the Coordinating Ministry for the Economy has formed a task force, one of which is to take care of deregulation. Deregulation is meant to simplify the rules to stimulate the arrival of investors. This team is also tasked with solving licensing problems between investors and the government. Even though the government is rarely doing deregulation, investor's confidence will not increase if many technical problems are not resolved. By solving various cases in the field, it is hoped

that investors' perceptions will change so that they are stimulated to invest. This can provide a positive signal for improving the business climate (Adiatma et al., 2019).

This year, the Investment Coordinating Board (BKPM) is targeting realized investment to grow by 14.4% from the 2015 target or up to IDR 594.8 trillion. This realization came from IDR 386.4 trillion Foreign Investment (PMA), up to 12.6% from last year, as well as from IDR 208.4 trillion Domestic Investors (PMDN), with an increase of 18.4% from the previous year's target. To achieve this target, BKPM set ten prioritized countries, including the United States, Australia, Singapore, Japan, South Korea, Taiwan, China, Malaysia, and the United Kingdom. The United States is a priority country with 2015 investment realization amounting to US $ 893 million for 261 projects (Adiatma et al., 2019; Patricia et al., 2016).

The popularity of stocks as an investment instrument has increased recently. One proof of this is that the number of stock investors reached 1.1 million in 2019, with an increase of 30% compared to 852,000 people in 2018. Shares are often considered an investment instrument that can generate relatively high returns. The advantages of investing in stocks often make investors tempted to start becoming stock investors. However, just like investing in general, the high potential returns from investing in stocks are also accompanied by high risks (Bünyamin et al., 2018). Therefore, stock investment is often referred to as a high-risk high-return instrument. This stock investment risk is something that is inherent or inseparable from stock investment activities (Artemkina et al., 2019). An investment plan should not only think about the benefits, but also the risks that come with it. The risk of investing in stocks needs to be known by investors as one of the considerations before making an investment decision. Without knowledge of this risk, stock investment can lead to disappointment, anger, and regret (Buberkoku, 2019; Alexander et al., 2006).

To minimize the risk, investors need to arrange portfolios or rebalance their investment portfolios. An investment portfolio is a collection of investment instruments owned by an investor or a group of investors (Ahmadi and Sitdhirasdr, 2016; Fachrudin and Fachrudin, 2015). Portfolios are created as a strategy to maximize the level of return in investing and to minimize risk. In compiling an investment portfolio, an investor has the opportunity to diversify. In an investment portfolio, there can be another portfolio in it (Bansal et al., 2014; Golafshani and Emamipoor, 2015). In a stock investment portfolio, investors can fill it with several types of stocks. If investors only put their investment funds in one share, for example, the potential risk to the investment funds will be high. Because if one share experiences a price decline, all the funds will in effect decrease (Baweja and Saxena, 2015; Hult et al., 2012; Strassberger, 2006). However, if the fund bought a number of shares or formed a portfolio,

it may be that not all of the shares in his portfolio will experience a price decline. Of course, the portfolio chosen by the investor is a portfolio that matches the preferences of the investor concerned with the return and risk that they can bear (Cochrane, 2014; Soeryana et al., 2017.a; 2017.b).

In economics and finance, the risk of loss can be measured using Value-at-Risk (VaR). VaR is the maximum loss that will not be passed for a probability which is defined as the level of confidence (confidence level) during a certain period of time (Boudt et al., 2013; Gambrah and Pirvu, 2014). VaR is usually used by securities institutions or investment banks to measure the market risk of their portfolio of assets, even though VaR is actually a general concept that can be applied to various things. VaR is widely applied in finance to quantitative risk management for various types of risk, including investment risk in the mining and energy sectors (Goh et al., 2011; Hooda and Stehlík, 2011).

Several investment portfolio optimization models involving a measure of VaR risk have been developed by previous researchers, including Gaivoronski and Pflug (2005), stating that Value-at-Risk (VaR) is an important measure and is widely used to determine the extent to which a particular portfolio is affected by the risk involved, inherent in financial markets. In this research, the aim is to present a portfolio calculation method that provides the smallest VaR, which can produce the expected return. Using this approach, the efficient Mean-VaR limit can be calculated. The analysis results show that the resulting efficient limits are quite different. An investor, who wishes to control his VaR, should not see a portfolio that lies outside the bounds of efficient VaR. Similar research has also been conducted by Plunus et al. (2015) and Ogryczak et al. (2015).

Sukono et al. (2017.a), discuss Mean-VaR portfolio optimization modeling with risk tolerance, for quadratic utility functions. In this study, it is assumed that the return on assets has a certain distribution, and portfolio risk is measured using Value-at-Risk (VaR). Therefore, the portfolio optimization process is carried out using the Mean-VaR model, and is carried out using the Lagrange multiplier method, as well as the Khun-Tucker method. The result of portfolio optimization modeling is a weight vector equation that depends on the mean vector return asset vector, the identity vector, and the matrix covariance between asset returns, as well as the risk tolerance factor.

Hashemi et al. (2016), conducted a research with the aim of evaluating various measurement tools to improve portfolio performance and asset selection using the Mean-VaR model. This paper focuses on the portfolio optimization process where the variance is replaced by risk (VaR). Furthermore, it is applied numerically with historical simulation techniques and Monte Carlo to calculate the risk value and determine the efficiency surface. The results of the analysis show that at first glance variance is a measure of risk. But in fact, both theory and practice show that variance is not a good measure of risk and has many weaknesses.

Sukono et al. (2017.b), discusses the issue of quadratic investment portfolios without risk-free assets based on Value-at-Risk. The aim is to formulate a model of maximizing portfolio return expectations and minimizing Value-at-Risk. They assume that investment portfolio risk is measured by Value-at-Risk. In his research, a quadratic investment portfolio weight vector determination model has been formulated without risk-free assets, and it has been applied to several stocks to obtain the optimum weight

composition. Based on the results of the analysis, it is concluded that the expected return of the portfolio does not only depend on the type of investor but also on the size of the investment and the risks faced.

Based on the description above, this paper intends to formulate a quadratic investment portfolio model based on Value-at-Risk (VaR) with risk-free assets, which is applied to analyze investments in stocks in the mining and energy sectors. The aim is to form an efficient surface curve of the investment portfolio, which is carried out based on data from 11 mining and energy sector stocks. The research conducted here is seen as a difference compared to the results of some of the studies that have been done above. Researches conducted by Gaivoronski and Pflug (2005) and Hashemi et al. (2016) have shown how important it is to use the VaR model for measuring financial risk, especially in investment portfolio analysis. Furthermore, in a research conducted by Sukono et al. (2017.a; 2017.b) the Mean-VaR portfolio optimization model has been formulated, but the investment portfolio analyzed does not involve risk-free assets. Besides that, the investor preferences are analyzed based on risk tolerance. In this research, in addition to risk being measured using VaR, the investment analysis carried out involves risk-free assets, and investor preferences are based on risk aversion. Thus, the model formulated in this study becomes one of the important alternatives that can be used in the analysis of investment portfolio optimization, especially investment in the mining and energy sectors.

## 2. MATERIALS AND METHODS

### 2.1. Materials
The material for modeling the quadratic investment portfolio optimization Mean-VaR with risk-free assets refers to the research papers conducted by Gaivoronski and Pflug (2005), Sukono et al. (2017.a), Hashemi et al. (2016), and Sukono et al. (2017.b). Furthermore, the data analyzed consists of 11 selected mining and energy sector stocks, which include the prices of shares: BSSR, BYAN, CITA, HRUM, MBAP, MDKA, MEDC, PSAB, PTBA, PTRO, and RUIS. The share price data is the monthly transaction value from the January 2017-November 2020 period, traded on the Indonesia Stock Exchange (IDX), which is accessed through the website: www.yahoofinance.com. Risk-free assets in this study are in the form of bank deposits with interest rates in accordance with Bank Indonesia regulations (Sukono et al., 2017.c). Data analysis was performed with the help of MS Excel 2010 and Matlab 7.0 software.

### 2.2. Methods
The quadratic investment portfolio optimization modeling Mean-VaR with risk-free assets is a model development from a research conducted by Sukono et al. (2017.b). Quadratic investment portfolio optimization is carried out using the Lagrange Multiplier technique and is based on the Khun-Tucker theorem. Furthermore, the model is used to analyze the investment portfolio on the 11 selected mining and energy stocks. In this investment portfolio analysis, it is assumed that individual investors or investment managers of an organization are influenced by the level of risk aversion where the values of this risk aversion level are generated in a simulation.

## 3. INVESTMENT PORTFOLIO MODELING

### 3.1. Investment Portfolio

Suppose $t_0 = 0$ time to start investing, and $t_1 = 1$ end time investing. Suppose an investor in making an investment forms a portfolio with an expected wealth value $E[V_1]$ at time $t_1 = 1$ is great. Because the value of wealth $V_1$ is fluctuating, it means that the risk (variance) of $Var[V_1]$ expected is minimal. Suppose $V_0$ the amount of the initial investment, and it is assumed that the portfolio consists of $n$ asset, where $n \geq 2$ a risky asset at a spot price $S_t^k$, where $t = 0$, 1 and $k = 1,\ldots,n$. Suppose that $S_0^k$ is known, and $S_1^k$ also allows ownership of bonds without coupons (risk free) with value $B_0$ at time $t = 0$, and is worth one unit at a time $t = 1$ (Sukono et al., 2017.b; 2018.a).

The ownership of risky assets represented by vectors $\mathbf{h} = (h_1,\ldots,h_n)^T \in \mathfrak{R}^n$, $h_k$ is the amount of wealth allocated to assets of $k$ during a certain period by the investor. Suppose $h_0$ indicates risk-free bond ownership, market value at time $t = 0$ and $t = 1$. The portfolios formed are:

$$h_0 B_0 + \sum_{k=1}^{n} h_k S_0^k \leq V_0 \text{ and } V_1 = h_0 + \sum_{k=1}^{n} h_k S_1^k .$$

If the investment does not have risk-free assets available, it means the amount of value is $h_0 = 0$. Next, take the initial value of the holdings in assets to $k$ is $w_k = h_k S_0^k$ and $w_0 = h_0 B_0$. Suppose that the investment portfolio weight, so that the current and future portfolio values can be expressed as:

$$w_0 + \sum_{k=1}^{n} w_k \leq V_0 \text{ and } w_0 \frac{1}{B_0} + \sum_{k=1}^{n} w_k \frac{S_1^k}{S_0^k} . \quad (1)$$

This can be seen from the determination of the optimal initial capital allocation $V_0$ that needs to know the expected value of $\mu$ and the variance-covariance matrix of $\mathbf{\pounds}$ and vector of $\mathbf{R}$, where (Sukono et al., 2017.b; 2018.b):

$$\mathbf{R}^T = \left( \frac{S_1^k}{S_0^k}, \ldots, \frac{S_1^n}{S_0^n} \right), \quad (2)$$

with $R_0 = 1/B_0$ and $\mathbf{w}^T = (w_1,\ldots,w_n)^T$, so it can be stated that $V_1 = w_0 R_0 + \mathbf{w}^T \mathbf{R}$, and therefore:

$$E[V_1] = w_0 R_0 + \mathbf{w}^T \mu, \quad (3)$$

and

$$Var[V_1] = \mathbf{w}^T \sum \mathbf{w}. \quad (4)$$

It is assumed that the variance-covariance matrix of $\sum = Cov(\mathbf{R}) = E[(\mathbf{R} - \mu)(\mathbf{R} - \mu)^T]$ is positive-definite, or $\mathbf{w}^T \sum \mathbf{w} > 0$ for all $\mathbf{w} \neq 0$. By definition, each variance-covariance matrix is symmetrical as well as positive-semidefinite, for all $\mathbf{w} \neq 0$, $\mathbf{w}^T \sum \mathbf{w} = Var(\mathbf{w}^T \mathbf{R}) \geq 0$. Therefore, let us just say $\sum$ is positive, it is definitely equivalent to an assumption that $\sum$ has an inverse, or it has the equivalent that all of its eigenvalues are $\sum > 0$ (Sukono et al., 2019; Ogryczak and Sliwinski, 2010).

### 3.2. Modeling of Mean-VaR Quadratic Investment Portfolio Optimization with Risk-free Assets

This section discusses the Mean-VaR quadratic investment portfolio optimization model with risk-free assets. It is assumed that investment portfolio risk is measured using Value-at-Risk (VaR). According to Lwin et al. (2017), the Value-at-Risk risk measurement model for portfolios is formulated as:

$$VaR_p = -V_0 \{\mu_p + z_\alpha \sigma_p\}.$$

Referring to equations (3) and (4), the Value-at-Risk for the portfolio can be expressed as:

$$VaR_p = -V_0 \{\mathbf{w}^T \mu + z_\alpha (\mathbf{w}^T \sum \mathbf{w})^{1/2}\}, \quad (5)$$

The sign (–) indicates a loss, $V_0$ indicates the initial wealth invested, and $z_\alpha$ indicates the percentile of the standard normal distribution when a significance level is determined $(1-\alpha)\%$ (Mustafa et al., 2015).

So, the objective function of the investment portfolio model is to maximize $\{\mu_p - \rho VaR_p\}$ or maximize $\left\{ w_0^T \mu_0 + \mathbf{w}^T \frac{1}{4} + \frac{c}{2V_0} V_0 \left\{ \mathbf{w}^T \frac{1}{4} + z_\alpha \left( \mathbf{w}^T \pounds \mathbf{w} \right)^{\frac{1}{2}} \right\} \right\}$, with $\rho = \frac{c}{2V_0}$, $w_0$ investment weights for risk-free assets, and the average return on risk-free assets (Pinasthika and Surya, 2014; Ghaemi et al., 2009). Therefore, the investment portfolio model can be expressed as:

$$\text{Maximize} \left\{ w_0^T \mu_0 + \left(1 + \frac{c}{2}\right) \mathbf{w}^T \mu + \frac{c}{2} z_\alpha \left( \mathbf{w}^T \sum \mathbf{w} \right)^{\frac{1}{2}} \right\} \quad (6)$$

Subject to $w_0 + \mathbf{w}^T \mathbf{I} \leq V_o$,

with $\mathbf{I}$ as a unit vector, and c a risk aversion level obtained in the following manner. Assume there are two investors with R and $\tilde{R}$, with $V_0$ as the initial wealth, then the value of the risk aversion level c can be determined by the following equation:

$$E[V_0 R] - \frac{c}{2V_0} VaR[V_0 R] = E[V_0 \tilde{R}] - \frac{c}{2V_0} VaR[V_0 \tilde{R}]$$

Thus, the value of c can be determined by:

$$c = \frac{2\{E[V_0 R] - E[V_0 \tilde{R}]\}}{\{VaR[V_0 R] - VaR[V_0 \tilde{R}]\}} \quad (7)$$

Furthermore, to find the solution of equation (6), the Lagrang Multiplier method is used. The Lagrange multiplier equation in equation (6) can be expressed as (Ghami et al., 2009):

$$L(\mathbf{w}, \lambda) = w_0^T \mu_0 + \left(1 + \frac{c}{2}\right) \mathbf{w}^T \mu + \frac{c}{2} z_\alpha \left( \mathbf{w}^T \sum \mathbf{w} \right)^{\frac{1}{2}}$$
$$+ \lambda \left( w_0 + \mathbf{w}^T \mathbf{I} - V_0 \right) \quad (8)$$

Based on the Khun-Tucker theorem, the necessary conditions for optimization in equation (8) can be done with the first derivative:

$$\frac{\partial L}{\partial \mathbf{w}} = \left(1 + \frac{c}{2}\right) \mu + \frac{c z_\alpha \sum \mathbf{w}}{2 \left( \mathbf{w}^T \sum \mathbf{w} \right)^{\frac{1}{2}}} + \lambda \mathbf{I} = 0 \quad (9)$$

$$\frac{\partial L}{\partial \lambda} = w_0 + \mathbf{w}^T \mathbf{I} - V_0 = 0 \, .$$

Based on equation (9) it can be expressed as:

$$\frac{c z_\alpha \sum \mathbf{w}}{2 \left( \mathbf{w}^T \sum \mathbf{w} \right)^{\frac{1}{2}}} = - \left\{ \left( 1 + \frac{c}{2} \right) \mu + \lambda \mathbf{I} \right\} \tag{10}$$

If equation (10) is multiplied by $2\mathbf{\Sigma}^{-1})/c z^\alpha$ then we get the equation:

$$\frac{\mathbf{w}}{\left( \mathbf{w}^T \sum \mathbf{w} \right)^{\frac{1}{2}}} = - \frac{\left( 1 + \frac{c}{2} \right) \mathbf{\Sigma}^{-1} \mu + \lambda \, \mathbf{\Sigma}^{-1} \mathbf{I}}{\frac{c}{2} z_\alpha} \tag{11}$$

Equation (11) if multiplied by $\mathbf{I}^T$ gets the results:

$$\frac{V_0 - w_0}{\left( \mathbf{w}^T \sum \mathbf{w} \right)^{\frac{1}{2}}} = - \frac{\left\{ \left( 1 + \frac{c}{2} \right) \mathbf{I}^T \, \mathbf{\Sigma}^{-1} \mu + \lambda \mathbf{I}^T \, \mathbf{\Sigma}^{-1} \mathbf{I} \right\}}{\frac{c}{2} z_\alpha}$$

or

$$\left( \mathbf{w}^T \sum \mathbf{w} \right)^{\frac{1}{2}} = - \frac{\frac{c (V_0 - w_0)}{2} z_\alpha}{\left\{ \left( 1 + \frac{c}{2} \right) \mathbf{I}^T \, \mathbf{\Sigma}^{-1} \mu + \lambda \mathbf{I}^T \, \mathbf{\Sigma}^{-1} \mathbf{I} \right\}} \tag{12}$$

Equation (12) is substituted into equation (11), and solving the substitution result will get the weight vector **w** as follows:

$$\mathbf{w} = \frac{(V_0 - \mathbf{w}_0) \left\{ \left( 1 + \frac{c}{2} \right) \mathbf{\Sigma}^{-1} \mu + \lambda \, \mathbf{\Sigma}^{-1} \mathbf{I} \right\}}{\left\{ \left( 1 + \frac{c}{2} \right) \mathbf{I}^T \, \mathbf{\Sigma}^{-1} \mu + \lambda \mathbf{I}^T \, \mathbf{\Sigma}^{-1} \mathbf{I} \right\}} \tag{13}$$

Equation (11) if multiplied by $\mathbf{w}^T \mathbf{\Sigma}$ get the equation:

$$\frac{c}{2} z_\alpha \left( \mathbf{w}^T \sum \mathbf{w} \right)^{\frac{1}{2}} = - \left\{ \left( 1 + \frac{c}{2} \right) \mu^T \mathbf{w} + \lambda (V_0 - w_0) \right\} \tag{14}$$

Equations (12) and (13) are substituted into equation (14), and solving the substitution results will get the following equation:

$$\left\{ (V_0 - w_0) \left( \mathbf{I}^T \, \mathbf{\Sigma}^{-1} \mathbf{I} \right) \right\} \lambda^2 + \left\{ (V_0 - w_0) \left( 1 + \frac{c}{2} \right) \right.$$

$$\left( \mathbf{I}^T \, \mathbf{\Sigma}^{-1} \mu + \mu^T \mathbf{\Sigma}^{-1} \mathbf{I} \right) \right\} \lambda$$

$$+ \left\{ (V_0 - w_0) \left( 1 + \frac{c}{2} \right)^2 \mu^T \mathbf{\Sigma}^{-1} \mu \right\} = 0 \tag{15}$$

Equation (15) is a quadratic equation, so the value can be calculated by the ABC formula as follows:

$$\lambda_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ad}}{2a}, \lambda > 0 \tag{16}$$

with

$$a = \left\{ (V_0 - w_0) \left( \mathbf{I}^T \, \mathbf{\Sigma}^{-1} \mathbf{I} \right) \right\}$$

$$b = \left\{ (V_0 - w_0) \left( 1 + \frac{c}{2} \right) \left( \mathbf{I}^T \, \mathbf{\Sigma}^{-1} \mu + \mu^T \, \mathbf{\Sigma}^{-1} \mathbf{I} \right) \right\}; \text{ and}$$

$$d = \left\{ (V_0 - w_0) \left[ \left( 1 + \frac{c}{2} \right)^2 \mu^T \mathbf{\Sigma}^{-1} \mu - \left( \frac{c z_\alpha}{2} \right)^2 \right] \right\}.$$

## 4. DATA ANALYSIS

### 4.1. Descriptive Statistic
Referring to the discussion of materials in section 2.1, the data analyzed consists of 11 selected mining and energy sector stocks, which include stock prices: BSSR, BYAN, CITA, HRUM, MBAP, MDKA, MEDC, PSAB, PTBA, PTRO, and RUIS. Furthermore, the price of these shares is determined by the return value of each share based on the principle of calculating stock returns as given in equation (2). The calculation of stock returns is carried out with the help of MS Excel 10 software, and the stock returns are determined by descriptive statistical values which include: mean, variance, and standard deviation. The results of the descriptive statistical calculations are given in Table 1.

Looking at the descriptive statistical values given in Table 1, it appears that the 11 mining and energy stocks analyzed have different mean and variance values. The smallest average return value is owned by PSAB stock which is 0.013523 with a variance of 0.026971, and the largest return average is owned by PTBA stock, which is 0.085473 with a variance of 0.350388. It appears that stocks that have a small average return are followed by a small variance (risk); on the other hand, stocks that have a large average return are followed by a large variance (risk). This shows that in investing in financial assets, an asset that promises a greater return will be followed by a greater risk that investors must face.

### 4.2. Portfolio Optimization Process
Furthermore, the average return values from Table 1 is used to form the mean return vector $\mu^T$ as follows:

$\mu^T$=[0.0199 0.0314 0.0374 0.0214 0.0272 0.0469 0.0353 0.0135 0.0854 0.0340 0.0135], and because it analyzed 11 stocks, the unit vector of the elements consist of a number of 11 as given $\mathbf{I}^T$ ==[1 1 1 1 1 1 1 1 1 1 1] (Kalfin et al., 2019). The variance values of stock returns, together with the covariance values between stock returns are used to form the variance-covariance matrix $\mathbf{\Sigma}$ as follows:

$$\Sigma = \begin{bmatrix}
0{,}0185 & 0{,}0027 & -0{,}0002 & 0{,}0025 & 0{,}0061 & 0{,}0030 & -0{,}0002 & -0{,}0009 & -0{,}0053 & 0{,}0029 & 0{,}0011 \\
0{,}0027 & 0{,}0242 & 0{,}0054 & 0{,}0033 & 0{,}0127 & 0{,}0001 & 0{,}0017 & -0{,}0005 & 0{,}0031 & 0{,}0031 & -0{,}0017 \\
-0{,}0002 & 0{,}0054 & 0{,}0288 & -0{,}0067 & -0{,}0003 & 0{,}0098 & -0{,}0073 & 0{,}0001 & 0{,}0015 & -0{,}0059 & -0{,}0034 \\
0{,}0025 & 0{,}0033 & -0{,}0067 & 0{,}0283 & 0{,}0064 & -0{,}0013 & 0{,}0142 & 0{,}0052 & -0{,}0021 & 0{,}0095 & 0{,}0054 \\
0{,}0061 & 0{,}0127 & -0{,}00003 & 0{,}0064 & 0{,}0199 & 0{,}0001 & 0{,}0044 & -0{,}0014 & -0{,}0075 & 0{,}0058 & 0{,}0021 \\
0{,}0030 & 0{,}0001 & 0{,}0098 & -0{,}0013 & 0{,}0001 & 0{,}0140 & 0{,}0043 & 0{,}0051 & -0{,}0003 & 0{,}0031 & -0{,}0011 \\
-0{,}0002 & 0{,}0017 & -0{,}0073 & 0{,}0142 & 0{,}0044 & 0{,}0043 & 0{,}0486 & 0{,}0031 & 0{,}0246 & 0{,}0158 & 0{,}0052 \\
-0{,}0009 & -0{,}0005 & 0{,}0001 & 0{,}0052 & -0{,}0014 & 0{,}0051 & 0{,}0031 & 0{,}0269 & -0{,}0033 & 0{,}0103 & -0{,}0006 \\
-0{,}0053 & 0{,}0031 & 0{,}0015 & -0{,}0021 & -0{,}0075 & -0{,}0003 & 0{,}0246 & -0{,}0033 & 0{,}3503 & 0{,}0333 & -0{,}0003 \\
0{,}0029 & 0{,}0031 & -0{,}0059 & 0{,}0095 & 0{,}0058 & 0{,}0031 & 0{,}0158 & 0{,}0103 & 0{,}0333 & 0{,}0238 & 0{,}0002 \\
0{,}0011 & -0{,}0017 & -0{,}0034 & 0{,}0054 & 0{,}0021 & -0{,}0011 & 0{,}0052 & -0{,}0006 & -0{,}0003 & 0{,}0002 & 0{,}0184
\end{bmatrix}$$

and the inverse matrix of $\Sigma$ that is $\Sigma^{-1}$ is as follows:

$$\Sigma^{-1} = \begin{bmatrix}
66{,}424 & 1{,}131 & 7{,}945 & -3{,}951 & -18{,}306 & -24{,}150 & 8{,}150 & 8{,}466 & 0{,}706 & -7{,}001 & -2{,}562 \\
1{,}347 & 71{,}314 & -17{,}776 & -2{,}127 & -47{,}850 & 14{,}448 & -2{,}242 & -3{,}893 & -1{,}625 & 2{,}031 & 10{,}666 \\
7{,}591 & -18{,}733 & 64{,}285 & 1{,}470 & 1{,}027 & -53{,}761 & 8{,}622 & -0{,}023 & -2{,}703 & 21{,}869 & 3{,}189 \\
-3{,}913 & -2{,}264 & 1{,}862 & 51{,}207 & -6{,}967 & 12{,}161 & -10{,}755 & -7{,}175 & 1{,}899 & -11{,}498 & -10{,}151 \\
-18{,}557 & -47{,}259 & -1{,}052 & -7{,}022 & 100{,}792 & 2{,}643 & 0{,}963 & 15{,}844 & 5{,}043 & -28{,}441 & -12{,}037 \\
-23{,}850 & 15{,}239 & -53{,}729 & 12{,}490 & 0{,}904 & 131{,}562 & -18{,}533 & -17{,}863 & 2{,}916 & -19{,}282 & 2{,}022 \\
8{,}094 & -2{,}348 & 8{,}545 & -10{,}810 & 1{,}240 & -18{,}473 & 32{,}267 & 7{,}978 & -0{,}626 & -15{,}961 & -5{,}862 \\
8{,}424 & -3{,}789 & -0{,}380 & -7{,}183 & 15{,}838 & -17{,}564 & 7{,}930 & 55{,}176 & 3{,}597 & -33{,}392 & -1{,}447 \\
0{,}710 & -1{,}558 & -2{,}795 & 1{,}914 & 4{,}954 & 2{,}994 & -0{,}641 & 3{,}582 & 4{,}010 & -9{,}664 & -1{,}140 \\
-7{,}061 & 1{,}557 & 22{,}379 & -11{,}619 & -27{,}725 & -19{,}720 & -15{,}866 & -33{,}271 & -9{,}654 & 100{,}392 & 11{,}901 \\
-2{,}557 & 10{,}562 & 3{,}382 & -10{,}166 & -11{,}931 & 1{,}859 & -5{,}832 & -1{,}429 & -1{,}143 & 11{,}941 & 61{,}945
\end{bmatrix}$$

**Table 1: Descriptive statistics of stock return data**

| No | Stock code | Averages | Variance | Standard Deviation |
|----|-----------|----------|----------|--------------------|
| 1. | BSSR | 0.019968 | 0.018552 | 0.136204876 |
| 2. | BYAN | 0.031440 | 0.024222 | 0.155634921 |
| 3. | CITA | 0.037400 | 0.028834 | 0.169805321 |
| 4. | HRUM | 0.021423 | 0.028336 | 0.168332973 |
| 5. | MBAP | 0.027229 | 0.019945 | 0.141225841 |
| 6. | MDKA | 0.046926 | 0.014093 | 0.118712360 |
| 7. | MEDC | 0.035373 | 0.048696 | 0.220671576 |
| 8. | PSAB | 0.013523 | 0.026971 | 0.164229531 |
| 9. | PTBA | 0.085473 | 0.350388 | 0.591935497 |
| 10. | PTRO | 0.034024 | 0.023890 | 0.154565332 |
| 11. | RUIS | 0.013529 | 0.018422 | 0.135728504 |

Vectors of $\mu^T$ and $I^T$, as well as the matrix $\Sigma^{-1}$ are then collectively used to determine the optimum weight of the investment portfolio. In this study, the risk-free assets used are bank deposits with an average return = 0.07 in accordance with the interest rate determined by Bank Indonesia, and it is assumed that the capital allocated to risk-free assets is 50% or $w_0 = 0.5$. Determination of the optimum weight was carried out by referring to equation (13), and was carried out with the help of Matlab 7.0 software (Napitupulu et al., 2018). In equation (13), the magnitude of the value is calculated using equation (16), and the values of c are generated simulated from the initial value of 1.9 with an increase of 0.05. The results of the investment portfolio optimization process based on the

Mean-Vary model with risk-free assets in equation (6) are summarized and presented in Table 2.

Taking into account of Table 2, it can be explained that the value of taken starts from 1.9 due to value $c < 1.9$ of generate investment portfolio weights , and with $i = 1,\ldots,11$, there is a negative value. This indicates a short sale (selling shares that are not his own). If it is assumed that short sales are not allowed, then the negative investment portfolio weights do not need to be analyzed again (Qin, 2015). In this research, the value of taken is given in intervals $1.9 \leq c \leq 5.00$, with an increase of 0.05. The optimization process resulted in the composition of the investment portfolio weights on 11 stocks with different values, resulting in the large average value of the portfolio returns $\mu_p$ and Value-at-Risk portfolios (VaR$_p$) to be different too, as shown in Table 2 column and column VaR$_p$.

## 5. DISCUSSION

The discussion in this section is more related to preferences which are described by the level of risk aversion of each investor, which in this study it is assumed that the investor concerned invests in 11 stocks in the mining and energy sectors. In this case the risk aversion level c is depicted from that lying in the interval of $1.9 \leq c \leq 5.00$. Based on the level of risk aversion in the interval of $1.9 \leq c \leq 5.00$ in increments of 0.05, and using the values given in Table 2 column $\mu_p$ and column VaR$_p$, a graph

**Table 2: Summary of the results of the investment portfolio optimization process**

| c | $w_1$ | $w_2$ | $w_3$ | $w_4$ | $w_5$ | $w_6$ | $w_7$ | $w_8$ | $w_9$ | $w_{10}$ | $w_{11}$ | $\sum w_i$ | $\mu_p$ | $VaR_p$ | $\mu_p/VaR_p$ |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.90 | 0.038 | 0.058 | 0.044 | 0.036 | 0.025 | 0.161 | 0.000 | 0.010 | 0.010 | 0.033 | 0.086 | 1.000 | 0.766522 | 0.038687 | 19.813191 |
| 1.95 | 0.038 | 0.058 | 0.044 | 0.036 | 0.025 | 0.160 | 0.000 | 0.010 | 0.010 | 0.033 | 0.086 | 1.000 | 0.766473 | 0.038637 | 19.837999 |
| 2.00 | 0.039 | 0.058 | 0.044 | 0.036 | 0.025 | 0.158 | 0.000 | 0.011 | 0.010 | 0.033 | 0.087 | 1.000 | 0.766427 | 0.038590 | 19.860876 |
| 2.05 | 0.039 | 0.057 | 0.045 | 0.035 | 0.025 | 0.157 | 0.000 | 0.012 | 0.010 | 0.033 | 0.087 | 1.000 | 0.766383 | 0.038547 | 19.882017 |
| 2.10 | 0.040 | 0.057 | 0.045 | 0.035 | 0.025 | 0.156 | 0.000 | 0.012 | 0.010 | 0.033 | 0.087 | 1.000 | 0.766341 | 0.038507 | 19.901591 |
| 2.15 | 0.040 | 0.057 | 0.045 | 0.035 | 0.025 | 0.154 | 0.001 | 0.013 | 0.010 | 0.032 | 0.088 | 1.000 | 0.766302 | 0.038469 | 19.919748 |
| 2.20 | 0.040 | 0.057 | 0.045 | 0.035 | 0.025 | 0.153 | 0.001 | 0.013 | 0.010 | 0.032 | 0.088 | 1.000 | 0.766264 | 0.038435 | 19.936621 |
| 2.25 | 0.041 | 0.057 | 0.046 | 0.035 | 0.025 | 0.152 | 0.001 | 0.014 | 0.010 | 0.032 | 0.088 | 1.000 | 0.766229 | 0.038403 | 19.952328 |
| 2.30 | 0.041 | 0.057 | 0.046 | 0.035 | 0.025 | 0.151 | 0.001 | 0.014 | 0.010 | 0.032 | 0.088 | 1.000 | 0.766195 | 0.038373 | 19.966972 |
| 2.35 | 0.041 | 0.057 | 0.046 | 0.035 | 0.025 | 0.150 | 0.001 | 0.015 | 0.010 | 0.032 | 0.089 | 1.000 | 0.766162 | 0.038345 | 19.980646 |
| 2.40 | 0.042 | 0.057 | 0.046 | 0.035 | 0.025 | 0.149 | 0.001 | 0.015 | 0.010 | 0.032 | 0.089 | 1.000 | 0.766131 | 0.038319 | 19.993434 |
| 2.45 | 0.042 | 0.057 | 0.047 | 0.035 | 0.025 | 0.148 | 0.001 | 0.015 | 0.010 | 0.032 | 0.089 | 1.000 | 0.766102 | 0.038295 | 20.005410 |
| 2.50 | 0.042 | 0.056 | 0.047 | 0.035 | 0.025 | 0.148 | 0.001 | 0.016 | 0.010 | 0.032 | 0.089 | 1.000 | 0.766073 | 0.038272 | 20.016641 |
| 2.55 | 0.043 | 0.056 | 0.047 | 0.035 | 0.025 | 0.147 | 0.001 | 0.016 | 0.010 | 0.032 | 0.090 | 1.000 | 0.766046 | 0.038250 | 20.027186 |
| 2.60 | 0.043 | 0.056 | 0.047 | 0.035 | 0.025 | 0.146 | 0.001 | 0.016 | 0.010 | 0.031 | 0.090 | 1.000 | 0.766020 | 0.038230 | 20.037099 |
| 2.65 | 0.043 | 0.056 | 0.047 | 0.035 | 0.024 | 0.145 | 0.001 | 0.017 | 0.010 | 0.031 | 0.090 | 1.000 | 0.765995 | 0.038211 | 20.046431 |
| 2.70 | 0.044 | 0.056 | 0.047 | 0.035 | 0.024 | 0.145 | 0.001 | 0.017 | 0.010 | 0.031 | 0.090 | 1.000 | 0.765971 | 0.038193 | 20.055224 |
| 2.75 | 0.044 | 0.056 | 0.047 | 0.034 | 0.024 | 0.144 | 0.001 | 0.017 | 0.009 | 0.031 | 0.090 | 1.000 | 0.765948 | 0.038176 | 20.063520 |
| 2.80 | 0.044 | 0.056 | 0.048 | 0.034 | 0.024 | 0.143 | 0.001 | 0.018 | 0.009 | 0.031 | 0.091 | 1.000 | 0.765925 | 0.038160 | 20.071354 |
| 2.85 | 0.044 | 0.056 | 0.048 | 0.034 | 0.024 | 0.143 | 0.002 | 0.018 | 0.009 | 0.031 | 0.091 | 1.000 | 0.765904 | 0.038145 | 20.078760 |
| 2.90 | 0.044 | 0.056 | 0.048 | 0.034 | 0.024 | 0.142 | 0.002 | 0.018 | 0.009 | 0.031 | 0.091 | 1.000 | 0.765883 | 0.038131 | 20.085767 |
| 2.95 | 0.045 | 0.056 | 0.048 | 0.034 | 0.024 | 0.141 | 0.002 | 0.019 | 0.009 | 0.031 | 0.091 | 1.000 | 0.765863 | 0.038117 | 20.092405 |
| 3.00 | 0.045 | 0.056 | 0.048 | 0.034 | 0.024 | 0.141 | 0.002 | 0.019 | 0.009 | 0.031 | 0.091 | 1.000 | 0.765844 | 0.038104 | 20.098698 |
| 3.05 | 0.045 | 0.056 | 0.048 | 0.034 | 0.024 | 0.140 | 0.002 | 0.019 | 0.009 | 0.031 | 0.091 | 1.000 | 0.765826 | 0.038092 | 20.104669 |
| 3.10 | 0.045 | 0.056 | 0.048 | 0.034 | 0.024 | 0.140 | 0.002 | 0.019 | 0.009 | 0.031 | 0.091 | 1.000 | 0.765808 | 0.038080 | 20.110340 |
| 3.15 | 0.045 | 0.056 | 0.049 | 0.034 | 0.024 | 0.139 | 0.002 | 0.020 | 0.009 | 0.031 | 0.092 | 1.000 | 0.765791 | 0.038069 | 20.115730 |
| 3.20 | 0.046 | 0.056 | 0.049 | 0.034 | 0.024 | 0.139 | 0.002 | 0.020 | 0.009 | 0.031 | 0.092 | 1.000 | 0.765774 | 0.038059 | 20.120857 |
| 3.25 | 0.046 | 0.056 | 0.049 | 0.034 | 0.024 | 0.138 | 0.002 | 0.020 | 0.009 | 0.031 | 0.092 | 1.000 | 0.765758 | 0.038049 | 20.125738 |
| 3.30 | 0.046 | 0.055 | 0.049 | 0.034 | 0.024 | 0.138 | 0.002 | 0.020 | 0.009 | 0.031 | 0.092 | 1.000 | 0.765742 | 0.038039 | 20.130388 |
| 3.35 | 0.046 | 0.055 | 0.049 | 0.034 | 0.024 | 0.137 | 0.002 | 0.020 | 0.009 | 0.031 | 0.092 | 1.000 | 0.765727 | 0.038030 | 20.134822 |
| 3.40 | 0.046 | 0.055 | 0.049 | 0.034 | 0.024 | 0.137 | 0.002 | 0.021 | 0.009 | 0.030 | 0.092 | 1.000 | 0.765712 | 0.038021 | 20.139051 |
| 3.45 | 0.046 | 0.055 | 0.049 | 0.034 | 0.024 | 0.136 | 0.002 | 0.021 | 0.009 | 0.030 | 0.092 | 1.000 | 0.765698 | 0.038013 | 20.143090 |
| 3.50 | 0.047 | 0.055 | 0.049 | 0.034 | 0.024 | 0.136 | 0.002 | 0.021 | 0.009 | 0.030 | 0.092 | 1.000 | 0.765684 | 0.038005 | 20.146947 |
| 3.55 | 0.047 | 0.055 | 0.049 | 0.034 | 0.024 | 0.136 | 0.002 | 0.021 | 0.009 | 0.030 | 0.093 | 1.000 | 0.765670 | 0.037997 | 20.150635 |
| 3.60 | 0.047 | 0.055 | 0.049 | 0.034 | 0.024 | 0.135 | 0.002 | 0.021 | 0.009 | 0.030 | 0.093 | 1.000 | 0.765657 | 0.037990 | 20.154163 |
| 3.65 | 0.047 | 0.055 | 0.049 | 0.034 | 0.024 | 0.135 | 0.002 | 0.021 | 0.009 | 0.030 | 0.093 | 1.000 | 0.765645 | 0.037983 | 20.157539 |
| 3.70 | 0.047 | 0.055 | 0.050 | 0.034 | 0.024 | 0.134 | 0.002 | 0.022 | 0.009 | 0.030 | 0.093 | 1.000 | 0.765632 | 0.037976 | 20.160773 |
| 3.75 | 0.047 | 0.055 | 0.050 | 0.034 | 0.024 | 0.134 | 0.002 | 0.022 | 0.009 | 0.030 | 0.093 | 1.000 | 0.765620 | 0.037970 | 20.163872 |
| 3.80 | 0.047 | 0.055 | 0.050 | 0.034 | 0.024 | 0.134 | 0.002 | 0.022 | 0.009 | 0.030 | 0.093 | 1.000 | 0.765609 | 0.037964 | 20.166843 |
| 3.85 | 0.048 | 0.055 | 0.050 | 0.034 | 0.024 | 0.133 | 0.002 | 0.022 | 0.009 | 0.030 | 0.093 | 1.000 | 0.765597 | 0.037958 | 20.169693 |
| 3.90 | 0.048 | 0.055 | 0.050 | 0.034 | 0.024 | 0.133 | 0.002 | 0.022 | 0.009 | 0.030 | 0.093 | 1.000 | 0.765586 | 0.037952 | 20.172428 |
| 3.95 | 0.048 | 0.055 | 0.050 | 0.034 | 0.024 | 0.133 | 0.002 | 0.022 | 0.009 | 0.030 | 0.093 | 1.000 | 0.765575 | 0.037947 | 20.175055 |
| 4.00 | 0.048 | 0.055 | 0.050 | 0.034 | 0.024 | 0.132 | 0.002 | 0.023 | 0.009 | 0.030 | 0.093 | 1.000 | 0.765565 | 0.037941 | 20.177579 |
| 4.05 | 0.048 | 0.055 | 0.050 | 0.034 | 0.024 | 0.132 | 0.002 | 0.023 | 0.009 | 0.030 | 0.094 | 1.000 | 0.765555 | 0.037936 | 20.180005 |
| 4.10 | 0.048 | 0.055 | 0.050 | 0.034 | 0.024 | 0.132 | 0.002 | 0.023 | 0.009 | 0.030 | 0.094 | 1.000 | 0.765545 | 0.037931 | 20.182339 |
| 4.15 | 0.048 | 0.055 | 0.050 | 0.034 | 0.024 | 0.132 | 0.002 | 0.023 | 0.009 | 0.030 | 0.094 | 1.000 | 0.765535 | 0.037927 | 20.184584 |
| 4.20 | 0.048 | 0.055 | 0.050 | 0.034 | 0.024 | 0.131 | 0.002 | 0.023 | 0.009 | 0.030 | 0.094 | 1.000 | 0.765525 | 0.037922 | 20.186745 |
| 4.25 | 0.048 | 0.055 | 0.050 | 0.034 | 0.024 | 0.131 | 0.002 | 0.023 | 0.009 | 0.030 | 0.094 | 1.000 | 0.765516 | 0.037918 | 20.188826 |
| 4.30 | 0.049 | 0.055 | 0.050 | 0.034 | 0.024 | 0.131 | 0.002 | 0.023 | 0.009 | 0.030 | 0.094 | 1.000 | 0.765507 | 0.037914 | 20.190831 |
| 4.35 | 0.049 | 0.055 | 0.050 | 0.033 | 0.024 | 0.130 | 0.002 | 0.023 | 0.009 | 0.030 | 0.094 | 1.000 | 0.765498 | 0.037910 | 20.192764 |
| 4.40 | 0.049 | 0.055 | 0.050 | 0.033 | 0.024 | 0.130 | 0.003 | 0.024 | 0.009 | 0.030 | 0.094 | 1.000 | 0.765490 | 0.037906 | 20.194627 |
| 4.45 | 0.049 | 0.055 | 0.051 | 0.033 | 0.024 | 0.130 | 0.003 | 0.024 | 0.009 | 0.030 | 0.094 | 1.000 | 0.765481 | 0.037902 | 20.196425 |
| 4.50 | 0.049 | 0.055 | 0.051 | 0.033 | 0.024 | 0.130 | 0.003 | 0.024 | 0.009 | 0.030 | 0.094 | 1.000 | 0.765473 | 0.037898 | 20.198159 |
| 4.55 | 0.049 | 0.055 | 0.051 | 0.033 | 0.024 | 0.129 | 0.003 | 0.024 | 0.009 | 0.030 | 0.094 | 1.000 | 0.765465 | 0.037895 | 20.199834 |
| 4.60 | 0.049 | 0.055 | 0.051 | 0.033 | 0.024 | 0.129 | 0.003 | 0.024 | 0.009 | 0.030 | 0.094 | 1.000 | 0.765457 | 0.037891 | 20.201451 |
| 4.65 | 0.049 | 0.055 | 0.051 | 0.033 | 0.024 | 0.129 | 0.003 | 0.024 | 0.009 | 0.030 | 0.094 | 1.000 | 0.765450 | 0.037888 | 20.203014 |
| 4.70 | 0.049 | 0.055 | 0.051 | 0.033 | 0.024 | 0.129 | 0.003 | 0.024 | 0.009 | 0.030 | 0.094 | 1.000 | 0.765442 | 0.037885 | 20.204524 |
| 4.75 | 0.049 | 0.055 | 0.051 | 0.033 | 0.024 | 0.129 | 0.003 | 0.024 | 0.009 | 0.030 | 0.094 | 1.000 | 0.765435 | 0.037882 | 20.205984 |
| 4.80 | 0.049 | 0.055 | 0.051 | 0.033 | 0.024 | 0.128 | 0.003 | 0.024 | 0.009 | 0.030 | 0.095 | 1.000 | 0.765428 | 0.037879 | 20.207396 |
| 4.85 | 0.049 | 0.055 | 0.051 | 0.033 | 0.024 | 0.128 | 0.003 | 0.025 | 0.009 | 0.030 | 0.095 | 1.000 | 0.765421 | 0.037876 | 20.208762 |
| 4.90 | 0.050 | 0.055 | 0.051 | 0.033 | 0.024 | 0.128 | 0.003 | 0.025 | 0.009 | 0.029 | 0.095 | 1.000 | 0.765414 | 0.037873 | 20.210083 |
| 4.95 | 0.050 | 0.054 | 0.051 | 0.033 | 0.024 | 0.128 | 0.003 | 0.025 | 0.009 | 0.029 | 0.095 | 1.000 | 0.765407 | 0.037870 | 20.211363 |
| 5.00 | 0.050 | 0.054 | 0.051 | 0.033 | 0.024 | 0.128 | 0.003 | 0.025 | 0.009 | 0.029 | 0.095 | 1.000 | 0.765400 | 0.037867 | 20.212602 |

of the efficiency of the investment portfolio can be made, as shown in Figure 1.

Each different level of risk aversion results in the mean return of the investment portfolio $\mu_p$ and investment portfolio risk $VaR_p$

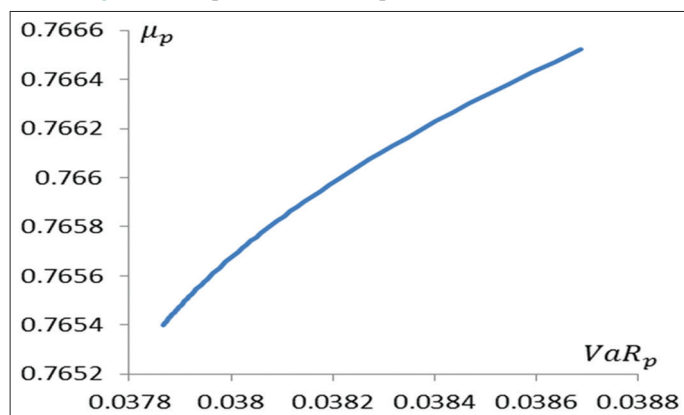**Figure 1:** Graph of investment portfolio efficient surface



**Figure 2:** The relationship between the level of risk aversion and the risk of the investment portfolio
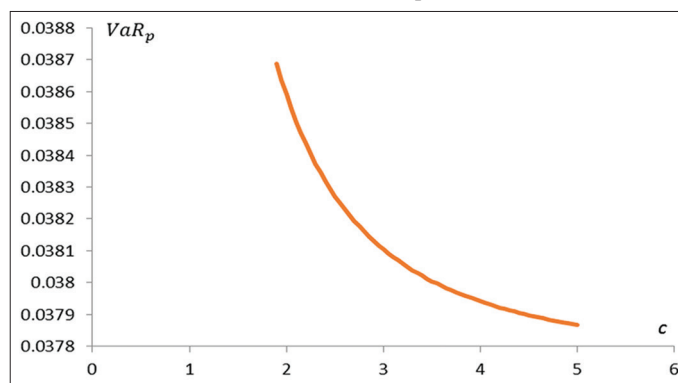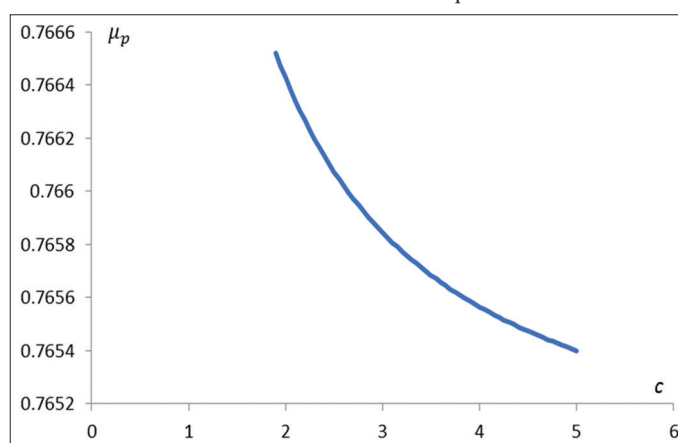


which is different. For rational investors, with the level of risk aversion they have, they will of course invest at points along the surface line of an efficient portfolio. Investors who invest beyond the points along the surface of an efficient portfolio can be viewed as irrational investors (Wang et al., 2016; Shakouri and Lee, 2016). Along the surface line of an efficient portfolio, an investment portfolio that has a minimum Value-at-Risk (VaR) risk occurs when the risk aversion level is $c = 1.9$ which yields the mean return of the investment portfolio $\mu_p = 0.766522$ and investment portfolio risk $VaR_p = 0.038687$ with the ratio value of $\mu_p/VaR_p = 19.813191$ as the smallest, causing this to often be referred to as the minimum portfolio. This minimum portfolio is generated when the investment is carried out with a weight vector composition of $\mathbf{w}^{Min} = 0.038\ 0.058\ 0.044\ 0.036\ 0.025\ 0.161\ 0.000\ 0.010\ 0.010\ 0.033\ 0.086)^T$. For the level of risk aversion in the interval $1.9 \le c \le 5.00$ with an increase of 0.05, the maximum portfolio occurs when the risk aversion level $c = 5.00$ which yields the mean return of the investment portfolio $\mu_p = 0.765400$ and investment portfolio risk $VaR_p = 0.037867$ with the ratio value $\mu_p/VaR_p = 20.212602$ is the largest, causing this to often be referred to as the maximum portfolio. This maximum portfolio is generated when the investment is made with a weight vector composition of $\mathbf{w}^{Max} = 0.050\ 0.054\ 0.051\ 0.033\ 0.024\ 0.128\ 0.003\ 0.025\ 0.009\ 0.029\ 0.095)^T$. The relationship between the level of risk aversion and investment portfolio risk can be noticed in the graph given in Figure 2.

The graph in Figure 2 shows the preference of an investor who has a small risk aversion; an investor tends to have the courage to face a relatively high investment risk. Furthermore, when the preference of an investor who has risk aversion is greater, then that investor has smaller courage to face investment risk. Thus, if the preference of an investor has a very high-risk aversion level, then an investor tends to have very little courage to face investment risk, causing the tendency not to invest because he is very afraid of the risk of loss.

The relationship between the level of risk aversion c and the mean return of the investment portfolio $\mu_p$ can be noticed the graph given in Figure 3.

The graph in Figure 3 shows that the mean return of the investment portfolio $\mu_p$ tenderness decreases, along with the increase in

**Figure 3:** The relationship between the level of risk aversion and the mean return of the investment portfolio



the value of the risk aversion level . This is consistent with the downward trend in the risk value of the investment portfolio $VaR_p$, in line with the increasing value of the risk aversion level $c$. In investing in financial assets in general, assets that have a high level of risk will also expect a large return. Conversely, assets that promise a small return expectation will generally be accompanied by a small risk as well.

## 6. CONCLUSION

This paper has discussed modeling of quadratic investment portfolio based on Value-at-Risk (VaR) with risk-free assets, which is applied to stocks of the mining and energy sector. Based on the results of the discussion, it can be concluded that the quadratic investment portfolio optimization model Mean-VaR has been formulated with risk-free assets and risk aversion. The results of data analysis on 11 mining and energy sector stocks have formed an efficient portfolio surface graph, with the minimum mean return of portfolio value of 0.766522 and the minimum risk of portfolio VaR of 0.038687, which occurs for a risk aversion level of 1.9. In this analysis, the surface of efficient portfolio graph has a mean return value of (maximum portfolio is 0.765400 and maximum portfolio risk is 0.037867), which occurs when the risk aversion level is 5.00. In addition, the results of the discussion show that the greater the level of risk aversion, the smaller the VaR portfolio

risk value will be, followed by the smaller the mean return value of the portfolio.

## 7. ACKNOWLEDGMENTS

## REFERENCES

Adiatma, J.C., Simamora, P., Gardian, P.I., Putra, R.P.M., Ragina, F.S., Siahaan, N. (2019), Energy Investment Needs in Indonesia: National Energy General Plan (Kebutuhan Investasi Energi di Indonesia: Rencana Umum Energi Nasional). Jakarta: Institute for Essential Services Reform.

Ahmadi, H., Sitdhirasdr, D. (2016), Portfolio optimization is one multiplication, the rest is arithmetic. Journal of Applied Finance and Banking, 6(1), 81-93.

Alexander, S., Coleman, T.F., Li, Y. (2006), Minimizing CVaR and VaR for a portfolio of derivatives. Journal of Banking and Finance, 30(2), 583-605.

Artemkina, L.R., Sopilko, N.Y., Myasnikova, O.Y., Eremina, I.U., Bondarchuk, N.V., Shamsheev, S.V. (2019), The model of upstream investment portfolio in the mature regions. International Journal of Energy Economics and Policy, 9(4), 173-180.

Bansal, Y., Kumar, S., Verma, P. (2014), Commodity futures in portfolio diversification: Impact on investor's utility. Global Business and Management Research, 6(2), 112-121.

Baweja, M., Saxena, R.R. (2015), Portfolio optimization with structured products under return constraint. Yugoslav Journal of Operations Research, 25(2), 221-232.

Boudt, K., Carl, P., Peterson, B.G. (2012), Asset allocation with conditional value-at-risk budgets. Journal of Risk, 15(3), 39-68.

Buberkoku, O. (2019), Do long-memory GARCH-type-value-at-risk models outperform none-and semi-parametric value-at-risk models? International Journal of Energy Economics and Policy, 9(2), 199-215.

Carnia, E. (2019), Optimization of the mean-absolute deviation portfolio investment in some mining stocks using the singular covariance matrix method. Journal of Physics: Conference Series,1315(1), 012002.

Cochrane, J.H. (2014), A mean-variance benchmark for intertemporal portfolio theory. The Journal of Finance, 69(1), 1-49.

Deller, S.C., Schreiber, A. (2012), Mining and community economic growth. Review of Regional Studies, 42(2), 121-141.

Devi, B., Prayogo, D. (2013), Mining and Development in Indonesia: An Overview of the Regulatory Framework and Policies. Action Research Report. Brisbane: International Mining for Development Center.

Er, B., Guneysu, Y., Ünal, H. (2018), Financing renewable energy projects: An empirical analysis for turkey. International Journal of Energy Economics and Policy, 8(6), 180-185.

Fachrudin, K.A., Fachrudin, H.T. (2015), The study of investment portfolio management and sustainability of property and real estate companies in Indonesia stock exchange. Procedia-Social and Behavioral Sciences, 211, 177-183.

Gaivoronski, A.A., Pflug, G. (2005), Value-at-risk in portfolio optimization: Properties and computational approach. Journal of

Risk, 7(2), 1-31.

Gambrah, P.S.N., Pirvu, T.A. (2014), Risk measures and portfolio optimization. Journal of Risk and Financial Management, 7(3), 113-129.

Ghaemi, R., Sun, J., Kolmanovsky, I.V. (2009), An integrated perturbation analysis and sequential quadratic programming approach for model predictive control. Automatica, 45(10), 2412-2418.

Goh, J.W., Lim, K.G., Sim, M., Zhang, W. (2012), Portfolio value-at-risk optimization for asymmetrically distributed asset returns. European Journal of Operational Research, 221(2), 397-406.

Golafshani, Z.Y., Emamipoor, S. (2015), Portfolio optimization using two methods of mean-variance analysis and mean risk in Tehran stock exchange. Technical Journal of Engineering and Applied Sciences, 2015(5), 128-133.

Gurrib, I., Elsharief, E., Kamalov, F. (2020), The effect of energy cryptos on efficient portfolios of key energy listed companies in the S&P composite 1500 energy index. International Journal of Energy Economics and Policy, 10(2), 179-193.

Hashemi, S.B., Azarpour, A.M., Navvabpour, H. (2016), Portfolio optimization by mean-value at risk framework. Applied mathematics, 7(1-17), 1.

Hooda, D.S., Stehlík, M. (2011), Portfolio analysis of investments in risk management. The Open Statistics and Probability Journal, 3(1), 21-26.

Hult, H., Lindskog, F., Hammarlid, O., Rehn, C.J. (2012), Risk and Portfolio Analysis: Principles and Methods. Berlin: Springer Science & Business Media.

Kitula, A.G.N. (2006), The environmental and socio-economic impacts of mining on local livelihoods in Tanzania: A case study of Geita District. Journal of Cleaner Production, 14(3-4), 405-414.

Lawal, A.I., Dahunsi, S.O., Babajide, A.A., Asaleye, A.J., Iseolorunkanmi, J.O., Inegbedion, H., Manasseh, C.O., Lawal-Adedoyin, B. (2020), Examining the effects of oil price long memory and exchange rate long memory on stock market behavior in Nigeria. International Journal of Energy Economics and Policy, 10(4), 430-436.

Lwin, K.T., Qu, R., MacCarthy, B.L. (2017), Mean-VaR portfolio optimization: A nonparametric approach. European Journal of Operational Research, 260(2), 751-766.

Mustafa, S., Bano, S., Hanif, M., Jamal, N. (2020), Reducing portfolio quadratic programming problem into regression problem: Stepwise algorithm. Punjab University Journal of Mathematics, 47(1), 127-134.

Napitupulu, H., Mohd, I.B., Hidayat, Y., Supian, S. (2018), Steepest descent method implementation on unconstrained optimization problem using C++ program. IOP Conference Series: Materials Science and Engineering, 332(1), 012024.

Ogryczak, W., Przyłuski, M., Sliwinski, T. (2015), Portfolio optimization with reward-risk ratio measure based on the conditional value-at-risk. Proceedings of the World Congress on Engineering and Computer Science, 2, 21-23.

Ogryczak, W., Sliwinski, T. (2010), Efficient portfolio optimization with conditional value at risk. In: Proceedings of the International Multiconference on Computer Science and Information Technology. United States: IEEE. p901-908.

Patricia, R.M., Sushanti, S., Priadarsini, N.W.R. (2016), Upaya Peningkatan Investasi Indonesia di Myanmar Melalui Diplomasi Ekonomi Pasca Demokratisasi Myanmar.

Pinasthika, N., Surya, B.A. (2014), Optimal portfolio analysis with risk-free assets using index-tracking and Markowitz mean-variance portfolio optimization model. Journal of Business and Management, 3(7), 737-751.

Plunus, S., Gillet, R., Hübner, G. (2015), Equivalent risky allocation: The new ERA of risk measurement for heterogeneous investors. American Journal of Industrial and Business Management, 5(6), 351-365.

Priyarsono, D.S., Tambunan, M., Firdaus, M. (2012), Perkembangan konsumsi dan penyediaan energi dalam perekonomian Indonesia. Jurnal Ilmu Ekonomi Pertanian Indonesia, 2(1), 97-119.

Qin, Z. (2015), Mean-variance model for portfolio optimization problem in the simultaneous presence of random and uncertain returns. European Journal of Operational Research, 245(2), 480-488.

Shakouri, M., Lee, H.W. (2016), Mean-variance portfolio analysis data for optimizing community-based photovoltaic investment. Data in Brief, 6, 840-842.

Soeryana, E., Fadhlina, N., Rusyaman, E., Supian, S. (2017.a), Mean-variance portfolio optimization by using time series approaches based on logarithmic utility function. IOP Conference Series: Materials Science and Engineering, 166(1), 012003.

Soeryana, E., Halim, N.B.A., Sukono, Rusyaman, E., Supian, S. (2017.b), Mean-Variance portfolio optimization by using non constant mean and volatility based on the negative exponential utility function. AIP Conference Proceedings, 1827(1), 020042.

Strassberger, M. (2005), Capital Requirement, Portfolio Risk Insurance, and Dynamic Risk Budgeting.

Sukono, Kartiwa, A., Subartini, B., Hidayat, Y., Bon, A.T. (2018.b), Estimation of conditional value-at-risk under assets liability model with non constant volatility. International Conference on Industrial Engineering and Operations Management, 2018, 2726-2733.

Sukono, Lesmana, E., Napitupulu, H., Hidayat, Y., Saputra, J., Ghazali, P.L.B. (2019), Mean-VaR portfolio optimisations: An application of multiple index models with non-constant volatility and long memory effects. International Journal of Innovation Creativity and Change, 9(12), 364-381.

Sukono, Sidi, P., Bon, A.T.B., Supian, S. (2017.a), Modeling of mean-VaR portfolio optimization by risk tolerance when the utility function is quadratic. AIP Conference Proceedings, 1827(1), 020035.

Sukono, Sidi, P., Susanti, D., Supian, S. (2017.b), Quadratic investment portfolio without a risk-free asset based on value-at-risk. Journal of Engineering and Applied Sciences, 12(19), 4846-4850.

Sukono, Supian, S., Napitupulu, H., Hidayat, Y., Putra, A.S. (2018.a), The application of genetic algorithm optimization on quadratic investment portfolio without a risk-free asset under value-at-risk. Journal of Physics: Conference Series, 1090(1), 012026.

Sukono, Suyudi, M., Islamiyati, F., Supian, S. (2017.c), Modelling of capital asset pricing by considering the lagged effects. IOP Conference Series: Materials Science and Engineering, 166, 012001.

Taušová, M., Čulková, K., Domaracká, L., Drebenstedt, C., Muchová, M.S., Koščo, J., Behúnová, A., Drevková, M., Benčöová, B. (2017), The importance of mining for socio-economic growth of the country. Acta Montanistica Slovaca, 22(4), 359-367.

Wang, Y., Chen, Y., Liu, Y. (2016), Modeling portfolio optimization problem by probability-credibility equilibrium risk criterion. Mathematical Problems in Engineering, 2016, 13.

Warburton, E. (2017), Resource nationalism in Indonesia: Ownership structures and sectoral variation in mining and palm oil. Journal of East Asian Studies, 17(3), 285-312.