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Book

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Time-Varying Parameter Four-Equation DSGE Model

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Abstract

We build in the time-varying parameter feature into the Sims et al. (2020) four-equation Dynamic Stochastic General Equilibrium (DSGE) model in this paper. We find that both parameters and impulse responses of the variables in the four-equation DSGE model exhibit significant variation over time. Allowing model parameters to vary over time also improves the model's forecasting performance.

JEL Classification: E32, C52, C53
Keywords: Four-Equation DSGE, Time-Varying Parameter, Forecasting

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1 Introduction

Dynamic stochastic general equilibrium (DSGE) models, which combines microfoundations (derived from the optimisation decisions of rational agents) with business cycle fluctuations, have become popular tools in the context of policy analysis and macroeconomic forecasting. In this regard, the textbook version of the three-equation New Keynesian (NK) model (see, for example, Woodford (2003), and Galí (2008)), has been of particular interest to academics and policymakers. This model basically comprises of a forward-looking Investment-Saving (IS) equation capturing aggregate demand, a Phillips curve portraying the aggregate supply-side of the economy, and a rule for the short-term interest rate, which in turn is the principal policy tool of the central bank.

However, despite of its several applications, the NK model has faced severe criticisms recently in the wake of the Global Financial Crisis (GFC) of 2007-2009. Firstly, since the model abstracts from the financial sector, it is unable to accommodate for the impact of developments (or disruption as witnessed during the GFC) in the financial sector on the rest of the economy. Secondly, the NK model is also incapable of providing a cost-benefit analysis of quantitative easing (QE) type policies, with these being among the first and most prominent of several unconventional interventions deployed by the central banks around the world to tackle the adverse impact of the GFC on the macroeconomy, once the policy rates reaches the Zero Lower Bound (ZLB). Note that QE again came to the fore following the outbreak of the COVID-19 pandemic.

Based on the seminal work of Bernanke et al. (1999), quite a bit of progress has been made in terms of exploiting financial channels in a DSGE framework to improve the fit of the model subjected to turmoil witnessed during the GFC (see, for example, Christiano et al. (2014), Del Negro and Schorfheide (2013), Del Negro et al. (2016)). Comparatively, however, the area of research in terms of incorporating QE into DSGE models is relatively nascent (see for example, Gertler and Karadi (2011, 2013); Carlstrom et al. (2017); Sims and Wu (2021)), and tends to lack the simplicity and transparency of the textbook three-equation NK model. Given this, Sims et al. (2020) bridged the gap between the complicated quantitative QE-based DSGE models with the simplicity of the three-equation NK model, upgraded to incorporate an explicit financial sector as well.

Specifically speaking, this of model Sims et al. (2020) incorporates financial intermediaries, short- and long-term bonds, credit market shocks, and a role for unconventional monetary policy, i.e., economically relevant central bank bond holdings. The linearized model now has four, rather than three, key equations as follows: The IS and Phillips curves, just as in the three equation benchmark, with the innovation being that credit shocks and central bank long bond holdings appear additively in both these equations. In addition to a rule for the short-term policy rate as in the three equation NK framework, the current model is closed with an additional equation involving the central bank's long bond portfolio. Using this framework, Sims et al. (2020) show that, in equilibrium, optimal monetary policy involves adjusting the short-term interest rate to neutralize natural rate shocks, but QE is used to offset credit market disruptions. Moreover, usage of QE is shown to significantly mitigate the welfare costs, associated with the fluctuations of inflation and the output gap, during a binding ZLB.

Indeed, incorporating a financial channel and QE in the textbook DSGE model is of immense importance in addressing the effects that structural changes in the underlying economy that extreme events, such as the GFC and the Coronavirus pandemic, might have had on the parameters and on exogenous shock processes of the NK framework. But one could, perhaps, also do more in this respect. Recall that, parameters of a DSGE model are considered to be structural ("deep") in the sense that these are invariant to both policy and structural shocks (Lucas, 1976). But, this does not imply that the parameters remain constant at all time scales, since long-term cultural or technological shifts might result in slow parameter variation, especially when one looks at lengthy sample periods (Cardani et al., 2019; Kapetanios et al., 2019). In this scenario, in spite of DSGE models focusing on business cycle frequency, parameter drift becomes potentially of great importance.

Notwithstanding the structural nature of the DSGE, due to the possibility of time-variation in the parameters when estimated over more than half a century of data, i.e., 1949-2021, the objective of our paper is to provide a time-varying parameter extension of the four-equation NK model discussed above. As far as estimation is concerned, we rely on the approach of Galvão et al. (2016), which is an extension and formalization of rolling-window estimation,[1] generalized by combining kernel-generated local likelihoods with appropriately chosen priors to generate a sequence of posterior distributions for the objects of interest over time.[2] In the process, we uncover significant variation over time in the parameters, and

[1]See for example, Canova (2009), Canova and Gambetti (2009), Castelnuovo (2012) in terms of application to the estimation of the parameters of DSGE models.

[2]When applied to structural models, the kernel approach assumes that, agents take variation in parameters as exogenous when forming their expectations about the future, instead of being endowed with perfect knowledge associated with the data generating process of the economy. This assumption assists in the process of estimation, and can be rationalized based on models involving learning, wherein agents form beliefs about the parameters by relying on the observation of past data.

the generated impulse responses of the variables associated with the estimated four-equation NK model. In addition, we find that, allowing the model parameters to vary over time, also improves the forecasting performance of the model, in particular for interest rate and long-term bond holdings.

To the best of our knowledge, this is the first paper to highlight the importance of incorporating time-variation in the four-equation NK model from the perspective of both structural and forecasting analyses. The remainder of the paper is organized as follows: Section 2 presents the basics of the four-equation NK model, with Section 3 discussing the data and the underlying methodology used for estimating this model. Section 4 is devoted to the empirical results, and Section 5 concludes the paper.

2 The Four-Equation Model

Following Sims et al. (2020), we consider the simple New Keynesian model with the following four equations:

$$x_t = E_t x_{t+1} - \frac{1-z}{\sigma}(r_t^s - E_t\pi_{t+1} - r_t^*) - z[\bar{b}^{FI}(E_t\theta_{t+1} - \theta_t) + \bar{b}^{CB}(E_t qe_{t+1} - qe_t)], \tag{1}$$

$$\pi_t = \gamma\zeta x_t - \frac{z\gamma\sigma}{1-z}[\bar{b}^{FI}\theta_t + \bar{b}^{CB} qe_t] + \beta E_t \pi_{t+1}, \tag{2}$$

$$r_t^s = \rho_r r_{t-1}^s + (1-\rho_r)[\phi_\pi \pi_t + \phi_x x_t] + s_r \epsilon_{r,t}, \tag{3}$$

$$qe_t = \rho_q qe_{t-1} - (1-\rho_q)[\lambda_\pi \pi_t + \lambda_x x_t] + s_q \epsilon_{q,t}, \tag{4}$$

where x_t is the inflation rate, x_t is the output gap, r_t^s is the short-term nominal interest rate, and r_t^* is the natural interest rate. The variables θ_t and qe_t capture credit conditions of the financial market and real market value of the central bank's long-term bond holdings, respectively. The parameter σ measures the inverse intertemporal elasticity of substitution, β is the discount factor, γ is the elasticity of inflation w.r.t. real marginal cost, and ζ is the elasticity of real marginal cost w.r.t. output gap. While these parameters carry the same meaning as in the traditional three-equation New Keynesian model, $\bar{b}^{FI}$ and $\bar{b}^{CB}$ are new parameters in this four-equation model that takes into account unconventional monetary policies; they measure the steady-state long-term holdings of financial intermediaries and the central bank, respectively, relative to the total outstanding long-term bonds. The parameter z stands for the fraction of children in the total population, as outlined in a full model in Sims et al. (2020). In the special case where $z = 0$, the four-equation model collapses to the traditional three-equation model and neither credit shocks nor the central bank's long-term bond holdings would matter for the dynamics of output and inflation.

Equation (1) is an augmented IS curve, (2) is a Phillips curve, (3) is the Taylor rule, and (4) is an endogenous QE rule which follows a similar Taylor-type rule that responds to inflation and output gap. The system of these four equations can be written in the form of a typical rational expectation model; see the appendix. ρ_r and ρ_q are autoregressive parameters; ϕ_π and λ_π capture the response of the short-term interest rate and long-term bond holdings to inflation; and ϕ_x and λ_x capture the response of the same two series to the output gap. The system is closed with another two AR(1) processes that guide the dynamics of the natural interest rate and credit shock, i.e.,

$$r_t^* = \rho_f r_{t-1}^* + s_f \epsilon_{f,t}, \tag{5}$$

$$\theta_t = \rho_\theta \theta_{t-1} + s_\theta \epsilon_{\theta,t}, \tag{6}$$

where the ϵ's are idiosyncratic shocks and the s's are their standard deviations.

3 Data and Methodology

Our sample includes quarterly data on the output gap (x_t), short-term interest rate (r_t^s), inflation rate π_t, and real value of the central bank's long-term bond holdings (qe_t) from 1949 to 2021. All data are retrieved from the Federal Reserve Economic Data (FRED) of the Federal Reserve Bank of St. Louis. The output gap is calculated as the percent deviation of real GDP from potential GDP. The short-term interest rate is the 3-month treasury bill secondary market rate. The inflation rate is calculated as the percent change in the GDP deflator. Each of these three variables is then demeaned. The real value of central bank's long-term bond holdings is calculated as the natural log of the total face value of U.S. Treasury securities held by the Federal Reserve maturing in over 10 years, deflated by the GDP deflator.

Given that our sample ranges over a long period of time that covers multiple phases of economic fluctuations, we use the Bayesian Local Likelihood (BLL) estimation method developed by Galvão et al. (2016) to explore the time-varying feature of the model parameters. Consider the state-space representation of a linearized rational expectation model with a state equation and a measurement equation:

$$\mathbf{z}_t = F(\Gamma_t)\mathbf{z}_{t-1} + G(\Gamma_t)\mathbf{v}_t, \tag{7}$$

$$\mathbf{y}_t = K(\Gamma_t) + H(\Gamma_t)\mathbf{z}_t, \tag{8}$$

where $\mathbf{z}_t$ is a $n \times 1$ vector of model variables, $\mathbf{v}_t$ is a $k \times 1$ vector of structural shocks, $\mathbf{y}_t$ is a $m \times 1$ vector of observables, and Γ_t is a vector of model parameters. The local likelihood of the sample, $\mathbf{y} := \{\mathbf{y}_j\}_{j=1}^{T}$, can be obtained by employing the Kalman filter in a recursive fashion,

$$L_t(\mathbf{y}|\Gamma_t) = \sum_{j=1}^{T} L(\mathbf{y}_j|\mathbf{y}_{j-1}, \Gamma_t)^{w_{tj}} \quad \text{for } t = 1, ..., T, \tag{9}$$

where w_{tj} is the tj-th element of a $T \times T$ weight matrix W. The weights are normalized to sum to $2B+1$ for each t, i.e.,

$$w_{tj} = (2B+1)\frac{\tilde{w}_{tj}}{\sum_{j=1}^{T} \tilde{w}_{tj}} \quad \text{for } j, t = 1, ..., T, \tag{10}$$

where $\tilde{w}_{tj}$ is given by a kernel function with a bandwidth parameter B.

The local likelihood, $L_t(\mathbf{y}|\Gamma_t)$, is augmented with the prior distribution of the structural model parameters to get the posterior distribution using the random walk Metropolis algorithm developed by Schorfheide (2000).

4 Empirical Results

4.1 Parameter estimates

Our calibrated parameter values in Table 1 follow those of Sims et al. (2020). The discount factor β pins down the average real interest rate over the sample period. The consumption share of child z is calibrated to match the share of durable consumption and private investment in aggregate private non-government domestic expenditure. The parameters γ and ζ are selected to imply a slope of the Phillips curve of 0.21.

Table 1: Model parameter calibration

Parameter	Description	Value
β	Discount factor	0.995
z	Consumption share of child	0.33
σ	Inverse elasticity of substitution	1
$\bar{b}^{FI}$	Weight on credit	0.7
$\bar{b}^{CB}$	Weight on quantitative easing	0.3
γ	Elasticity of inflation w.r.t. marginal cost	0.086
ζ	Elasticity of marginal cost w.r.t. output gap	2.49

The remaining parameters are estimated using the algorithm described in Section 3. Table 2 presents their prior distributions. While the Taylor rule parameters ϕ_π and ϕ_x follow standard prior distributions as in the literature, we use a loose prior for the QE rule parameters λ_π and λ_x. In their calibration exercise, Sims et al. (2020) vary the values of λ_π and λ_x between 0 and 15. We therefore choose a Normal prior distribution with mean 5 and a large standard deviation 10. The autoregressive parameters and standard deviations of the shock processes follow standard prior distributions as in the literature.

Table 2: Model parameter prior distribution

Parameter	Description	Distribution	Mean	St. Dev.
ϕ_π	Taylor rule inflation parameter	Normal	1.5	0.1
ϕ_x	Taylor rule output gap parameter	Normal	0	0.1
λ_π	QE rule inflation parameter	Normal	5	10
λ_x	QE rule output gap parameter	Normal	5	10
ρ_r	Taylor rule AR parameter	Beta	0.8	0.1
ρ_q	QE rule AR parameter	Beta	0.8	0.1
ρ_f	Natural rate AR parameter	Beta	0.8	0.1
ρ_θ	Credit AR parameter	Beta	0.8	0.1
s_r	Interest rate shock St. Dev.	Inverse Gamma	0.1	2
s_q	QE shock St. Dev.	Inverse Gamma	0.1	2
s_f	Natural rate shock St. Dev.	Inverse Gamma	0.1	2
s_θ	Credit shock St. Dev.	Inverse Gamma	0.1	2

The time-varying pattern of parameters estimated from a Normal kernel is depicted in Figure 1. The figure plots the posterior mean of each parameter in solid blue line and 90 percent confidence intervals of the posterior distribution in dashed red lines. The Taylor rule inflation parameter, ϕ_π, is relatively stable over the entire sample period; it varies between 1.5 in the early 1950s and 1.6 around the Great Recession. Other parameters, however, exhibit more variation over time. The Taylor rule output gap parameter, ϕ_x, varies between 0.2 and 0.4 prior to the 1990s and declines to near zero thereafter. Around the same point in time, the response of QE to output gap, λ_x, increases dramatically. The AR parameter in the Taylor rule, ρ_r, decreases between the 1950s and 1990s, and then starts to increase until it reaches the peak at the end of the sample period. The AR parameter of the natural rate shock, ρ_f, fluctuates over time and exhibits a downward trend. Both AR parameters of the QE rule and credit shock, ρ_q and ρ_θ, are significantly higher in the post-2000 period. The standard deviations of structural shocks also vary significantly over time, especially those of the QE shock (s_q) and credit shock (s_θ).[3]

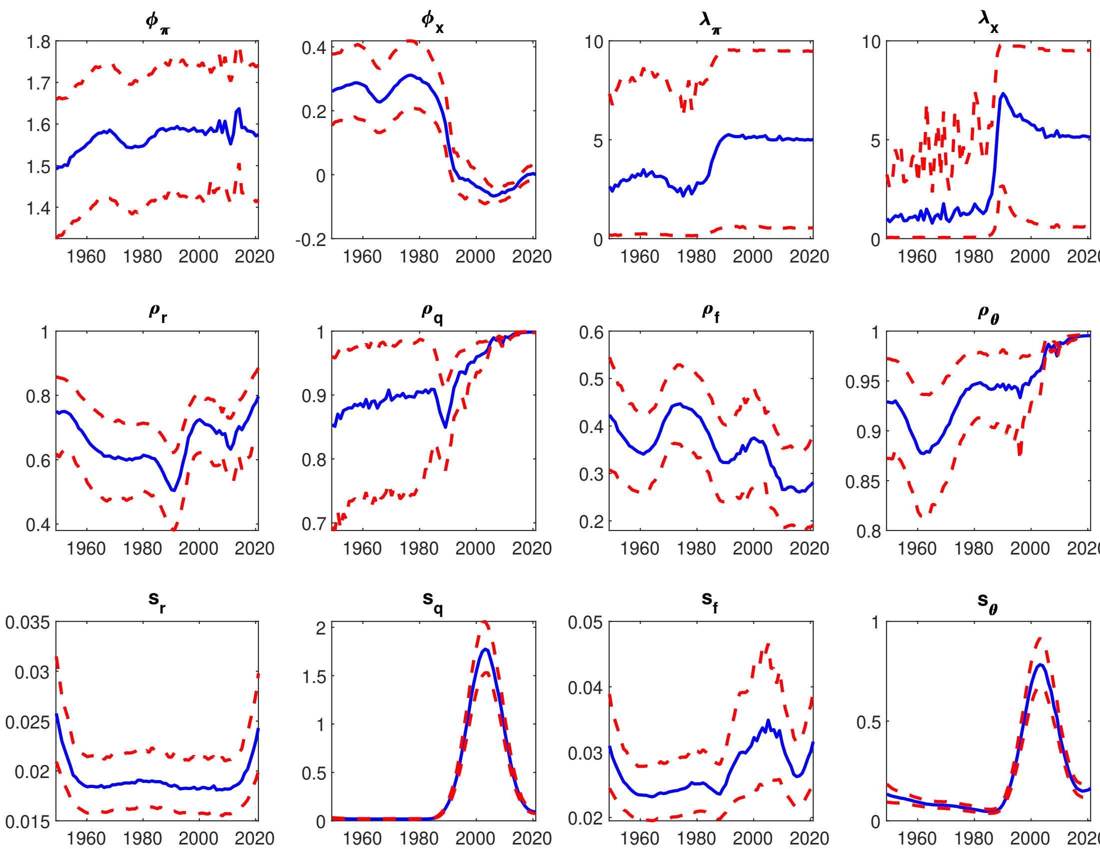

Figure 1: Time-varying parameter estimates

As a comparison, we also estimate the model assuming constant value parameters. Figure 2 reports the constant value parameter estimates in solid red lines and their 90 percent confidence intervals in

[3]As a robustness check, we also use a rolling-window estimation strategy rather than a Normal kernel and similar results can be found in Figure A1 in the appendix.

dashed red lines. It also plots the time-varying parameter estimates as in Figure 1 in solid blue lines. For most of the model parameters, the time-varying parameter estimate fluctuates out of the 90 percent confidence intervals of its time-invariant estimate, which suggest that a time-varying parameter version of the model is more adequate compared to its constant value parameter counterpart.

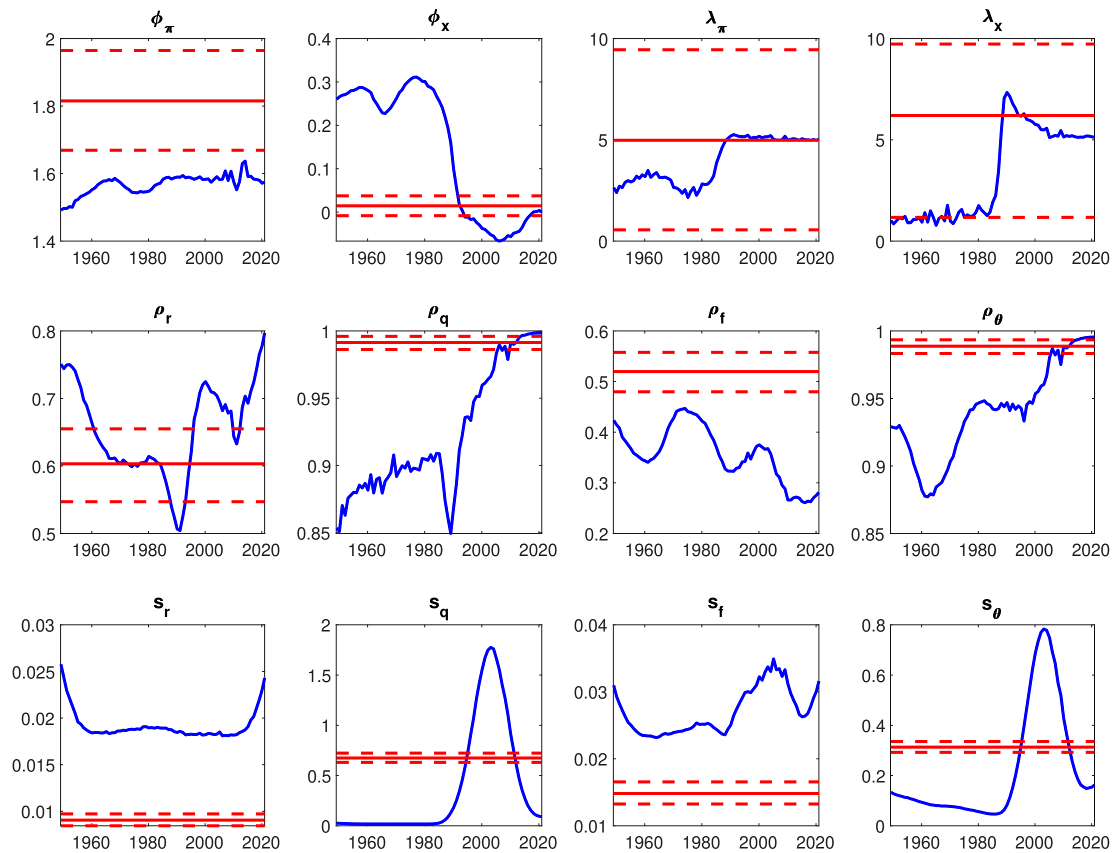

Figure 2: Time-invariant parameter estimates

4.2 Impulse responses

In this subsection, we present the time-varying impulse responses of the main model variables to each of the key structural shocks. Figure 3 plots the impulse responses to a one-standard-deviation interest rate shock ϵ_r. Following a positive interest rate shock, interest rate increases and both output gap and inflation decrease. This shock's impacts on output gap, interest rate, and inflation exhibit significant time variation, but they are short-lived in general. However, the shock has a long-lasting positive impact on central bank's long-term bond holdings during the Great Recession.

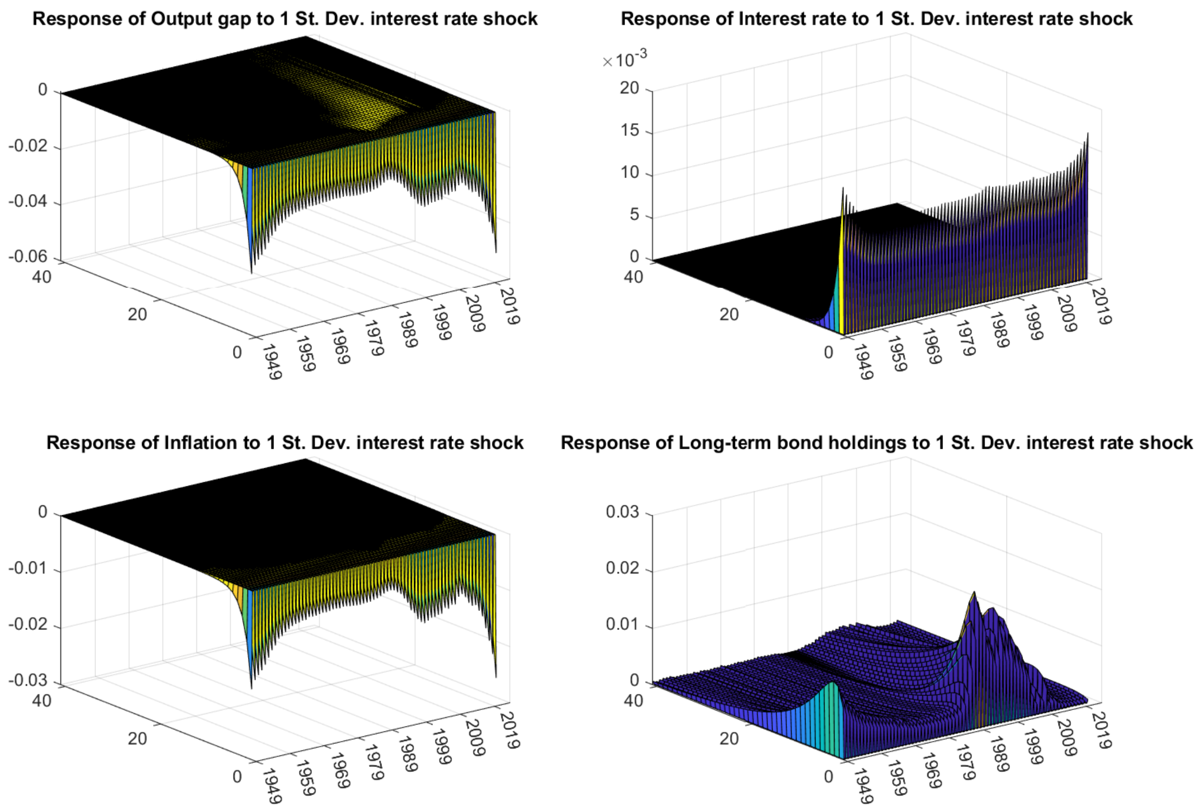

Figure 3: Time-varying impulse responses to the interest rate shock

Figure 4 plots the impulse responses to a one-standard-deviation QE shock ϵ_q. Such a shock is irrelevant in the traditional three-equation DSGE model but plays an important role in the four-equation model proposed by Sims et al. (2020). The purchase of long-term bonds by the central bank, starting in the early 2000s, significantly affects output gap and inflation. During the Great Recession period, a positive QE shock of one standard deviation increases output gap and inflation by a maximum of 10 and 1.5 percentage points, respectively. The positive QE shock promotes aggregate demand and therefore leads to an increase in the short-term interest rate.

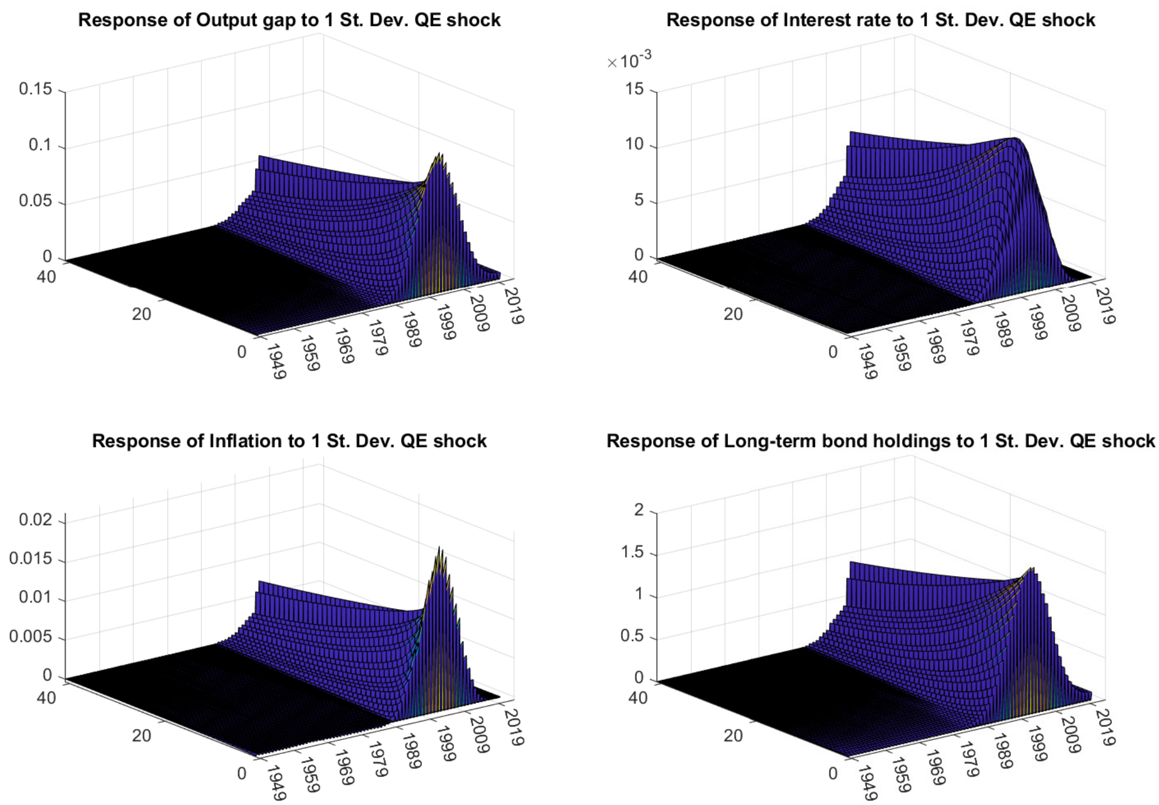

Figure 4: Time-varying impulse responses to the QE shock

Figure 5 shows the impulse responses to a one-standard-deviation credit shock ϵ_θ. The credit shock in the financial market crowds out the long-term holdings of the central bank. Just like the QE shock, the credit shock significantly increases output gap and inflation. It also causes an increase in the short-term interest rate due to its positive impact on the aggregate demand.

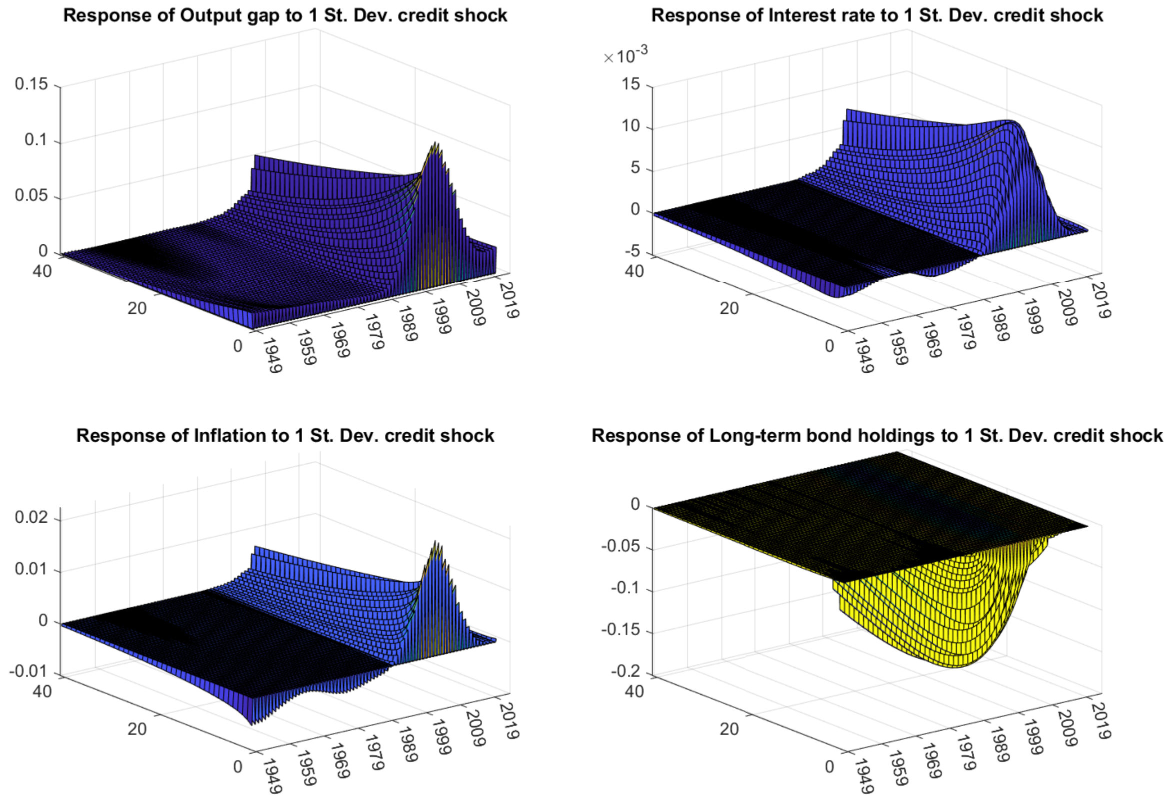

Figure 5: Time-varying impulse responses to the credit shock

4.3 Forecast evaluation

The time-varying estimation of the four-equation DSGE model confirms that model parameters exhibit significant variation over time. In this subsection, we explore the 1- to 8-quarter-ahead forecasting performance of the time-varying parameter model relative to the constant value parameter model. We choose to split the sample into an estimation sample between 1949Q1 and 2007Q4 and an evaluation sample between 2008Q1 and 2019Q4. We estimate the model parameters using data up to the last period of the in-sample and then generate projections 1 to 8 quarters ahead. Table 3 reports evaluates the forecasting performance using root mean squared forecast errors (RMSFEs) for output gap, interest rate, inflation, and long-term bond holdings of the central bank over the entire evaluation sample and two sub-samples. Values smaller than one imply more accurate forecasts of the time-varying parameter model relative to its time-invariant counterpart.

Table 3: Forecasting evaluation

Variable	h=1	h=2	h=3	h=4	h=5	h=6	h=7	h=8
Evaluation sample: 2008Q1 to 2019Q4								
Output gap	1.56*	1.35	1.47	1.58	1.70	1.87	2.00	2.26
Interest rate	0.53*	0.60*	0.66*	0.68*	0.72	0.78	0.83	0.89
Inflation	1.10	1.12	1.20	1.34	1.44	1.49	1.55	1.53
Long-term bond holdings	0.68*	0.66*	0.66*	0.66*	0.66*	0.66*	0.66*	0.66*
Evaluation sample: 2008Q1 to 2013Q4								
Variable	h=1	h=2	h=3	h=4	h=5	h=6	h=7	h=8
Output gap	1.59	1.76	1.91	2.08	2.33	2.68	3.08	3.53
Interest rate	0.57*	0.67*	0.76	0.79	0.85	0.93	1.00	1.08
Inflation	1.25	1.37	1.54	1.73	1.84	2.00	2.13	1.99
Long-term bond holdings	0.90	0.88	0.88	0.88	0.88	0.88	0.88	0.88
Evaluation sample: 2014Q1 to 2019Q4								
Variable	h=1	h=2	h=3	h=4	h=5	h=6	h=7	h=8
Output gap	1.50*	1.09	1.13	1.15	1.20	1.25	1.31	1.41
Interest rate	0.48*	0.47*	0.48*	0.48*	0.49*	0.50*	0.51*	0.51*
Inflation	0.91	0.83	0.79	0.81	0.84	0.87	0.88	1.02
Long-term bond holdings	0.17*	0.19*	0.20*	0.19*	0.20*	0.20*	0.20*	0.20*

The table reports ratios of RMSFEs of the time-varying parameter model relative to the time-invariant parameter model. * indicates rejection of the null of equal performance against the two-sided alternative at the 10 percent level using the Diebold and Mariano (1995) test.

Over the entire evaluation sample and both sub-samples, the time-varying parameter model and time-invariant parameter model perform equally well for the forecasts of inflation. The former version significantly under-performs the latter at the 10 percent level for the 1-quarter ahead forecasts of output gap over the entire evaluation sample and between 2014Q1 and 2019Q4, but not for the 2008Q1-2013Q4 period. However, for the forecasts of short-term interest rate and long-term bond holdings, which are the key features of the four-equation model, the time-varying parameter version significantly outperforms the constant value parameter model at various forecast horizons, especially over the 2014Q1-2019Q4 period.

5 Conclusion

The recently proposed four-equation New Keynesian (NK) DSGE model by Sims et al. (2020) aims to capture the effects of unconventional monetary policy, besides an explicit role of the financial sector on the macroeconomy, which are absent in the traditional three-equation model. We extend this framework by building the time-varying parameter feature. We find that both parameters and impulse responses of the variables of the four-equation DSGE model exhibit significant variation over time. Moreover, the time-varying parameter four-equation model significantly outperforms its constant value parameter counterpart in terms of forecasts of the short-term interest rate and long-term bond holdings of the central bank, whereas both versions perform equally well for the forecasts of output gap and inflation. In the process, our paper highlights the importance of incorporating time-variation into an extended NK DSGE model, over and above the influence of the financial sector and quantitative easing, when aiming to capture structural changes in the economy and policy-making associated with more than half a century of data involving the US.

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Appendix

Let $\mathbf{z}_t = [\pi_t, x_t, r^s_{t-1}, qe_{t-1}]'$, the system of the four-equation New Keynesian model can be written as:

$$\mathbf{B}_1 E_t \mathbf{z}_{t+1} = \mathbf{B}_2 \mathbf{z}_t, \tag{A1}$$

where

$$\mathbf{B}_1 = \begin{pmatrix} \beta & 0 & 0 & -\frac{z\gamma\sigma}{1-z}\bar{b}^{CB} \\ \frac{1-z}{\sigma} - z\bar{b}^{CB}(1-\rho_q)\lambda_\pi & 1 - z\bar{b}^{CB}(1-\rho_q)\lambda_x & -\frac{1-z}{\sigma} & z\bar{b}^{CB}(1-\rho_q) \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \tag{A2}$$

and

$$\mathbf{B}_2 = \begin{pmatrix} 1 & -\gamma\zeta & 0 & 0 \\ 0 & 1 & 0 & 0 \\ (1-\rho_r)\phi_\pi & (1-\rho_r)\phi_x & \rho_r & 0 \\ (1-\rho_q)\lambda_\pi & (1-\rho_q)\lambda_x & 0 & \rho_q \end{pmatrix}. \tag{A3}$$

Then we have:

$$E_t \mathbf{z}_{t+1} = \mathbf{B}_1^{-1} \mathbf{B}_2 \mathbf{z}_t. \tag{A4}$$

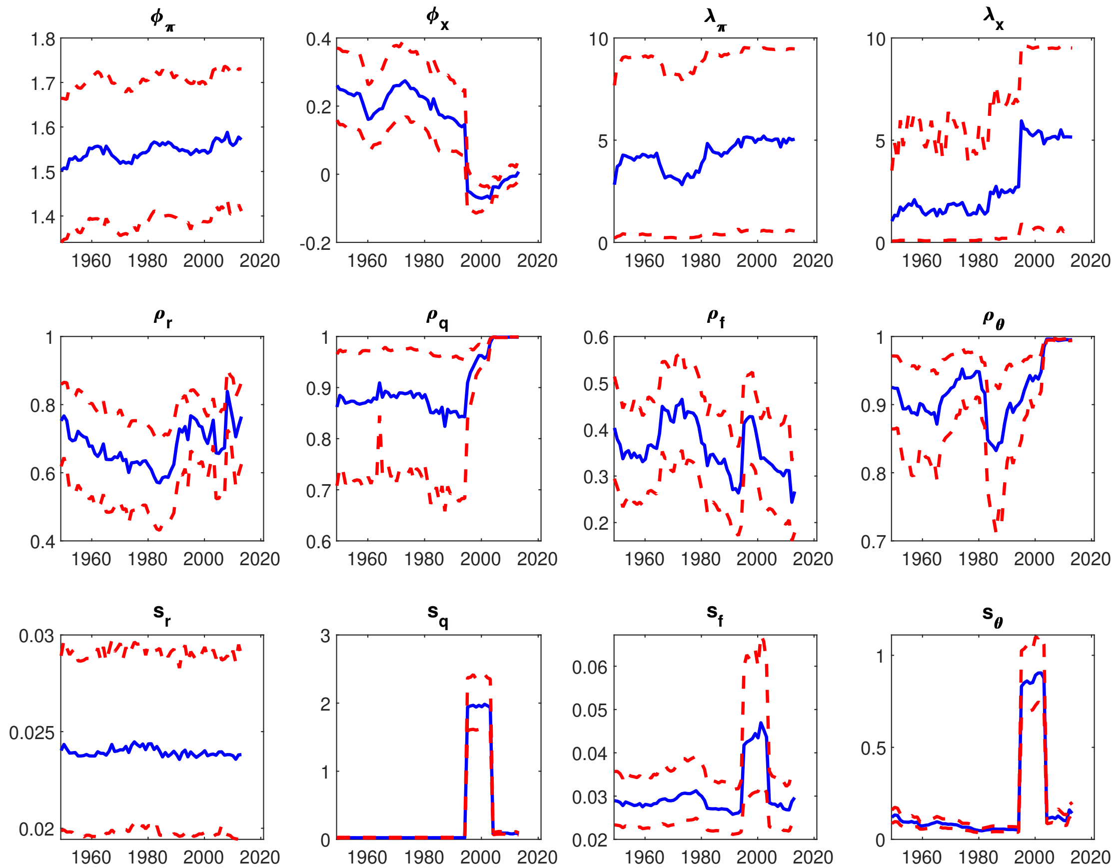

Figure A1: Rolling-window parameter estimates